Cooperative Retransmission in Heterogeneous Cellular Networks

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***Abstract*—This paper provides an analysis to compare the benefits of spatial and spatiotemporal cooperation between base stations in the presence of interference in a heterogeneous cellular network. The focus of the paper is the *cooperative retransmission* scenario, where a set of randomly located base stations that are selected based on their average received power levels, possibly belonging to different network tiers, jointly transmit data in each transmission. Using tools from stochastic geometry, an integral expression for the network coverage probability is derived in the scenario where the typical user receives one retransmission in case of failure to decode the message in the first transmission. An integral expression for the coverage probability is also derived for the case when the typical user is able to perform maximum ratio combining (MRC) of the received copies in two transmissions. Numerical evaluation illustrates that temporal transmission is often better than spatial cooperation in terms of backhaul overhead and coverage probability. It also shows that there are only small gains due to MRC compared to cooperative retransmission without MRC.**

I. Introduction

The problem of base station cooperation in wireless networks has recently gained more importance due to an increasing demand for data traffic over cellular networks [1]. One of the solutions to address this demand is the deployment of heterogeneous networks—networks of small base stations (BSs) along with the existing macro ones. However, deploying more BSs introduces larger intercell interference, which may offset the gain from smaller distances between the BSs and the user. Recently, heterogeneous networks have been studied by modeling different network tiers as a Poisson point process (PPP) to use the tools from stochastic geometry to characterize the outage/coverage probability of the network, e.g., [2]–[4]. Under this model, it has been shown that the coverage probability at the typical user with 0 dB threshold is just 56% if the user connects only to the nearest BS [5]. Therefore, the most recent discussions in the LTE cellular standard bodies center around the proposals of coordinated multipoint (CoMP) techniques [6], where BSs communicate with each other over a backhaul link to limit the intercell interference and exploit the benefits of distributed multiple antenna systems [7], hence increasing the network throughput.

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BSs can cooperate with each other either in space (spatial cooperation) or in space and time (spatiotemporal cooperation) using CoMP techniques. If only one BS transmits in each time slot, we call it temporal transmission. In spatial cooperation, BSs cooperate proactively to send the message to the user in a single transmission and hence, they always use extra resources due to cooperation for all of the users even if it is not needed. For example, cell-interior users may be able to decode the message when served by only one BS, while cell-edge users may always need cooperation [5]. Hence, BSs end up wasting resources. In spatiotemporal cooperation, BSs can adapt based on the response from the user in the first transmission and use extra resources during retransmission, i.e., BSs use extra resources only when it is needed. Hence, spatiotemporal cooperation serves all of the users fairly. Also, the user can employ MRC in case of retransmission to increase its coverage probability. In this paper, we focus on *cooperative retransmission* where we first transmit the message using a few (or just one) cooperating BSs and if the user is not able to decode the message, the message is retransmitted using more cooperating BSs.

Spatial cooperation has been studied for different CoMP techniques, see [5] and the references therein. In case of spatiotemporal cooperation, it is important to account for temporal correlation of the interference since the locations of the BSs do not change over different transmissions. The authors of [8]–[13] used tools from stochastic geometry to better understand the interference correlation in a single-tier wireless network. Similar tools were used in [14], [15] to study the benefits of cooperative relaying in a multi-user scenario. [16] studies the effect of interference correlation on the performance of MRC in a SIMO setting.

This paper presents a tractable stochastic geometry–based model for studying the interplay between spatial and spatiotemporal BS cooperation in the downlink of a K-tier heterogeneous network. While spatial BS cooperation may be necessary to send time-sensitive information to the receiver, spatiotemporal BS cooperation may use fewer BSs to serve the user and hence reduce the backhaul overhead in distributing the message to other BSs. Although this model can in principle be used to analyze arbitrary retransmission schemes, the paper focuses on the *cooperative retransmission* scenario. Fig. 1

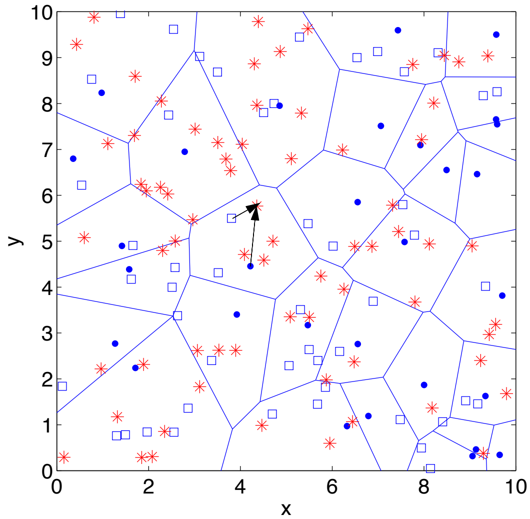

Fig. 1. Two-tier heterogeneous network with Voronoi cells of tier-1 where dots and squares denote the base stations from tier-1 and tier-2, respectively, and stars denote users, which are uniformly distributed. Here, the typical user connects to two base stations with strongest average received power.

shows one possible realization of a two-tier network where users are uniformly distributed and denoted by stars. Assuming that cooperating BSs do not have channel state information (CSI), and that a user connects to the set of cooperating BSs that results in the maximum average received power in each transmission, we derive closed integral-form expressions for the coverage probability in two different cases:

- *Case 1:* The receiver is not capable of performing MRC in the case of retransmission.
- *Case 2:* The receiver employs MRC in the case of retransmission.

