Reliability Analysis of V2V Communications on Orthogonal Street Systems

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***Abstract*—The analysis of vehicle-to-vehicle communications is generally limited to vehicles on street segments, which are modeled using 1-D point processes. However, it is essential to model the intersections which are crucial for vehicle safety. In this paper, we focus on orthogonal street systems involving intersections with Poisson distributed vehicles on each street. We derive analytical expressions for the success probabilities of two types of users—the typical general user and the typical intersection user. We show that the the orthogonal street system shares some properties of both 1-D and 2-D Poisson networks. Specifically, the vehicles on the street system behave like 1-D and 2-D Poisson point processes of vehicles in the high-reliability and low-reliability regimes, respectively. Also, we deduce that the success probability of the typical general/intersection user is upper bounded by the minimum of the success probabilities of the 1-D and 2-D Poisson networks.**

***Index Terms*—Poisson point process, success probability, stochastic geometry, vehicle-to-vehicle communication.**

I. Introduction

A. Motivation

Vehicle-to-vehicle (V2V) communication holds the key to reliable and safer semi-autonomous or fully-autonomous driving. Vehicles can exchange the information required for safety such as their speed, brake status, position, etc. to other vehicles. Such a communication can alert the vehicles about the events happening in their vicinity which even the best sensors installed in the vehicles may fail to anticipate [1]. The key factors that affect the successful message transmission between two vehicles are their interfering vehicles, locations, and the channel characteristics. The vehicle locations further depend on the geometry of the streets as the street system restricts the possible locations of the vehicles and the interferers. The streets may be either well-structured as in Manhattan city or irregular and possess different topologies in different parts of a city. Hence it is of great importance to include the street geometry while analyzing V2V communications. The uncertainty in the vehicle locations can be conveniently modeled using random point processes. Stochastic geometry provides the mathematical tools to analyze such networks.

B. Related Work

Street systems have been included in wireless network models to study the impact of street segments on the network quality-of-service and global cost analysis in dense areas (see [2] and references therein). As for V2V communications, most prior works focus on the 1-D highway model [3]–[6], which limits the analysis to a street consisting of one or many lanes without intersections. Though the 1-D models provide insights on the vehicular network behavior on a single segment, it is insufficient to understand the overall network behavior. A preliminary analysis on GPS traces of Beijing taxis performed in [7] demonstrates that (i) the taxi locations do not form a 2-D Poisson point process as claimed in [3] and (ii) it is desirable to model the intersections which are critical for vehicle safety. The authors in [8] show that the proximity of a user to an intersection reduces the chances of successful packet reception. Reference [9] models roads as random line processes and vehicle locations as Poisson point processes, and studies the nearest-neighbor connectivity properties of the vehicular network. The general case of a $d-$dimensional Poisson bipolar network not involving street systems is studied in [10, Sec. 5.2]. Each transmitter pairs with a dedicated receiver at distance b in any random direction. The success probability of the typical user in this setting is

$$p_{\rm s} = \exp(-c_d \lambda' b^d \theta^{\delta'} \Gamma(1+\delta')\Gamma(1-\delta')), \tag{1}$$

where c_d denotes the volume of a d-dimensional unit ball, b is the link distance between each transmitter and its dedicated receiver, λ' is the transmitter intensity, $\delta' = d/\alpha$ and α is the path loss exponent. Note that $c_1 = 2$ and $c_2 = \pi$.

C. Contributions

In this paper, we consider orthogonal street systems, *i.e.,* grids formed by horizontal and vertical streets involving intersections. We model the spatial randomness of vehicles on each street using Poisson point processes and focus on the communication between vehicle pairs. Our contributions are:

- We provide exact analytical expressions for the (i) success probability of the typical general user, which corresponds to the average fraction of the successful transmissions between vehicles, and (ii) success probability of the typical intersection user, which corresponds to the average fraction of the successful transmissions when the receivers are at intersections.
- To gain crisp insights on the vehicular network behavior, we provide an asymptotic analysis and show that in the high-reliability regime, the vehicular network behaves like a 1-D Poisson network, whereas in the low-reliability regime, it behaves like a 2-D Poisson network.

