A Throughput-Optimum Adaptive ALOHA MAC Scheme for Full-Duplex Wireless Networks

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***Abstract*—This paper proposes an adaptive and distributed MAC scheme for wireless networks with full-duplex radios. Full-duplex (FD) radios can exchange data simultaneously using the same frequency band and potentially double the throughput. On the other hand, it will inevitably cause extra interference to the network to use FD transmission instead of half-duplex (HD) transmission. Hence, there is a tradeoff between interference and throughput for wireless networks with FD radios. In order to mitigate the interference and maximize the network throughput, we propose the Optimum Adaptive ALOHA (OA-ALOHA) MAC scheme based on the nodes' local information about their neighbors. We derive each node's optimal transmit probability that maximizes the throughput and achieves proportional fairness. Our numerical results show that if the network is using the proposed adaptive MAC scheme, significantly better network throughput can be achieved than with the non-adaptive MAC schemes. HD can achieve around 50% maximum throughput gain using adaptive ALOHA while FD more than 70%. Moreover, a Simplified Adaptive ALOHA (SA-ALOHA) MAC scheme is also included that is based on an approximation of the optimum transmit probability and achieves comparable performance.**

I. Introduction

FD communication has attracted a lot of attention in the last few years due to the fact that it can potentially double the throughput if the self-interference can be well mitigated. FD radios have been successfully prototyped in laboratory environments by different research groups [1]–[4]. The key elements to the success are novel analog and digital self-interference cancellation techniques and/or spatially separated transmit and receive antennas as proposed in the literature. The basic idea of FD transmission is to let the receive chain of a node remove the self-interference caused by the known signal from its transmit chain, so that reception can be concurrent with transmission.

Many works have theoretically studied the throughput gain using information-theoretical approaches such as [5], [6]. The throughput gain has been illustrated via extensive simulation for a cellular system with FD base station and HD mobile users in [7]. The throughput has been analyzed using stochastic geometry for wireless networks with FD radios in [8]. A novel idea of hybrid-duplex cell networks for future heterogeneous cellular networks has been presented in [9]. [10] and [11] are focused on the throughput gain of FD over HD in CSMA-based wireless networks. In particular, the Martern hard-core process has been extended to study the the throughput performance of CSMA-based wireless networks with FD radios in [11]. In [12], the throughput for FD wireless network with imperfect self-interference cancellation has been investigated. The analytical result shows that if the network is adopting an ALOHA protocol, the maximal throughput is achieved by scheduling all concurrently transmitting nodes to work in either FD mode or HD mode depending on one simple condition. Moreover, the impact of imperfect self-interference cancellation on the throughput gain, transmission range, and other metrics has been quantified.

The claim that FD can double the throughput may be achievable for point-to-point communication when the self-interference is perfectly cancelled. However, when FD radios are used in a wireless network, additional interference will be introduced. Hence, there is a strong need for a MAC protocol tailored for a wireless network of radios with both FD and HD capabilities and an intelligent, distributed and adaptive scheme to switch between FD and HD operation based on different network configurations so as to better manage the interference. An adaptive ALOHA MAC scheme for HD networks has been studied in [13], [14]. The analysis of a proportionally fair and locally adaptive spatial ALOHA MAC scheme is presented in [13] while the adaptive ALOHA scheme is analyzed with full information about the network topology in [14].

In this paper, we propose an adaptive MAC scheme that achieves the optimum throughput performance given local information for wireless networks with FD radios. A simplified approximation of the proposed MAC scheme is also presented to make it easy to implement the adaptive FD MAC scheme. The MAC scheme proposed is distributed and only depends on the node's local spatial information such as the distances to its neighbors within some range. The throughput performance is compared with the non-adaptive MAC schemes for both HD and FD networks and the adaptive HD MAC scheme in [13] and shows significant improvement overall.