In both the cases, the coverage probability is independent of the number of network tiers, network tier density, and available power (see Theorems 1 and 2). The results are used to quantify the benefits of cooperation and retransmission. Numerical evaluation illustrates that temporal transmission is often better than spatial cooperation in terms of backhaul overhead and coverage probability. It also shows that there are only small gains by employing MRC.

Throughout the paper, we denote by $\|\mathbf{u}\|_p$ the L^p-norm of a vector $\mathbf{u} = (u_1, u_2, \ldots, u_n) \in \mathbb{R}^n$, i.e., $\|\mathbf{u}\|_p = \left(\sum_{i=1}^{n} |u_i|^p\right)^{1/p}$, and we drop the subscript p in the special case $p = 2$ of Euclidean distance. The function $(z)^+$ equals z for $z > 0$ and zero otherwise.

II. System Model

A. Heterogeneous Network Model

We consider a heterogeneous wireless network composed of K independent network tiers of BSs with different deployment densities and transmit powers. It is assumed that the BSs belonging to the jth tier have transmit power P_j and are spatially distributed according to a two-dimensional homogeneous PPP Φ_j of density λ_j, $j = 1, \ldots, K$. We focus on the typical user, that is without loss of generality assumed to be located at the origin $(0, 0) \in \mathbb{R}^2$. A subset of the total ensemble of BSs cooperate by jointly transmitting a message to the typical user. If the user is not able to decode the message in the first transmission, we assume that the negative acknowledgement (NACK) is heard by all cooperating BSs and the message is retransmitted. We denote by $\mathcal{C}_i \subset \bigcup_{j=1}^{K} \Phi_j$ the set of the cooperating BSs in the i-th transmission with $|\mathcal{C}_i| = n_i$. In this paper, we only consider one retransmission, i.e., $i \in \{1, 2\}$, and we assume that the network operates in the interference-limited regime, i.e., the background thermal noise power is negligible compared to the total aggregate interference power. The received channel output at the typical user in the i-th transmission can be written as

$$\sum_{x \in \mathcal{C}_i} \frac{P_{\nu(x)}^{1/2}}{\|x\|^{\alpha/2}} h_x^{(i)} X + \sum_{x \in \mathcal{C}_i^c} \frac{P_{\nu(x)}^{1/2}}{\|x\|^{\alpha/2}} h_x^{(i)} X_x^{(i)}, \; i = 1, 2 \quad (1)$$

where $\nu(x)$ is the index of the network tier to which BS located at $x \in \mathbb{R}^2$ belongs, i.e., $\nu(x) = j$ iff $x \in \Phi_j$; $h_x^{(i)}$ denotes the random fading coefficient between the BS located at x and the user located at the origin; $\alpha > 2$ denotes the path loss exponent; X denotes the channel input symbol that is sent by the cooperating BSs in $\mathcal{C}_i$; $\mathcal{C}_i^c := \bigcup_{j=1}^{K} \Phi_j \setminus \mathcal{C}_i$ denotes the BSs that are not in the set of cooperating BSs during the i-th transmission; $X_x^{(i)}$ denotes the channel input symbol sent by the BS located at $x \in \mathcal{C}_i^c$. Throughout the paper it is assumed that the fading coefficients $h_x^{(i)}$ are i.i.d. $\sim \mathcal{CN}(0, 1)$ independent of everything else (Rayleigh fading) for each transmission, a legitimate assumption in a rich scattering environment.

Assuming that the $X_x^{(i)}$ and X in (1) are independent zero-mean random variables of unit variance, the resulting signal-to-interference-ratio (SIR) at the typical user during the i-th transmission due to (1) for a given realization of the PPPs and the fading coefficients is given by

$$\mathrm{SIR}_i = \frac{\left|\sum_{x \in \mathcal{C}_i} P_{\nu(x)}^{1/2} \|x\|^{-\alpha/2} h_x^{(i)}\right|^2}{\sum_{j=1}^{K} P_j I_j^{(i)}}, \; i = 1, 2 \quad (2)$$

where we defined

$$I_j^{(i)} := \sum_{x \in \Phi_j \setminus \mathcal{C}_i} |h_x^{(i)}|^2 \|x\|^{-\alpha} \quad (3)$$

as the aggregate interference power due to the non-cooperating BSs in tier j during i-th transmission. Notice that the interference terms in each transmission $I_j^{(i)}$ are correlated since the distances between the typical user and the interfering BSs do not change in the time frame of a transmission which occurs milliseconds apart from each other.