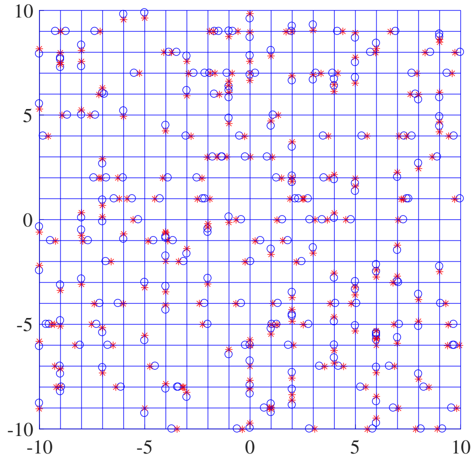

Figure 1. A realization of an orthogonal street system (PPPSG) with transmitters and receivers on each street denoted by 'o' and '$*$'. $\lambda_1 = 1$, $p = 0.3$, $b = 0.25$, and $s = 1$.

II. Network Model

We consider a vehicular network model that consists of a square (orthogonal) grid formed by horizontally and vertically oriented streets. The transmitting vehicles on each street form independent 1-D homogeneous Poisson point processes (PPPs) with intensity λ_1. Each transmitter has a dedicated receiver at a distance b, $i.e.$, the network is a Poisson bipolar network [10]. We formally define the *PPP on a square grid* (PPPSG) below.

Definition 1 (PPPSG)**.** *Let*

$$L_\ell^\varphi \triangleq \{(x,y) \in \mathbb{R}^2 \colon x\cos\varphi + y\sin\varphi = \ell\}$$

denote a line in $\mathbb{R}^2$, where $\ell \in \mathbb{R}$ is the location and $\varphi \in [0,\pi)$ is the direction of the line. For example, the x axis $(\mathbb{R},0)$ is $L_0^{\pi/2}$, and the y axis $(0,\mathbb{R})$ is L_0^0.

Let $\mathcal{P}_\ell^\varphi$ denote a 1-D PPP of intensity λ_1 on the line L_ℓ^φ. For different ℓ or φ, the processes are independent. Then

$$\mathcal{P}_\mathbb{Z} \triangleq \bigcup_{k\in\mathbb{Z}} \mathcal{P}_k^0 \cup \mathcal{P}_k^{\pi/2}$$

is a system of horizontally and vertically oriented 1-D PPPs, where exactly one of the coordinates of each point is an integer.

To make the model stationary and of variable intensity with respect to the inter-street spacing s, the PPPSG model is defined as

$$\mathcal{V} \triangleq s(\mathcal{P}_\mathbb{Z} + U),$$

where $s > 0$ and U is uniform on $[0,1)^2$.

On a horizontally oriented street, the receiver is either on the left or right of the transmitter, whereas on a vertically oriented street, the receiver is either above or below the transmitter. Fig. 1 depicts the model. Each transmitter transmits with probability p to its receiver. Then the intensity of active transmitters on each street is $\lambda = \lambda_1 p$. We can also obtain an grid of 1-D PPPs by quantizing either of the coordinates of each point of a 2-D PPP with equal probability.

Definition 2 (PPPSG—alternative definition)**.** *Let $q_\mathbb{Z} \colon \mathbb{R}^2 \mapsto \mathbb{R}^2$ be the random quantization function defined as*

$$q_\mathbb{Z}((u,v)) \triangleq Q_{(u,v)}(u,\lfloor v\rfloor) + (1-Q_{(u,v)})(\lfloor u\rfloor, v),$$

where Q_x, $x \in \mathbb{R}^2$, is a random field of independent Bernoulli random variables with mean $1/2$ and $\lfloor z\rfloor$ is the largest integer smaller than or equal to z.