II. Network Model

Consider a Poisson bipolar network [15] that consists of a Poisson point process (PPP) of nodes $\{x_i\} \subset \mathbb{R}^2$ with density λ and another set of nodes $\{y_i\}$ where y_i is at distance R from x_i in a uniformly random direction. In other words, the network consists of links, each having two nodes x_i and y_i communicating with each other, and $\|x_i - y_i\| = R$ for all i. The point process $\{y_i\}$ is also a PPP. Thus, a Poisson bipolar network consists of two dependent PPPs, denoted as

$\Phi_1 = \{x_i\}$ and $\Phi_2 = \{y_i\}$. Let $\hat{\Phi} = \Phi_1 \cup \Phi_2$. All nodes in $\hat{\Phi}$ are assumed to have FD capability. Define the "partner function" $m(x)$ for $x \in \hat{\Phi}$ such that $m(x)$ is the unique point $y \in \hat{\Phi}$ s.t. $\|x - m(x)\| = R$. As a result, $x \in \Phi_1 \Rightarrow m(x) \in \Phi_2$ and $x \in \Phi_2 \Rightarrow m(x) \in \Phi_1$. Also, $m(m(x)) \equiv x$. From the definition of the partner function, $y_i = m(x_i)$ and $x_i = m(y_i)$. We consider a network that consists of a mixture of HD and FD transmissions. Therefore, each link in the Poisson bipolar network has three states: silence, HD and FD. Silence means that a link is inactive and hence there is no transmission between two nodes in the link. HD means that either x transmits messages to $m(x)$ or vice versa. FD means that x and $m(x)$ transmit to each other at the same time. Assume that each node x chooses to transmit independently with probability p_x. Then the link is HD with probability $p_x \left(1 - p_{m(x)}\right) + p_{m(x)} \left(1 - p_x\right)$, FD with probability $p_x p_{m(x)}$ and silent with probability $\left(1 - p_x\right)\left(1 - p_{m(x)}\right)$.

Consider the SIR model where a transmission attempt from x to y is considered successful if

$$\mathrm{SIR}_y = \frac{S_y}{I_y} > \theta,$$

where

$$S_y = P_x h_{xy} l(x, y) = P_x h_{xy} \|x - y\|^{-\alpha},$$

$$I_y = \sum_{z \in \Phi \setminus \{x\}} P_z h_{yz} l(z, y) = \sum_{z \in \Phi \setminus \{x\}} P_z h_{zy} \|z - y\|^{-\alpha},$$

$\Phi \subset \hat{\Phi}$ is the set of transmitting nodes in a given time slot, P_x is the transmit power at node $x \in \Phi$, θ is the SIR threshold, and h_{xy} and h_{zy} are the fading power coefficients with mean 1 from the desired transmitter x and the interferer z to y, respectively. We focus on the Rayleigh fading case for both the desired link and interferers. The path loss function $l(x, y) = \|x - y\|^{-\alpha}$, where $\alpha > 2$ is the path-loss exponent. We assume that the transmit power $P_x \equiv 1$ for all $x \in \Phi$. Also, the self-interference in the FD links is assumed to be cancelled perfectly.

III. Link Throughput in Adaptive ALOHA

In our network model, the throughput of node x is $p_{m(x)} q_x$ while the throughput of its partner node $m(x)$ is $p_x q_{m(x)}$, where q_x is the conditional success probability at x given that its partner node $m(x)$ is transmitting and given the point process $\hat{\Phi}$.

Definition 1. *In a wireless network described by $\hat{\Phi}$, the link throughput consisting of nodes x and $m(x)$ is defined as $p_x q_{m(x)} + p_{m(x)} q_x$.*

Instead of assuming a network-wide transmit probability p as in conventional ALOHA, adaptive ALOHA allows each node to transmit with different probabilities. There are two factors in the throughput expression, the success probability and the transmit probability. In the following, we discuss the two factors.

A. Success Probability

The success probability at node x is defined as $q_x = \mathbb{P}\left(\mathrm{SIR}_x \geq \theta \mid \hat{\Phi}\right)$ at node x given that $m(x)$ is transmitting.

Lemma 1. *Given a wireless network described by $\hat{\Phi}$, the conditional success probability q_x is*

$$q_x = \prod_{y \in \hat{\Phi} \setminus \{x, m(x)\}} \left(1 - \frac{p_y}{1 + b_{yx}}\right), \tag{1}$$

where $b_{yx} = \frac{\|y - x\|^\alpha}{\theta R^\alpha}$.

Proof: Assume that e_x is the indicator function that takes value one if node $x \in \hat{\Phi}$ chooses to transmit and zero otherwise, i.e., $p_x = \mathbb{P}\left(e_x = 1\right) = \mathbb{P}\left(x \text{ transmits}\right)$.