B. Set of cooperating BSs

The set of cooperating BSs in the i-th transmission $\mathcal{C}_i$ consists of the n_i BSs in $\bigcup_{j=1}^{K} \Phi_j$ with the *strongest* received power averaged over fading, as depicted in Fig. 1 and it is given as

$$\mathcal{C}_i = \arg \max_{(x_1,\ldots,x_{n_i})\subset\cup_{j=1}^K \Phi_j} \sum_{k=1}^{n_i} \frac{P_{\nu(x_k)}}{\|x_k\|^\alpha}. \tag{4}$$

Notice that the BSs in $\mathcal{C}_i$ belong in general to different network tiers. This setup is applicable to a heterogeneous wireless network where users keep a list of the neighboring BSs with the strongest received power to initiate handoff requests. If we assume $n_1 \le n_2$, $\mathcal{C}_1 \subseteq \mathcal{C}_2$ since the distances between the BSs and the typical user as well as the transmit powers of BSs do not change in different transmissions.

C. Definition of coverage probability

Throughout the paper, we focus on the coverage probability as the performance metric. Depending on whether the user employs MRC or not, we consider two cases.

1) Case 1: Retransmission without MRC: In this case, the coverage probability provided by the cooperative retransmission with n_1 cooperating BSs in the first transmission and n_2 cooperating BSs in the second transmission, P_{n_1,n_2} at a receiver located at the origin with coverage threshold θ is defined as

$$\begin{aligned}&\mathbb{P}(\mathrm{SIR}_1 > \theta) + \mathbb{P}(\mathrm{SIR}_2 > \theta \mid \mathrm{SIR}_1 < \theta)\cdot\mathbb{P}(\mathrm{SIR}_1 < \theta)\\ &\triangleq \mathsf{P}^{(1)}_{n_1}(\theta) + \mathsf{P}^{(2)}_{n_2}(\theta)\cdot(1-\mathsf{P}^{(1)}_{n_1}(\theta)),\end{aligned} \tag{5}$$

where $\mathsf{P}^{(1)}_{n_1}(\theta)$ denotes the probability that it is able to decode the message successfully due to n_1 cooperating BSs in the first transmission for threshold θ; $\mathsf{P}^{(2)}_{n_2}(\theta)$ denotes the probability that the typical user is able to decode the message successfully due to retransmission by n_2 cooperating BSs for the threshold θ given that the first transmission was unsuccessful.

2) Case 2: Retransmission with MRC: In this case, the receiver is able to perform MRC of two received copies of the message in two transmissions. Similar to the analysis in [16], we can get the combined SIR due to MRC as $\mathrm{SIR}_1 + \mathrm{SIR}_2$. Hence for a given threshold θ, we can define the coverage probability $\mathsf{P}^{\mathrm{MRC}}_{n_1,n_2}$ as

$$\begin{aligned}&\mathbb{P}(\mathrm{SIR}_1 > \theta) + \mathbb{P}(\mathrm{SIR}_1 + \mathrm{SIR}_2 > \theta \mid \mathrm{SIR}_1 < \theta)\cdot\mathbb{P}(\mathrm{SIR}_1 < \theta)\\ &= \mathbb{P}(\mathrm{SIR}_1 > \theta) + \mathbb{P}(\mathrm{SIR}_1 + \mathrm{SIR}_2 > \theta,\ \mathrm{SIR}_1 < \theta).\end{aligned} \tag{6}$$

This method is called *chase combining*; it is one of the methods of soft combining in hybrid automatic repeat request (HARQ).

III. Coverage Probabilities

In this section, we first derive a computable expression for the coverage probability (5) at the typical user for the case when the user does not employ MRC in Theorem 1. Second, we derive the coverage probability for the case when the user is capable of performing MRC in Theorem 2.

A. Case 1: Retransmission without MRC

We prove the following result for the case when receiver does not employ MRC for two received copies of the message.

Theorem 1: Let the set $\mathcal{C}_i$ be defined as in (4). Then, the coverage probability P_{n_1,n_2} in (5) is

$$\mathsf{P}_{n_1,n_2} = g_1(n_1,\theta) + g_1(n_2,\theta) - g_2(n_1,n_2,\theta), \tag{7}$$

where $g_1(n,\theta)$ is given by

$$\int\limits_{\substack{0<u_1<\ldots\\ \ldots<u_n<\infty}} \exp\left(-u_n\left(1+2\frac{F(\|\tilde{\mathbf{u}}_{\mathbf{n},\mathbf{n}}\|^{1/2}_{\alpha/2}\,\theta^{-1/\alpha})}{\|\tilde{\mathbf{u}}_{\mathbf{n},\mathbf{n}}\|_{\alpha/2}\,\theta^{-2/\alpha}}\right)\right)\mathrm{d}\mathbf{u};$$

with $\tilde{\mathbf{u}}_{\mathbf{m},\mathbf{n}} := \left(\frac{u_n}{u_1}, \frac{u_n}{u_2}\ldots, \frac{u_n}{u_m}\right)$ and $F(x) := \int_x^\infty \frac{r}{1+r^\alpha}\mathrm{d}r$. $g_2(n_1,n_2,\theta)$ for $n_1 \le n_2$ is given by