Let $\mathcal{P} \subset \mathbb{R}^2$ be a stationary 2-D PPP of intensity λ_2 and define

$$\mathcal{P}_\mathbb{Z} \triangleq q_\mathbb{Z}(\mathcal{P}).$$

The PPPSG follows as

$$\mathcal{V} \triangleq s(\mathcal{P}_\mathbb{Z} + U),$$

as in the first definition.

We also define the (scaled) quantization function $q \colon s(\mathcal{P} + U) \mapsto s(\mathcal{P}_\mathbb{Z} + U)$ as

$$q(x) \triangleq s(q_\mathbb{Z}(x) + U).$$

The quantization levels correspond to the streets, which are integers for the model shown in Fig. 1. For example, given integer quantization levels, $U = (0,0)$, and $s = 1$, a point $(4.93, 1.16)$ in a 2-D PPP $\mathcal{P}$ can be quantized either as $(5, 1.16)$, displacing the point to a vertical street or as $(4.93, 1)$, displacing it to a horizontal street. On quantizing a 2-D PPP $\mathcal{P}$ defined on $[0,M]^2$, we obtain M horizontal and M vertical streets, each of length M. Random translation U does not affect the number of streets and their lengths. Scaling by a factor s results in a square grid of size $s \times M$. The intensity of active transmitters in $\mathcal{P}$ is $\lambda_2 p$. Equating the total expected number of active transmitters in $\mathcal{P}$ and its scaled quantized version $\mathcal{V}$, we obtain

$$\lambda_1 \times p \times (s \times M) \times 2M = \lambda_2 \times p \times (s \times M)^2, \quad (2)$$

which implies $\lambda_1 = \lambda_2 s/2$. For $s = 1$, displacing each of the points of a 2-D PPP to either horizontal or vertical streets results in streets each with transmitters at half the intensity of a 2-D PPP.

To the PPPSG, we add a receiver at the origin $(0,0)$. On averaging over the PPPSG, this receiver becomes the typical receiver (user). In the PPPSG, we consider two types of typical users: the typical general user and the typical intersection user. For the typical general user, we condition on the translation $U = (u_1, u_2)$ such that the origin is not an intersection, *i.e.*, $u_1 = 0, u_2 \in (0,1)$. In the case of the typical intersection user, the intersection falls at the origin, *i.e.*, $u_1 = u_2 = 0$. Note that the term 'typical user' refers to both the general and intersection users, unless otherwise stated. Let r_j denotes the distance between the typical user and the nearest location on the jth street, *i.e.*, the perpendicular distance. Without loss of generality, we order the perpendicular distances r_j such that $r_0 \leq r_1 \leq \ldots$, where $r_0 = 0$. Figs. 2 and 3 show the typical general and intersection users and the interferers on the same as well as different streets.

For the typical user at the origin and its transmitter at x, the received signal power S is $h_x\ell(x)$, where the channel power

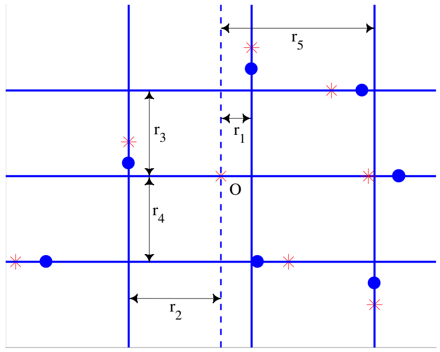

Figure 2. Interferers (filled circles) from the same street and different streets to the typical general user at the origin O. $r_0 = 0$, $r_1 < r_2 < r_3 = r_4 < r_5$.