Letting $H_x = \left\{(h_{yx}, e_y), y \in \hat{\Phi} \setminus \{x, m(x)\}\right\}$, we have

$$\begin{aligned}
&\mathbb{P}(\mathrm{SIR}_x \geq \theta \mid H_x, \hat{\Phi}) \\
&= \mathbb{P}\left\{ h_{m(x)x} \geq \theta R^\alpha \left(\sum_{y \in \hat{\Phi} \setminus \{x, m(x)\}} \|y - x\|^{-\alpha} h_{yx} e_y \right) \right\} \\
&\stackrel{(a)}{=} \exp\left(-\theta R^\alpha \sum_{y \in \hat{\Phi} \setminus \{x, m(x)\}} \|y - x\|^{-\alpha} h_{yx} e_y \right) \\
&= \prod_{y \in \hat{\Phi} \setminus \{x, m(x)\}} \exp\left(-\frac{h_{yx} e_y}{b_{yx}} \right)
\end{aligned}$$

where (a) comes from the fact that $h_{m(x)x}$ is exponential. Therefore,

$$\begin{aligned}
q_x &= \mathbb{E}_{H_x}\left[\mathbb{P}\left(\mathrm{SIR}_x \geq \theta \mid H_x, \hat{\Phi} \right) \right] \\
&= \prod_{y \in \hat{\Phi} \setminus \{x, m(x)\}} \left(\frac{p_y}{1 + \frac{1}{b_{yx}}} + 1 - p_y \right) \\
&= \prod_{y \in \hat{\Phi} \setminus \{x, m(x)\}} \left(1 - \frac{p_y}{1 + b_{yx}} \right).
\end{aligned}$$

■

B. Transmit Probability

For wireless networks with FD radios as studied in this paper, we propose an adaptive ALOHA MAC policy in which each node $z \in \hat{\Phi}$ sets its transmit probability p_z based on the local spatial information "seen" from z. The "local spatial information" can be formalized using the notion of stopping set $S_z = S_z(\hat{\Phi})$ [13]. S_z is the region where the locations of nodes in node sets $\hat{\Phi}$ are known to the node z, i.e., z knows $\hat{\Phi} \cap S_z$. For example, it can be a disk centered at z with radius r. For a given local set, as in [13], the following class of MAC policies with local spatial information S_z is considered:

$$p_z = \psi(\hat{\Phi}) = \psi(S_z(\hat{\Phi}) \cap \hat{\Phi}), \tag{2}$$

where $z \in \hat{\Phi}$. $\psi(\cdot)$ is a function that takes point patterns as its argument and has range $[0, 1]$. It means that each node z will choose its transmit probability using the same MAC policy evaluated using its local spatial information.

IV. Throughput-Optimum Adaptive ALOHA

In this section, we derive the adaptive ALOHA strategy that maximizes the network throughput. Specifically, we would like to maximize the normalized sum of the logarithms of the node throughputs, which leads to proportional fairness [13].

A. Optimum Transmit Probability

The goal is to maximize the spatial average of the logarithmic throughput, given by

$$T = \lim_{r\to\infty} \frac{1}{\lambda\pi r^2} \sum_{x\in\hat{\Phi}\cap B_o(r)} \log(p_{m(x)}q_x), \tag{3}$$

where $B_z(r)$ denotes the disk of radius r centered at z. The sample path average T equals $\mathbb{E}^o\left[\log(p_{m(o)}q_o)+\log(p_o q_{m(o)})\right]$ due to the ergodicity of the PPP. $\mathbb{E}^o$ is the expectation with respect to $\mathbb{P}^o$, where $\mathbb{P}^o$ is the Palm distribution of the point process $\hat{\Phi}$. Under $\mathbb{P}^o$ there is a node located at the origin. As a result, our goal is to solve the following optimization problem:

$$\begin{aligned} &\text{maximize} \quad T = \mathbb{E}^o\left[\log(p_{m(o)}q_o)+\log(p_o q_{m(o)})\right] && (4)\\ &\text{subject to} \quad\quad 0 < p_y \le 1,\ y\in\hat{\Phi}, && (5)\end{aligned}$$

where the transmit probabilities p_y satisfy (2). This is a proportional fair ALOHA problem with local spatial information S similar to that in [13].