$$\int\limits_{\substack{0<u_1<\ldots\\ \ldots<u_{n_2}<\infty}} \frac{\exp\left(-2u_{n_2}G\left(\frac{\theta}{\|\tilde{\mathbf{u}}_{\mathbf{n_1},\mathbf{n_2}}\|^{\alpha/2}_{\alpha/2}}, \frac{\theta}{\|\tilde{\mathbf{u}}_{\mathbf{n_2},\mathbf{n_2}}\|^{\alpha/2}_{\alpha/2}}\right)\right)}{e^{u_{n_2}}\prod_{i=n_1+1}^{n_2}\left(1+\theta\|\tilde{\mathbf{u}}_{\mathbf{n_1},\mathbf{i}}\|^{-\alpha/2}_{\alpha/2}\right)}\mathrm{d}\mathbf{u},$$

where $G(x,y) := \int_1^\infty \left(1-\frac{1}{(1+xr^{-\alpha})\cdot(1+yr^{-\alpha})}\right) r\ \mathrm{d}r$ which can be expressed in terms of $F(x)$ as

$$G(x,y) = \begin{cases} \frac{x^{1+2/\alpha}F(x^{-1/\alpha}) - y^{1+2/\alpha}F(y^{-1/\alpha})}{x-y}., & x \ne y;\\ (1+2/\alpha)x^{2/\alpha}F(x^{-1/\alpha}) + \frac{x}{\alpha(1+x)}, & x = y.\end{cases} \tag{8}$$

Proof: See Appendix A. ■

The result in Theorem 1 is not limited to the case $n_1 \le n_2$. The coverage probability for $n_1 > n_2$ can be obtained by interchanging n_1 and n_2 in the above expression. The result in Theorem 1 only depends on the number of cooperating BSs n_1 and n_2, the threshold θ, and the path loss exponent α. Hence, we can draw similar conclusions on the fact that Equation (7) is independent of the number of network tiers K, and their respective power levels and deployment densities as in [5, Theorem 1].

Notice that the expression of coverage probability in Theorem 1 is a consequence of the inclusion-exclusion formula in set theory applied to $\mathsf{P}_{n_1,n_2} = \mathbb{P}(\bigcup_{i=1}^2\{\mathrm{SIR}_i > \theta\})$ with $g_1(n_i,\theta) = \mathbb{P}(\mathrm{SIR}_i > \theta)$ and $g_2(n_1,n_2,\theta) = \mathbb{P}(\mathrm{SIR}_1 > \theta,\ \mathrm{SIR}_2 > \theta)$.

It should also be remarked that $F(x)$ in (7) and (8) can not be expressed in closed form in general. However, closed-form expressions exist for specific values of $\alpha > 2$. For example, it can be easily verified that if $\alpha = 3$, then

$$F(x) = \frac{1}{6}\log\left(1+\frac{3x}{1-x+x^2}\right) + \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{3}}{2x-1}\right),$$

while for $\alpha = 4$, $F(x) = \frac{1}{2}\tan^{-1}(x^{-2})$. Also, making use of Theorem 1, it is possible to derive an expression for the coverage probability in the case of no cooperation.

Corollary 1: In the special case $n_1 = n_2 = 1$, i.e., when the typical user connects to a single BS and in case of failure in the first transmission, the message is retransmitted by only that BS, the coverage probability simplifies to

$$\mathsf{P}_{1,1} = \frac{2}{1+2\theta^{2/\alpha}F(\theta^{-1/\alpha})} - \frac{1}{1+2G(\theta,\theta)}. \tag{9}$$

In the special case $\alpha = 4$, (9) admits the closed form expression

$$\frac{2}{1+\sqrt{\theta}\tan^{-1}(\sqrt{\theta})} - \frac{1}{1+\frac{3}{2}\sqrt{\theta}\tan^{-1}(\sqrt{\theta})+\frac{\theta}{2(1+\theta)}}.$$

Thus far, we have assumed the same SIR threshold for both transmisions. But, it may be sensible to use different thresholds in each transmission. For example, in speech compression, we may send coarser content encoded at lower rate in the retransmission to increase the reliability. If we consider the thresholds θ_1 for the first transmission and θ_2 for the second transmission, the coverage probability P_{n_1,n_2} for Case 1 can be generalized to

$$\begin{aligned}&\mathbb{P}(\mathrm{SIR}_1 > \theta_1) + \mathbb{P}(\mathrm{SIR}_2 > \theta_2 \mid \mathrm{SIR}_1 < \theta_1)\cdot\mathbb{P}(\mathrm{SIR}_1 < \theta_1)\\ &= \mathsf{P}^{(1)}_{n_1}(\theta_1) + \mathsf{P}^{(2)}_{n_2}(\theta_1,\theta_2)\cdot(1-\mathsf{P}^{(1)}_{n_1}(\theta_1)).\end{aligned} \tag{10}$$

We prove the following result in Proposition 1.