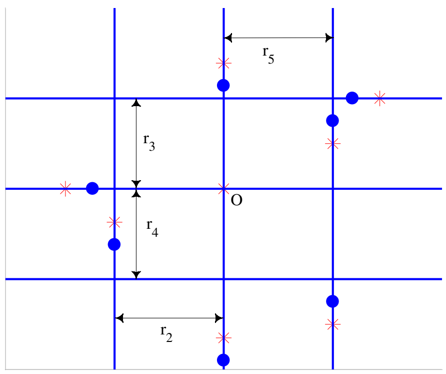

Figure 3. Interferers (filled circles) from the same street and different streets to the typical intersection user at the origin O. $r_0 = r_1 = 0$, $r_2 = r_3 = r_4 = r_5$.

gain h_x is exponentially distributed with mean 1 and $\ell(w) = \|w\|^{-\alpha}$ is the standard path loss function with exponent α. The received interference power at the origin is the sum of all the interference powers from the other transmitters on the same as well as the different streets. Let I_j denote the interference from the street at perpendicular distance r_j from the typical user. Thus the total interference power is $I = \sum_{j\in\mathbb{N}_0} I_j$, where $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$. Accordingly, the signal-to-interference ratio SIR at the typical user is

$$\mathsf{SIR} = \frac{S}{\sum\limits_{j\in\mathbb{N}_0} I_j} = \frac{h_x b^{-\alpha}}{\sum\limits_{j\in\mathbb{N}_0}\sum\limits_{z\in\mathcal{V}_j} h_z\|z\|^{-\alpha}}, \tag{3}$$

where $\mathcal{V}_j$ represents the set of active transmitters on the jth street at a time instant and $\mathcal{V} = \bigcup_{j\in\mathbb{N}_0}\mathcal{V}_j$. The transmission is considered successful when the SIR exceeds a certain threshold θ.

III. Success Probability

In this section, we derive the success probabilities of (i) the typical general user, which equals the average fraction of users who achieve SIR greater than θ, and (ii) the typical intersection user, which equals the average fraction of users who achieve SIR greater than θ when they are at intersections. The success probability (reliability) of the typical user is defined as

$$\begin{aligned} p_{\mathrm{s}} &\triangleq \mathbb{P}(\mathsf{SIR} > \theta) = \mathbb{P}(S > I\theta) && (4)\\ &= \mathbb{P}(h_x > \theta b^\alpha I) = \mathbb{E}_I(\exp(-\theta b^\alpha I)) && (5)\\ &\overset{(a)}{=} \prod_{j\in\mathbb{N}_0} \mathbb{E}_{I_j}(\exp(-\theta b^\alpha I_j)) \overset{(b)}{=} \prod_{j\in\mathbb{N}_0} \mathcal{L}_{I_j}(\theta b^\alpha) && (6)\end{aligned}$$

where (a) follows from the independence of the 1-D PPPs and (b) follows from the definition of Laplace transform. First, we will find the interference from the same street and from a different street. Using the results obtained, we will evaluate the success probabilities of the typical general and intersection users.

A. Interference from the Same Street

Based on our ordering of perpendicular distances, I_0 denotes the interference from the same street where the typical general user lies (see Fig. 2). The Laplace transform of the interference from the same street is

$$\begin{aligned} \mathcal{L}_{I_0}(\theta b^\alpha) &= \mathbb{E}_{I_0}(\exp(-\theta b^\alpha I_0))\\ &= \mathbb{E}\Bigg[\prod_{z\in\mathcal{V}_0} \mathbb{E}_h\left(\exp\left(-\theta b^\alpha h_z\|z\|^{-\alpha}\right)\right)\Bigg]\\ &= \mathbb{E}\Bigg[\prod_{z\in\mathcal{V}_0} \frac{1}{1+\theta b^\alpha |z|^{-\alpha}}\Bigg]\\ &\overset{(c)}{=} \exp\Bigg(-\lambda\int_{-\infty}^{\infty} \frac{1}{1+\left(\frac{u^2}{b^2\theta^\delta}\right)^{1/\delta}}\mathrm{d}u\Bigg)\\ &\overset{(d)}{=} \exp\Bigg(-\lambda b\theta^{\delta/2}\int_0^\infty \frac{1}{\left(1+v^{1/\delta}\right)\sqrt{v}}\mathrm{d}v\Bigg)\\ &= \exp(-2\lambda b\theta^{\delta/2}\Gamma(1+\delta/2)\Gamma(1-\delta/2)), \end{aligned} \tag{7}$$