Theorem 1. *Define $\hat{p}_z$ as the solution of the fixed-point equation*

$$\frac{1}{p_z} = \sum_{y\in\hat{\Phi}\cap S_z(\hat{\Phi})\setminus\{o,m(o)\}} \frac{1}{1+b_{zy}-p_z} - \frac{\partial F(p_z,z)}{\partial p_z}, \tag{6}$$

where $z\in\{o,m(o)\}$ and

$$F(p,z) = 2\lambda \int_{y\in\mathbb{R}^2\setminus S_z(\hat{\Phi})} \log\left(1-\frac{p}{1+|y-z|^{\alpha}/\theta R^{\alpha}}\right)dy.$$

Also, define a_z as

$$a_z = \sum_{y\in\hat{\Phi}\cap S_z(\hat{\Phi})\setminus\{x,m(x)\}} \frac{1}{b_{zy}} - \frac{\partial F(1,z)}{\partial p_z}. \tag{7}$$

Given the wireless network described by $\hat{\Phi}$, the optimal solution to the maximization problem in (4) is $p_z^{\text{opt}} = \hat{p}_z$ if $a_z > 1$ and $p_z^{\text{opt}} = 1$ if $a_z \le 1$.

For $S_z(\hat{\Phi}) = B_z(r)$, (6) can be simplified to

$$\frac{1}{p_z} = \sum_{y\in\hat{\Phi}\cap S_z(\hat{\Phi})\setminus\{x,m(x)\}} \frac{1}{1+b_{zy}-p_z} + 4\pi\lambda R^2\cdot \int_{\frac{r}{R}}^{\infty} \frac{s}{1-p_z+s^{\alpha}/\theta}ds. \tag{8}$$

Proof: From (1), we have

$$\begin{aligned} T &= \mathbb{E}^o\Bigg[\log p_{m(o)} + \sum_{y\in\hat{\Phi}\setminus\{o,m(o)\}} \log\left(1-\frac{p_y}{1+b_{yo}}\right) + \\ &\qquad \log p_o + \sum_{y\in\hat{\Phi}\setminus\{o,m(o)\}} \log\left(1-\frac{p_y}{1+b_{ym(o)}}\right)\Bigg] \\ &\overset{(a)}{=} \mathbb{E}^o\Bigg[\log p_{m(o)} + \sum_{y\in\hat{\Phi}\setminus\{o,m(o)\}} \log\left(1-\frac{p_o}{1+b_{oy}}\right) + \\ &\qquad \log p_o + \sum_{y\in\hat{\Phi}\setminus\{o,m(o)\}} \log\left(1-\frac{p_{m(o)}}{1+b_{m(o)y}}\right)\Bigg],\end{aligned}$$

where (a) holds due to the mass transport principle [16, Page 65]. The two sums in the last expression can be split into two terms depending on whether $y\in S_z$ or $y\notin S_z$. For example, given $S_z(\hat{\Phi})$ where $z\in\{o,m(o)\}$, we have

$$\begin{aligned}&\mathbb{E}^o\left[\sum_{y\in\hat{\Phi}\setminus\{o,m(o)\}} f(y)\right] = \\ &\quad \mathbb{E}^o\left[\sum_{y\in\hat{\Phi}\cap S_o(\hat{\Phi})\setminus\{o,m(o)\}} f(y) + \sum_{y\in\hat{\Phi}\setminus S_o(\hat{\Phi})} f(y)\right],\end{aligned}$$

where $f(y) = \log\left(1-\frac{p_o}{1+b_{oy}}\right)$. $\hat{\Phi} = \Phi_1\cup\Phi_2$ is a stationary point process with density 2λ. By Campbell's formula [15, Page 79], we have

$$\mathbb{E}^o\left[\sum_{y\in\hat{\Phi}\setminus S_o(\hat{\Phi})} \log\left(1-\frac{p_o}{1+b_{oy}}\right)\right] = F(p_o,o).$$

Hence

$$\begin{aligned}&\mathbb{E}^o\left[\sum_{y\in\hat{\Phi}\setminus\{o,m(o)\}} \log\left(1-\frac{p_o}{1+b_{oy}}\right)\right] = \\ &\mathbb{E}^o\left[\sum_{y\in\hat{\Phi}\cap S_o(\hat{\Phi})\setminus\{o,m(o)\}} \log\left(1-\frac{p_o}{1+b_{oy}}\right) + F(p_o,o)\right].\end{aligned}$$