Proposition 1: For different coverage thresholds θ_1 and θ_2 for the first and the second transmission respectively, the coverage probability P_{n_1,n_2} in (10) is

$$g_1(n_1,\theta_1) + g_1(n_2,\theta_2) - g_4(n_1,n_2,\theta_1,\theta_2), \tag{11}$$

where $g_4(n_1,n_2,\theta_1,\theta_2)$ for $n_1 \le n_2$ is given by

$$\int\limits_{\substack{0<u_1<\dots\\ \dots<u_{n_2}<\infty}} \frac{\exp\left(-2u_{n_2}G\left(\frac{\theta_1}{\|\tilde{\mathbf{u}}_{\mathbf{n_1},\mathbf{n_2}}\|^{\alpha/2}_{\alpha/2}},\ \frac{\theta_2}{\|\tilde{\mathbf{u}}_{\mathbf{n_2},\mathbf{n_2}}\|^{\alpha/2}_{\alpha/2}}\right)\right)}{e^{u_{n_2}}\prod_{i=n_1+1}^{n_2}\left(1+\theta_1\|\tilde{\mathbf{u}}_{\mathbf{n_1},\mathbf{i}}\|^{-\alpha/2}_{\alpha/2}\right)}\,\mathrm{d}\mathbf{u},$$

The coverage probability for $n_1 > n_2$ can be obtained by interchanging n_1, n_2, and θ_1, θ_2. Making use of Proposition 1, we can also derive the coverage probability without retransmission by letting $\theta_1 \to \infty$ and substituting $n_2 = n$. This way, we recover the result in [5, Theorem 1]:

In the special case when the n BSs cooperate to send the message to the typical receiver and there is no retransmission, the coverage probability simplifies to

$$\int\limits_{\substack{0<u_1<\dots\\ \dots<u_n<\infty}} \exp\left(-u_n\left(1+2\frac{F(\|\tilde{\mathbf{u}}\|^{1/2}_{\alpha/2}\,\theta^{-1/\alpha})}{\|\tilde{\mathbf{u}}\|_{\alpha/2}\,\theta^{-2/\alpha}}\right)\right)\,\mathrm{d}\mathbf{u}. \tag{12}$$

B. Case 2: Retransmission with MRC

The following theorem addresses the case when the typical user is able to perform MRC on the two received copies of the desired message.

Theorem 2: Let the set $\mathcal{C}_i$ be defined as in (4). Then, the coverage probability $\mathsf{P}^{\mathsf{MRC}}_{n_1,n_2}$ with $n_1 \le n_2$ in (6) is

$$g_1(n_1,\theta) + \int\limits_0^\infty \left[g_3(n_1,n_2,z,(\theta-z)^+) - g_3(n_1,n_2,z,\theta)\right]\mathrm{d}z, \tag{13}$$

where $g_3(n_1,n_2,z,a)$ is given by

$$\int\limits_{\substack{0<u_1<\dots\\ \dots<u_{n_2}<\infty}} \frac{2u_{n_2}e^{-u_{n_2}}H\left(\frac{z}{\|\tilde{\mathbf{u}}_{\mathbf{n_2},\mathbf{n_2}}\|^{\alpha/2}_{\alpha/2}},\frac{a}{\|\tilde{\mathbf{u}}_{\mathbf{n_1},\mathbf{n_2}}\|^{\alpha/2}_{\alpha/2}}\right)}{\|\tilde{\mathbf{u}}_{\mathbf{n_2},\mathbf{n_2}}\|^{\alpha/2}_{\alpha/2}\prod_{i=n_1+1}^{n_2}\left(1+\frac{a}{\|\tilde{\mathbf{u}}_{\mathbf{n_1},\mathbf{i}}\|^{\alpha/2}_{\alpha/2}}\right)} \times\exp\left(-2u_{n_2}G\left(\frac{z}{\|\tilde{\mathbf{u}}_{\mathbf{n_2},\mathbf{n_2}}\|^{\alpha/2}_{\alpha/2}},\frac{a}{\|\tilde{\mathbf{u}}_{\mathbf{n_1},\mathbf{n_2}}\|^{\alpha/2}_{\alpha/2}}\right)\right)\mathrm{d}\mathbf{u}$$

with $H(x,y) := \frac{\partial}{\partial x}G(x,y) = \int_1^\infty \frac{r^{1-\alpha}}{(1+xr^{-\alpha})^2\cdot(1+yr^{-\alpha})}\mathrm{d}r$.

Proof: Due to the limited space, we only provide an outline of the proof here. The first term in (6) equals $g_1(n_1,\theta)$ from Theorem 1. To compute the second term in (6), we can first condition on SIR_2 and $\bigcup_{j=1}^K \Phi_j$. Following similar steps as in the proof of Theorem 1 in Appendix A, we obtain the coverage probability given SIR_2 and $\bigcup_{j=1}^K \Phi_j$. Then, we can calculate the probability distribution function (pdf) of SIR_2 given $\bigcup_{j=1}^K \Phi_j$ and take the expectation with respect to SIR_2. Finally, we take the expectation over the PPPs. ■

Again, similar to the result in Theorem 1, the coverage probability is independent of the number of network tiers K and their respective power levels and deployment densities.