where $\lambda = \lambda_1 p$, $\delta = 2/\alpha$, (c) follows from the probability generating functional (PGFL) of the PPP, and (d) results from the change of variable $v = \frac{u^2}{b^2\theta^\delta}$. For the typical intersection user, $r_0 = r_1 = 0$, as shown in Fig. 3. Then $I_0 = I_1$ in distribution, and the Laplace transform of the interference from both the streets is

$$\begin{aligned} \prod_{j=0}^{1}\mathcal{L}_{I_j}(\theta b^\alpha) &= \mathcal{L}^2_{I_0}(\theta b^\alpha)\\ &= \exp(-4\lambda b\theta^{\delta/2}\Gamma(1+\delta/2)\Gamma(1-\delta/2)). \end{aligned} \tag{8}$$

B. Interference from a Different Street

The Laplace transform of the interference from a different street for the typical general/intersection user is given by

$$\begin{aligned} \mathcal{L}_{I_j}(\theta b^\alpha) &= \mathbb{E}_{I_j}(\exp(-\theta b^\alpha I_j))\\ &= \mathbb{E}\Bigg[\prod_{z\in\mathcal{V}_j} \mathbb{E}_h\left(\exp\left(-\theta b^\alpha h_z\|z\|^{-\alpha}\right)\right)\Bigg]\\ &= \mathbb{E}\Bigg[\prod_{z\in\mathcal{V}_j} \frac{1}{1+\theta b^\alpha \|z\|^{-\alpha}}\Bigg]\end{aligned}$$

$$\overset{(e)}{=} \exp\left(-\lambda \int_{-\infty}^{\infty} \frac{1}{1+\left(\frac{r_j^2+u^2}{b^2\theta^\delta}\right)^{1/\delta}} \mathrm{d}u \right)$$

$$\overset{(f)}{=} \exp\left(-\lambda b\theta^{\delta/2} \int_{\frac{r_j^2}{b^2\theta^\delta}}^{\infty} \frac{1}{\left(1+v^{1/\delta}\right)\sqrt{v-\frac{r_j^2}{b^2\theta^\delta}}} \mathrm{d}v \right), \tag{9}$$

where $\lambda = \lambda_1 p$, $\delta = 2/\alpha$, (e) applies the PGFL of the PPP, and (f) is due to the change of variable $v = \frac{r_j^2+u^2}{b^2\theta^\delta}$. Note that (7) can be obtained from (9) by setting $r_j = 0$. For $\alpha = 2$ and 4, (9) simplifies to

$$\mathcal{L}_{I_j}(\theta b^2) = \exp\left(\frac{-\pi\lambda b^2\theta}{\sqrt{r_j^2+b^2\theta}} \right), \text{and}$$

$$\mathcal{L}_{I_j}(\theta b^4) = \exp\left(\frac{-\pi\lambda b^2\sqrt{\theta}\sin\left(\frac{1}{2}\arctan\left(\frac{b^2\sqrt{\theta}}{r_j^2}\right)\right)}{\left(r_j^4+b^4\theta\right)^{\frac{1}{4}}} \right),$$

respectively.

C. Success Probabilities for General and Intersection Users

Now, we can express the success probabilities of the typical general and intersection users using (6) as shown in Lemma 1.

Lemma 1. *The success probability of the typical general/intersection user in the PPPSG is given by*

$$p_\mathrm{s} = \exp(-2m\lambda b\theta^{\delta/2}\Gamma(1+\delta/2)\Gamma(1-\delta/2))\times \prod_{j\geq m} \exp\left(-\lambda b\theta^{\delta/2} \int_{\frac{r_j^2}{b^2\theta^\delta}}^{\infty} \frac{1}{\left(1+v^{1/\delta}\right)\sqrt{v-\frac{r_j^2}{b^2\theta^\delta}}} \mathrm{d}v \right), \tag{10}$$

where $\lambda = \lambda_1 p$, $\delta = 2/\alpha$, $m = 1$ for the typical general user and $m = 2$ for the typical intersection user.