Similarly, we have

$$\begin{aligned}&\mathbb{E}^o\left[\sum_{y\in\hat{\Phi}\setminus\{o,m(o)\}} \log\left(1-\frac{p_{m(o)}}{1+b_{m(o)y}}\right)\right] = \\ &\quad \mathbb{E}^o\Bigg[\sum_{y\in\hat{\Phi}\cap S_{m(o)}(\hat{\Phi})\setminus\{o,m(o)\}} \log\left(1-\frac{p_{m(o)}}{1+b_{m(o)y}}\right) + \\ &\qquad\qquad F(p_{m(o)},m(o))\Bigg].\end{aligned}$$

The derivative of the expression under the expectation of T w.r.t. p_z is

$$\frac{1}{p_z} - \sum_{y \in \hat{\Phi} \cap S_z(\hat{\Phi}) \setminus \{o, m(o)\}} \frac{1}{1 + b_{zy} - p_z} + \frac{\partial F(p_z, z)}{\partial p_z}, \quad (9)$$

where $z \in \{o, m(o)\}$ and

$$\frac{\partial F(p, z)}{\partial p} = -2\lambda \int_{y \in \mathbb{R}^2 \setminus S_z(\hat{\Phi})} \frac{1}{1 + |y - z|^{\alpha} / \theta R^{\alpha} - p} dy.$$

Therefore, given the local spatial information $S_z(\hat{\Phi})$, the optimal p_z that maximizes the spatial average of the logarithmic throughput T is determined by the fixed-point equation (6). Since $p_z^{\text{opt}} \in (0, 1]$, p_z^{opt} is the solution to (6) if $a_z > 1$ in (7) and $p_z^{\text{opt}} = 1$ otherwise. The existence and uniqueness of the solution to (6) come from the fact that the LHS of (6) decreases from ∞ to 1 w.r.t. p_z on $(0, 1]$ while the RHS increases to a_z and from the continuity of these two functions.[1]

Next, consider the special case $S_z(\hat{\Phi}) = B_z(r)$ which means that each node knows the location information of all nodes in $\hat{\Phi}$ in a disk of radius r centered at itself. The second term of the RHS of (6) is

$$\frac{\partial F(p_z, z)}{\partial p_z} = -4\pi\lambda R^2 \int_{r/R}^{\infty} \frac{s}{1 - p_z + s^{\alpha}/\theta} ds, \quad (10)$$

since $\frac{\partial F(p_z, z)}{\partial p_z}$ is the same for $z \in \{o, m(o)\}$. ■

For $\alpha = 4$ and $S_z(\hat{\Phi}) = B_z(r)$, the second term of the RHS in (8) can be further simplified as

$$\begin{aligned} \frac{\partial F(p_z, z)}{\partial p_z} &= -4\pi\lambda R^2 \int_{r/R}^{\infty} \frac{s}{1 - p_z + s^{\alpha}/\theta} ds \\ &= -\frac{2\pi\lambda R^2 \sqrt{\theta}}{\sqrt{1 - p_z}} \left(\frac{\pi}{2} - \tan^{-1} \left(\frac{(r/R)^2}{\sqrt{\theta (1 - p_z)}} \right) \right). \end{aligned}$$

B. Optimum Adaptive ALOHA

If the node can obtain neighborhood information, such information can be used to improve the performance of the MAC scheme. In this subsection, we propose a novel MAC scheme that adaptively chooses the transmit probability such that the throughput is maximized based on the previous analysis.

Note that all the above results are derived in Theorem 1 based on the "typical link" that consists of o and $m(o)$. For any node $z \in \hat{\Phi}$ with its associated node $m(z)$, we define the Optimum Adaptive ALOHA (OA-ALOHA) by letting each node in the network choose its optimal transmit probability by solving

$$\frac{1}{p_z} = \sum_{y \in \hat{\Phi} \cap S_z(\hat{\Phi}) \setminus \{z, m(z)\}} \frac{1}{1 + b_{zy} - p_z} - \frac{\partial F(p_z, z)}{\partial p_z}. \quad (11)$$

The key assumption used here is that each node knows the locations of its neighbors within its local set S. Such information can be obtained via neighbor discovery [17]. Each node z then determines its transmit probability based on its local spatial information S_z.

[1]A similar argument is used in [13]. However, the difference from [13] is that p_z^{opt} depends on both Φ_1 and Φ_2 due to the full-duplex setup instead of only Φ_1.

Given a wireless network described by $\hat{\Phi}$, the procedure to calculate the transmit probability for each node is first to evaluate the values a_z in (7). If a_z is less than 1, the corresponding optimal transmit probability is set to be 1; otherwise, the optimal transmit probability for each node in the network is determined by solving (11).