Making use of Theorem 2, it is possible to derive the expression of coverage probability for different cases of n_1 and n_2.

Corollary 2: In the special case when $n_1 = n_2 = n$, i.e., the number of cooperating BSs does not change between two transmissions, the coverage probability with MRC simplifies to

$$\mathsf{P}^{\mathsf{MRC}}_{n,n} = \int_0^\infty g_3(n,n,z,(\theta-z)^+)\mathrm{d}z. \tag{14}$$

Using the fact that $H(x,y) = \frac{\partial}{\partial x}G(x,y)$ gives us $g_1(n,\theta) = \int_0^\infty g_3(n,n,z,\theta)\mathrm{d}z$ and reduces (13) into (14).

Corollary 3: In the special case when $n_1 = n_2 = 1$, i.e., there is no cooperation between BSs during the two transmissions, the coverage probability at the typical user with MRC, $\mathsf{P}^{\mathsf{MRC}}_{1,1}$ simplifies to

$$\frac{1}{1+2\theta^{2/\alpha}F(\theta^{-1/\alpha})} + \int_0^\theta \frac{2H(z,\theta-z)}{(1+2G(z,\theta-z))^2}\mathrm{d}z. \tag{15}$$

In comparison to the case when there is no retransmission in [5, Corollary 1], we can see that there is a gain in coverage probability due to retransmission, given by the integral in the above expression.

C. Numerical Evaluation

Here, we present numerical evaluations of the integral expressions for the coverage probability derived in this paper. We focus on the case of $n_1 = n_2$ with $\alpha = 4$. Fig. 2 illustrates the effect of threshold θ on the coverage probability and compares the coverage probabilities for Case 1 and 2 with the case without retransmission as described in Corollary III-A for $n_1 = n_2 = 1$ and 2. Using this figure, we can compare the case when two BSs cooperate in the first transmission and there is no retransmission (spatial cooperation) with the case when one BS transmits the message and retransmits it again in case of failure in the first transmission (temporal transmission). In the first case, two resource blocks are used while in the second case, the expected number of used resource blocks is $1 + 1 \cdot \mathbb{P}(\mathrm{SIR}_1 < \theta) = 2 - \frac{1}{1+\sqrt{\theta}\tan^{-1}(\sqrt{\theta})}$, which is less than two resource blocks. Also, the second case with MRC provides a higher coverage probability than the first case upto the threshold of 5 dB and both the coverage probabilities are comparable thereafter. Hence, temporal transmission can provide a higher coverage probability than spatial cooperation while also using fewer resource blocks on average and eliminating the backhaul overhead in distributing the message to the other BSs. Also, notice that the slope of the curve for no retransmission with $n = 2$ cooperating BSs is less than the slope of the curves for Case 1 and Case 2 with $n_1 = n_2 = 1$, which suggests that we get diversity gain due to retransmission similar to the result in [13, Proposition 3]. Therefore, temporal transmission is often better than spatial cooperation. This figure also shows that Case 2 provides a relative gain of 9% for $n_1 = n_2 = 1$ and 5% for $n_1 = n_2 = 2$ compared to Case 1 at threshold of 0 dB. It means that we do not gain much by employing MRC.

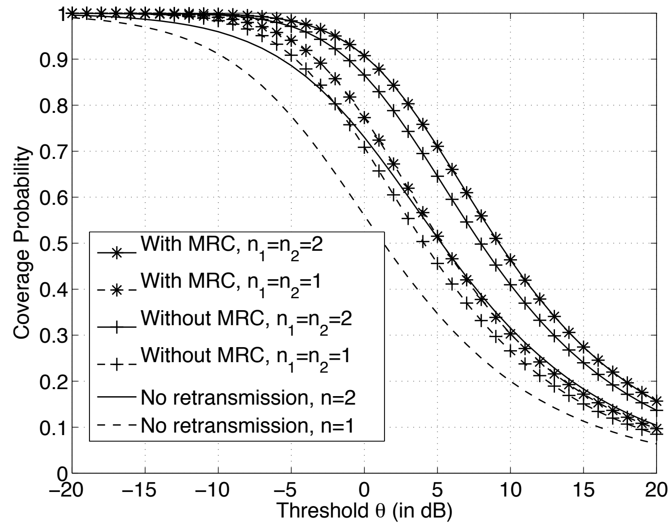

Fig. 2. Coverage probabilities with and without retransmission using (7), (9), (14), (15) and (12) with $\alpha = 4$.

IV. Conclusion

In this paper, we considered the problem of cooperative retransmission in heterogeneous wireless networks. We derived an integral expression for the coverage probability in two cases based on whether the receiver can employ MRC or not. The analysis presented in this paper can be used to compare the benefits of spatial and spatiotemporal cooperation and numerical results show that temporal transmission is often better than spatial cooperation in terms of average number of used resource blocks, backhaul overhead and coverage probability.