Proof: Substituting the Laplace transform of the interference from the same street(s) (7), and (8), and from a different street (9) in (6), we obtain the result (10). ∎

A closed-form expression for (10) exists only for specific modeling parameters. To gain insights on the impact of interference from different streets on p_s, we focus on the asymptotic regimes $\theta \to 0$ and $\theta \to \infty$.

IV. Understanding Vehicular Network Behavior

A. Asymptotic Reliability Analysis

First, we will study how the interferers affect p_s as $\theta \to 0$, which corresponds to the high-reliability regime.

Theorem 1. *As $\theta \to 0$, the PPPSG behaves as*

$$1 - p_\mathrm{s} \sim 2m\lambda b\theta^{\delta/2}\Gamma(1+\delta/2)\Gamma(1-\delta/2),$$

where $\lambda = \lambda_1 p$, $\delta = 2/\alpha$, $m = 1$ for the typical general user and $m = 2$ for the typical intersection user.

Proof: Let $K = \Gamma(1+\delta/2)\Gamma(1-\delta/2)$ and $\forall j \geq 0$,

$$F_j(\theta) = \exp\left(-\lambda b\theta^{\delta/2} \int_{\frac{r_j^2}{b^2\theta^\delta}}^{\infty} \frac{1}{\left(1+v^{1/\delta}\right)\sqrt{v-\frac{r_j^2}{b^2\theta^\delta}}} \mathrm{d}v \right).$$

Note that $\prod_{j<m} F_j(\theta) = \exp(-2m\lambda b\theta^{\delta/2}K)$, for $m \in \{1,2\}$. Then we can express (10) as

$$p_\mathrm{s} = \exp\left(-2m\lambda b\theta^{\delta/2}K - \sum_{j\geq m} \ln F_j(\theta) \right). \tag{11}$$

For small θ, we can approximate (11) using Taylor's series as

$$\lim_{\theta\to 0} \frac{1-p_\mathrm{s}}{\theta^{\delta/2}} = 2m\lambda bK + \lim_{\theta\to 0} \sum_{j\geq m} \frac{\ln F_j(\theta)}{\theta^{\delta/2}} \tag{12}$$

Observe that $\frac{\ln F_j(\theta))}{\theta^{\delta/2}} \to 0$ as $\theta \to 0$ since the lower limit of the integral $\frac{r_j^2}{b^2\theta^\delta} \to \infty$. Hence the limit (12) reduces to

$$\lim_{\theta\to 0} \frac{1-p_\mathrm{s}}{\theta^{\delta/2}} = 2m\lambda bK,$$

and thus

$$p_\mathrm{s} \sim 1 - 2m\lambda b\theta^{\delta/2}\Gamma(1+\delta/2)\Gamma(1-\delta/2), \quad \theta \to 0. \tag{13}$$

This completes the proof. ∎

Similarly, we can approximate the success probability of the typical user in the d-dimensional Poisson network (1) using Taylor's series as

$$p_\mathrm{s} \sim 1 - c_d\lambda' b^d\theta^{\delta'}\Gamma(1+\delta')\Gamma(1-\delta'), \quad \theta \to 0. \tag{14}$$

Comparing (13) and (14), we observe that $\lambda = \lambda'$, $d = 1$ ($c_1 = 2$), $\delta' = \delta/2 = 1/\alpha$, and $m = 1$ since a 1-D Poisson bipolar network models a single street. For the typical intersection user, as it lies at the intersection of two streets, and each street forms an independent 1-D Poisson bipolar network, $m = 2$, and

$$p_\mathrm{s} \sim 1 - 4\lambda b\theta^{\delta/2}\Gamma(1+\delta/2)\Gamma(1-\delta/2), \quad \theta \to 0. \tag{15}$$

Remark 1. *The PPPSG behaves like a 1-D Poisson bipolar network as $\theta \to 0$. In this regime, the effect of the interference from different streets ($r_j \neq 0$) is negligible, and it is sufficient to consider only the interference from the same street(s) as seen from* (13)-(15).