C. Simplified Adaptive ALOHA

As one can imagine, for such an adaptive MAC scheme, when a node has many neighbors, it will transmit with a small probability; otherwise, it will transmit with a high probability. That is because the RHS of (11) increases as more nodes are added. As a result, the transmit probability decreases. In some networks, it is desirable to avoid solving (11), for example if the computational power is limited. Hence we also propose a simplified adaptive ALOHA (SA-ALOHA) scheme for FD networks, which is only based on the number of nodes in S but preserves most of the performance benefits of OA-ALOHA. Using the insight above, SA-ALOHA determines the transmit probability based on the number of neighbors in a manner that is informed by OA-ALOHA.

To derive SA-ALOHA, let us look at the first term of the RHS in (11). For any $y \in \hat{\Phi} \cap S_z(\hat{\Phi}) \setminus \{z, m(z)\}$, i.e., the neighbors of node z in $S_z(\hat{\Phi})$, $0 \leq \|y - z\| \leq r$ for $S_z(\hat{\Phi}) = B_z(r)$. Hence, $\frac{1}{1 + r^{\alpha}/\theta R^{\alpha} - p_z} \leq \frac{1}{1 + b_{zy} - p_z} \leq \frac{1}{1 - p_z}$. As a result, lower and upper bounds of the transmit probability can be obtained by solving

$$\frac{1}{p_z^{\text{lb}}} = \frac{N_z}{1 - p_z^{\text{lb}}} - \frac{\partial F(p_z^{\text{lb}}, z)}{\partial p_z^{\text{lb}}} \quad (12)$$

and

$$\frac{1}{p_z^{\text{ub}}} = \frac{N_z}{1 + r^{\alpha}/\theta R^{\alpha} - p_z^{\text{ub}}} - \frac{\partial F(p_z^{\text{ub}}, z)}{\partial p_z^{\text{ub}}}, \quad (13)$$

where N_z is the number of node z's neighbors in $B_z(r)$ and p_z^{lb} and p_z^{ub} are lower and upper bounds of p_z. Both bounds only depend on the number of neighbors of the node z instead of the exact locations of the neighbors which makes the expression and implementation much simpler.

Based on the above observations, we propose the SA-ALOHA to work in the following way: the SA-ALOHA scheme lets node z with FD capability transmit independently with transmit probability

$$p_z = \alpha_1 p_z^{\text{lb}} + \alpha_2 p_z^{\text{ub}}, \quad (14)$$

where $\alpha_1 + \alpha_2 = 1$ with $\alpha_1 \geq 0$ and $\alpha_2 \geq 0$. By doing so, the SA-ALOHA only uses the number of the neighbors for a node to obtain the transmit probability as the bounds do. Since the probability that node z's neighbor is within distance $r/2$ given that the neighbor is within distance r is $1/4$, the distances of the neighbors to the node are closer to the upper bound r than the lower bound 0. It means that the optimal transmit probability should be closer to p_z^{ub} than to p_z^{lb}. Therefore, we choose $\alpha_1 = 1/4$ and $\alpha_2 = 3/4$ in the above equation.

Figure 1 compares the transmit probability averaged over 10000 simulations, the bounds of the transmit probability and the simplified approximation as a function of N_z for different SIR threshold values. The optimal transmit probability of node

z from (11) always lies between the lower and upper bounds. Even if two nodes have the same number of neighbors, their optimal transmit probabilities might be different due to the different locations of their neighbors. However, the transmit probability from (14) is a good approximation to the optimal transmit probability. Moreover, the bounds get tighter as the SIR threshold increases, which makes the difference of the optimal transmit probabilities between nodes with the same number of neighbors smaller. As a result, the proposed SA-ALOHA has a transmit probability that approaches the optimal transmit probability as the SIR threshold increases.