Appendix A
Proof of Theorem 1 and Proposition 1

For every $i = 1, \ldots, K$, let $\Xi_i = \{\|x\|^\alpha / P_i, x \in \Phi_i\}$ denote the normalized path loss between each BS in Φ_i and the typical user located at the origin. By the mapping theorem [17, Theorem 2.34], Ξ_i is a PPP with intensity $\lambda_i(x) = \lambda_i \frac{2\pi}{\alpha} P_i^{2/\alpha} x^{2/\alpha - 1}$, $x \in \mathbb{R}^+$. From the independence of the PPPs $\Phi_1, \cdots, \Phi_K$, it follows that $\Xi_1, \cdots, \Xi_K$ are also independent and thus the process $\Xi = \bigcup_{i=1}^K \Xi_i$ is a non-homogeneous PPP with density $\lambda(x) = \sum_{i=1}^K \lambda_i(x)$. Without loss of generality, suppose that the elements of Ξ are indexed in increasing order, such that $\|x_1\|^\alpha / P_{\nu(x_1)} \leq \|x_2\|^\alpha / P_{\nu(x_2)} \leq \|x_3\|^\alpha / P_{\nu(x_3)} \leq \cdots$, and define $\gamma_k = \|x_k\|^\alpha / P_{\nu(x_k)}$ as the normalized path loss between the typical user and the k-th BS in the ordered list.

The expression for $\mathsf{P}_{n_1}^{(1)}(\theta_1)$ in (11) has been proved in [5, Theorem 1] as $g_1(n_1, \theta_1)$. Assuming $n_1 \leq n_2$, the normalized path loss of the cooperating BSs in $\mathcal{C}_2$ is given by $\boldsymbol{\gamma} = \{\gamma_1, \ldots, \gamma_{n_2}\}$. Then, by defining $g_k^{(i)} := |h_{x_k}^{(i)}|^2$ for $i = 1, 2$, $\mathbf{g} = \left(\mathbf{g^{(1)}}, \mathbf{g^{(2)}}\right)$, interference in the i-th transmission as $I^{(i)} = \sum_{k > n_i} g_k^{(i)} \gamma_k^{-1}$, $\mathsf{P}_{n_2}^{(2)}(\theta_1, \theta_2)$ can be written as:

$$\mathsf{P}_{n_2}^{(2)} = \mathbb{P}\Big(\mathrm{SIR}_2 > \theta_2 \mid \mathrm{SIR}_1 < \theta_1\Big) = \frac{\mathbb{P}\Big(S_2 > \theta_2 I^{(2)},\, S_1 < \theta_1 I^{(1)}\Big)}{1 - \mathsf{P}_{n_1}^{(1)}(\theta_1)} \tag{16}$$

where we define $S_i = \left|\sum_{k \leq n_i} \gamma_k^{-1/2} h_k^{(i)}\right|^2$. Using the fact that $h_k^{(1)}$ and $h_k^{(2)}$ are mutually independent and the fact that S_i is exponentially distributed with mean $\sum_{k=1}^{n_i} \gamma_k^{-1}$ because of the Rayleigh fading assumption, the numerator in the above expression can be expressed as

$$\mathsf{E}_{\boldsymbol{\gamma}, \Xi, \mathbf{g}}\left[\exp\left(-\frac{\theta_2 I^{(2)}}{\sum_{k \leq n_2} \gamma_k^{-1}}\right) \cdot \left(1 - \exp\left(-\frac{\theta_1 I^{(1)}}{\sum_{k \leq n_1} \gamma_k^{-1}}\right)\right)\right]$$
$$= \mathsf{E}_{\boldsymbol{\gamma}, \Xi, \mathbf{g}}\left[\exp\left(-\frac{\theta_2 I^{(2)}}{\sum_{k \leq n_2} \gamma_k^{-1}}\right)\right]$$

$$- \mathsf{E}_{\boldsymbol{\gamma},\Xi,\mathbf{g}}\left[\exp\left(-\frac{\theta_2 I^{(2)}}{\sum_{k\leq n_2}\gamma_k^{-1}}-\frac{\theta_1 I^{(1)}}{\sum_{k\leq n_1}\gamma_k^{-1}}\right)\right]. \quad (17)$$

The first term in the above expression equals $g_1(n_2,\theta_2)$ defined in (7) as proved in [5, Theorem 1]. The second term can be expressed as

$$\mathsf{E}_{\boldsymbol{\gamma}}\,\mathsf{E}_{\Xi,\mathbf{g}}\left[e^{-\frac{\theta_2\sum_{k>n_2} g_k^{(2)}\gamma_k^{-1}}{\sum_{k\leq n_2}\gamma_k^{-1}}-\frac{\theta_1\sum_{k>n_1} g_k^{(1)}\gamma_k^{-1}}{\sum_{k\leq n_1}\gamma_k^{-1}}}\;\Big|\;\gamma_1,\ldots,\gamma_{n_2}\right]$$
$$=\int\limits_{\substack{0<\gamma_1<\ldots\\ \ldots<\gamma_{n_2}<\infty}}\mathsf{E}_{\Xi,\mathbf{g}}\left[e^{-\frac{\theta_2\sum_{k>n_2} g_k^{(2)}\gamma_k^{-1}}{\sum_{k\leq n_2}\gamma_k^{-1}}-\frac{\theta_1\sum_{k>n_1} g_k^{(1)}\gamma_k^{-1}}{\sum_{k\leq n_1}\gamma_k^{-1}}}\;\Big|\;\boldsymbol{\gamma}\right]\times$$
$$f_{\boldsymbol{\Gamma}}(\boldsymbol{\gamma})\,\mathrm{d}\boldsymbol{\gamma}, \quad (18)$$