Next, we analyze the low-reliability regime, where $\theta \to \infty$.

Theorem 2. *As $\theta \to \infty$, the success probability of the typical user in the PPPSG behaves as*

$$p_\mathrm{s} \sim \exp(-\pi\lambda_2 pb^2\theta^\delta\Gamma(1+\delta)\Gamma(1-\delta)),\ \theta \to \infty, \tag{16}$$

where $\lambda_2 p = 2\lambda_1 p/s = 2\lambda/s$, and $\delta = 2/\alpha$.

Proof: The success probability of the typical general/intersection user in the PPPSG from (3) and (5) is

$$p_\mathrm{s} = \mathbb{P}\left(h_x > b^\alpha \sum_{z\in\mathcal{V}} h_z \|\theta^{-1/\alpha}z\|^{-\alpha} \right). \tag{17}$$

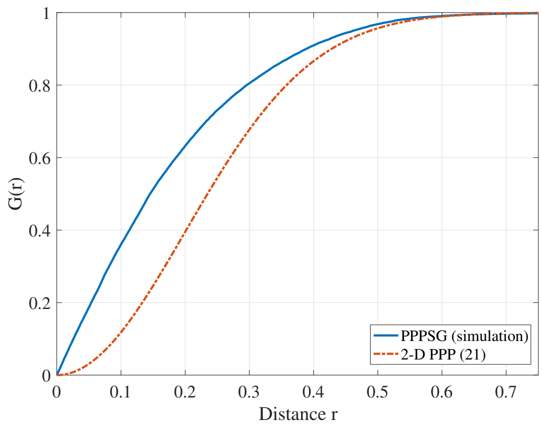

Figure 4. Nearest neighbor distance distributions $G(r)$ for PPPSG and 2-D PPP. The equation number is given in parentheses. $\lambda_1 = 2$, $p = 1$, and $s = 1$.

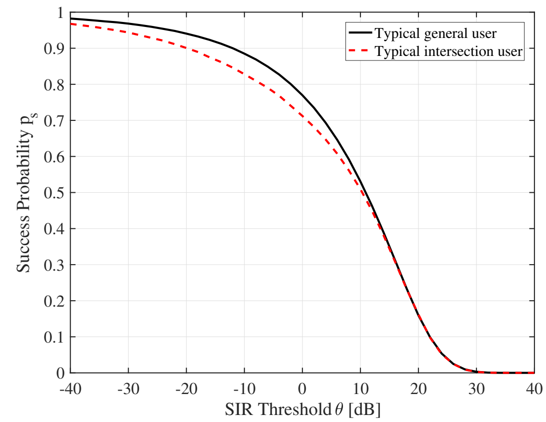

Figure 5. Success probabilities of the typical general and intersection users.

If $q(\cdot)$ denotes the quantized version of each point of a 2-D PPP $\mathcal{P}$ (see Def. 2), then (17) can be equivalently written as

$$p_{\rm s} = \mathbb{P}\bigg(h_x > b^\alpha \sum_{z\in\mathcal{P}} h_z \|\theta^{-1/\alpha} q(z)\|^{-\alpha}\bigg). \tag{18}$$

Similarly, for a stationary 2-D PPP $\mathcal{P}$, the success probability of the typical user at the origin $p'_{\rm s}$ can be expressed as

$$p'_{\rm s} = \mathbb{P}\bigg(h_x > b^\alpha \sum_{z\in\mathcal{P}} h_z \|\theta^{-1/\alpha} z\|^{-\alpha}\bigg). \tag{19}$$