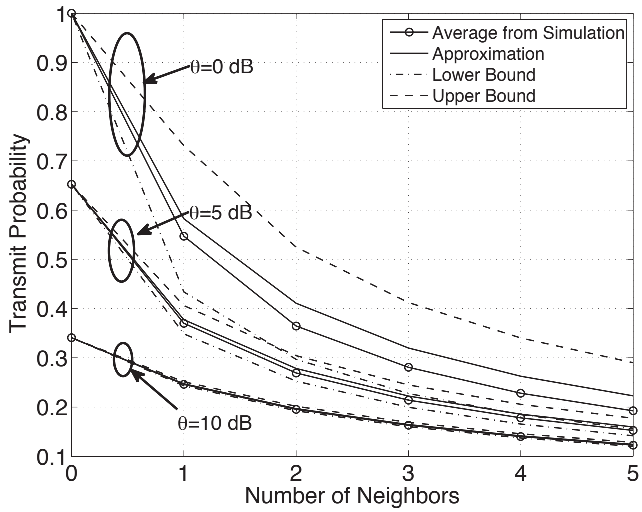

Figure 1: Comparison of bounds of the optimal transmit probability, its approximation and the average transmit probability from 10000 realizations as a function of the number of neighbors under different SIR threshold θ within $B_z(r)$ for $\alpha = 4$, $\lambda = 0.1$, $R = 1$, $r = 1$.

V. SIMULATION RESULTS

In the following, we present simulation results for the proposed adaptive MAC schemes, OA-ALOHA and SA-ALOHA. For comparison, we also consider the non-adaptive MAC scheme where each node in the network transmits with the same probability regardless of its local spatial information. This fixed transmit probability is obtained by setting $S = \emptyset$. Figure 2 compares the average link throughput between the novel adaptive ALOHA MAC schemes and non-adaptive ALOHA schemes. For comparison, the average link throughputs of the adaptive and non-adaptive ALOHA MAC schemes for wireless networks with only HD radios are also plotted. As seen, the adaptiveness improves the link throughput performance no matter if it is for HD or FD. Hence it pays off to flexibly choose different transmit probabilities for different nodes depending on their neighborhood information, i.e., a node in a dense neighborhood transmits with small probability while a node in a sparse neighborhood transmits frequently. It is also shown that the FD network outperforms the HD network in particular for small SIR thresholds and that the throughput gain is further improved using the OA-ALOHA scheme. The choice of r in the local set affects the throughput performance, as expected. As shown in Figure 2, the average link throughput curve is higher when $r = 2R$ than that when $r = R$ for $\theta > 1$. For smaller SIR thresholds, knowing the neighbors that are closer than the partner node is enough, for both HD and FD.

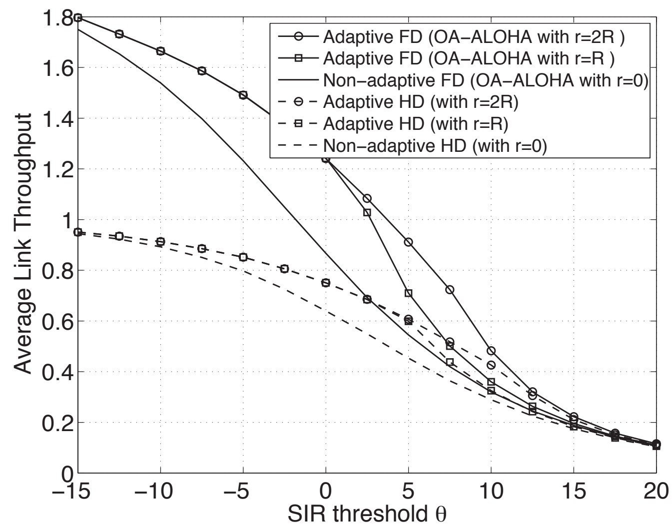

Figure 2: Comparison of the average link throughput between adaptive and non-adaptive ALOHA MAC schemes as a function of the SIR threshold θ for $\alpha = 4$, $\lambda = 0.1$, $R = 1$. The simulations are averaged over 10000 realizations.

The throughput gains of adaptive ALOHA over non-adaptive ALOHA for both FD and HD are plotted in Figure 3. The throughput gain is defined as the ratio of average link throughput of adaptive ALOHA for FD (HD) networks over that of non-adaptive ALOHA for FD (HD) networks. So FD networks have a higher throughput gain when using the adaptive ALOHA as illustrated in Figure 3. The OA-ALOHA can achieve more than 70% throughput gain while the adaptive ALOHA for HD networks can only achieve about 50%.