where $f_{\boldsymbol{\Gamma}}(\boldsymbol{\gamma})$ is the joint distribution of $\boldsymbol{\gamma}$ which can be obtained by following the similar steps as in the derivation of the joint distribution of the nearest points in a homogeneous PPP [18]. It can be easily verified that for any $0<\gamma_1<\ldots<\gamma_{n_2}<\infty$, the joint distribution of $\boldsymbol{\gamma}$ is given by

$$f_{\boldsymbol{\Gamma}}(\boldsymbol{\gamma})=e^{-\pi\sum_{i=1}^{K}\lambda_i P_i^{\delta}\gamma_{n_2}^{\delta}}\prod_{i=1}^{n_2}\left(\sum_{j=1}^{K}\pi\lambda_j\delta P_j^{\delta}\gamma_i^{\delta-1}\right) \quad (19)$$

with $\delta=2/\alpha$. Given $\boldsymbol{\gamma}$, the expected value inside the integral in (18) can be expressed as

$$\mathsf{E}_{\Xi,\mathbf{g}}\left[\exp\left(-\frac{\theta_1\sum_{k=n_1+1}^{n_2} g_k^{(1)}\gamma_k^{-1}}{\sum_{k\leq n_1}\gamma_k^{-1}}\right)\times\right.$$
$$\left.\exp\left(-\sum_{j>n_2}\left(\frac{g_j^{(2)}\theta_2}{\sum_{k\leq n_2}\gamma_k^{-1}}+\frac{g_j^{(1)}\theta_1}{\sum_{k\leq n_1}\gamma_k^{-1}}\right)\gamma_j^{-1}\right)\right]$$
$$\overset{(a)}{=}\prod_{i=n_1+1}^{n_2}\left(1+\frac{\theta_1\gamma_i^{-1}}{\sum_{k\leq n_1}\gamma_k^{-1}}\right)^{-1}\times$$
$$\mathsf{E}_{\Xi}\left[\prod_{j>n_2}\left(1+\frac{\theta_2\gamma_j^{-1}}{\sum_{k\leq n_2}\gamma_k^{-1}}\right)^{-1}\cdot\left(1+\frac{\theta_1\gamma_j^{-1}}{\sum_{k\leq n_1}\gamma_k^{-1}}\right)^{-1}\right]$$
$$\overset{(b)}{=}\prod_{i=n_1+1}^{n_2}\left(1+\frac{\theta_1\gamma_i^{-1}}{\sum_{k\leq n_1}\gamma_k^{-1}}\right)^{-1}\times$$
$$e^{-\int_{\gamma_{n_2}}^{\infty}\left(1-\left(1+\frac{\theta_2 x^{-1}}{\sum_{k\leq n_2}\gamma_k^{-1}}\right)^{-1}\cdot\left(1+\frac{\theta_1 x^{-1}}{\sum_{k\leq n_1}\gamma_k^{-1}}\right)^{-1}\right)\lambda(x)\mathrm{d}x}$$
$$\overset{(c)}{=}\prod_{i=n_1+1}^{n_2}\left(1+\frac{\theta_1\gamma_i^{-1}}{\sum_{k\leq n_1}\gamma_k^{-1}}\right)^{-1}\times$$
$$\exp\left(-2\pi\sum_{i=1}^{K}\lambda_i P_i^{2/\alpha}\gamma_{n_2}^{2/\alpha}G\left(\frac{\theta_1\gamma_{n_2}^{-1}}{\sum_{k=1}^{n_1}\gamma_k^{-1}},\frac{\theta_2\gamma_{n_2}^{-1}}{\sum_{k=1}^{n_2}\gamma_k^{-1}}\right)\right), \quad (20)$$

where (a) uses the fact that $g_k^{(1)}$ and $g_k^{(2)}$ are mutually independent and exponentially distributed with unit mean; (b) is due to the probability generating functional for a PPP [17, Theorem 4.9]; (c) follows from the transformation $x=\gamma_{n_2}t^{\alpha}$ and the definition of $G(x,y)$ in (7). Substituting (20) in (18) and using the transformation $u_i=\pi\sum_{j=1}^{K}\lambda_j P_j^{2/\alpha}\gamma_i^{2/\alpha}$ gives us the second term in (17) as $g_4(n_1,n_2,\theta_1,\theta_2)$ defined in (11), and we already know the value of the first term in (17). Substituting (17) in (16), we have the value of $\mathsf{P}_n^{(2)}$ and hence we get the desired result in (11). Substituting $\theta_1=\theta_2=\theta$ gives us the result in Theorem 1.

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