The SA-ALOHA MAC scheme approximates the OA-ALOHA scheme well as illustrated in Figure 4. The difference comes from the fact that the simplified MAC scheme only uses the number of the neighbors for each node to derive the transmit probability while the OA-ALOHA MAC scheme uses the locations of the neighboring nodes. SA-ALOHA achieves near-optimum performance but it is much easier to implement. As shown, when the SIR threshold is greater than 0 dB for $r = R$ and 10 dB for $r = 2R$, the average link throughputs of SA-ALOHA and OA-ALOHA are almost equal. The reason is the following: when $\theta \gg 1$, $r^\alpha/\theta R^\alpha \ll 1$, which implies that (13) approaches (12) as $\theta \to \infty$. Therefore, the difference between the bounds is getting smaller and the optimal transmit probability is getting closer to both its bounds as the SIR threshld θ increases. Moreover, as illustrated in Figure 1, as the transmit probability from the SA-ALOHA scheme is also getting closer to both bounds as the SIR thresholds increases and hence to the optimal transmit probability from the OA-ALOHA. This explains why the approximation is so good when θ is large.

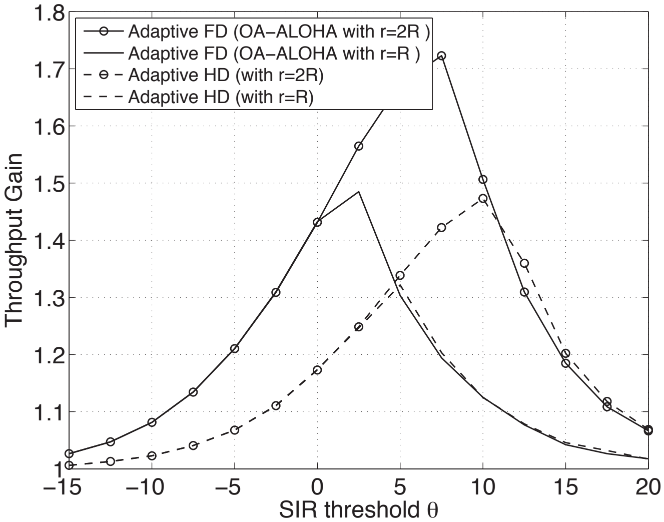

Figure 3: Comparison of throughput gain between adaptive and non-adaptive ALOHA MAC schemes as a function of the SIR threshold θ for $\alpha = 4$, $\lambda = 0.1$, $R = 1$. The simulations are averaged over 10000 realizations.

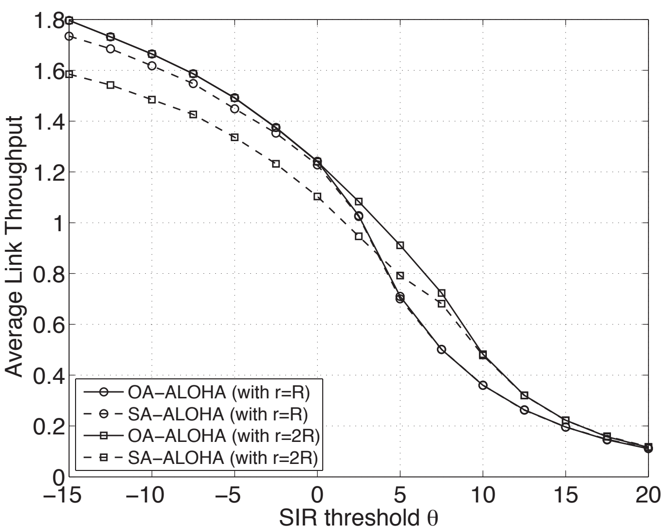

Figure 4: Comparison of the average link throughput between OA-ALOHA and SA-ALOHA MAC schemes as a function of the SIR threshold θ for $\alpha = 4$, $\lambda = 0.1$, $R = 1$. The simulations are averaged over 10000 realizations.

VI. CONCLUSION

In this paper, we proposed and analyzed an adaptive MAC scheme for FD wireless networks. In this MAC scheme, each node determines its transmit probability by using its local spatial information and maximizing the network-wide sum of the logarithms of the throughputs. A simplified version of the adaptive MAC scheme was also provided with less computational complexity and communication overhead but comparable performance. In contrast to the non-adaptive ALOHA MAC scheme where each node transmits with the same probability, the adaptive ALOHA MAC schemes exploit its information about its neighbors and efficiently mitigate interference and thus improve the network throughput. Moreover, the adaptive ALOHA leads to a larger improvement with FD than with HD. The maximum gain is around 50% for HD but more than 70% for FD.

ACKNOWLEDGEMENT

The partial support of the U.S. NSF (grant ECCS 1231806) is gratefully acknowledged.

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