Each point in $\mathcal{P}$ is displaced at most by $s/2$, where s is the spacing between the streets. Using the Cauchy-Schwarz inequality, we can obtain the lower bound $|\|z\| - \|q(z)\|| \leq \|z - q(z)\| \leq s/2$ on the distance between the point and its quantized version. On multiplying by $\theta^{-1/\alpha}$, we obtain

$$|\|\theta^{-1/\alpha} z\| - \|\theta^{-1/\alpha} q(z)\|| \to 0, \quad \theta \to \infty. \tag{20}$$

Applying (20) to (18) and (19), we infer that the interference experienced by the typical general/intersection user in the PPPSG tends to that of in a 2-D PPP as $\theta \to \infty$. Note that the type of user does not matter since the quantization does not affect the points of $\mathcal{P}$ as $\theta \to \infty$, *i.e.*, there is no difference between a general user and an intersection user. Hence the success probability in the PPPSG tends to that in a 2-D Poisson network $\mathcal{P}$. From (2), we find the intensity of active transmitters in $\mathcal{P}$ as $\lambda_2 p = 2\lambda_1 p/s$. Setting $d = 2$ ($c_2 = \pi$), $\lambda' = \lambda_2 p$, and $\delta' = \delta = 2/\alpha$ in (1), we obtain (16). ■

Remark 2. *The PPPSG behaves like a 2-D Poisson bipolar network as $\theta \to \infty$.*

B. Comparison to Poisson Point Processes

From our asymptotic analysis, we infer that the vehicular network shares some properties of both 1-D and 2-D Poisson networks. Here, we provide heuristic arguments that generalize the behavior of the vehicular network for all θ. For a 2-D PPP of intensity λ_2, conditioned that there exists a point on the origin, the probability that the nearest neighbor is within a distance r is [10]

$$G_{\rm PPP}(r) = 1 - \exp(-\lambda_2 \pi r^2). \tag{21}$$

Due to the quantization (see Def. 2), the PPPSG is a Cox process and thus exhibits clustering behavior, which means that $G_{\rm PPPSG}(r) \geq G_{\rm PPP}(r)$ for all r. Fig. 4 shows the nearest neighbor distance distribution curves for the 2-D PPP and the vehicular network (obtained through simulations), which supports our clustering argument. For the same channel distribution, as the number of neighbors to the typical user within a distance r increases, the interference increases, which leads to lower success probability. Let I' denote the interference to the typical user in $\mathcal{P}$. Using (5), for $I \geq I'$, we get

$$\mathbb{E}_I(\exp(-\theta b^\alpha I)) \leq \mathbb{E}_{I'}(\exp(-\theta b^\alpha I')),$$

which implies that the success probability in the PPPSG is less than or equal to that of in a 2-D Poisson network.

The success probability $p_{\rm s}$ in (10) is the product of the Laplace transforms of the interference from the same and different streets. We can infer that $p_{\rm s}$ is less than or equal to the Laplace transform of the interference from the same street(s), *i.e.*, $p_{\rm s} \leq \exp(-2m\lambda b\theta^{\delta/2}\Gamma(1+\delta/2)\Gamma(1-\delta/2))$. Hence the success probability of the vehicular network is upper bounded by that of the 1-D and 2-D Poisson bipolar networks.

Remark 3. *The success probability in the PPPSG is upper bounded by the minimum of the success probabilities in the corresponding 1-D and 2-D Poisson bipolar networks. As $\theta \to 0$, and $\theta \to \infty$, the bound gets tight.*

V. Results and Discussion

In this section, we present the numerical evaluations of the success probabilities of the typical general and intersection users, and validate the asymptotic analysis. We assume $\lambda_1 = 1$, $p = 0.3$, $\lambda = \lambda_1 p = 0.3$, $b = 0.25$, $s = 1$, and $\alpha = 4$. Fig. 5 shows the success probabilities of the typical general and intersection users. The success probability of the typical general user is higher than that of the typical intersection user. As two streets pass through the intersection, the probability

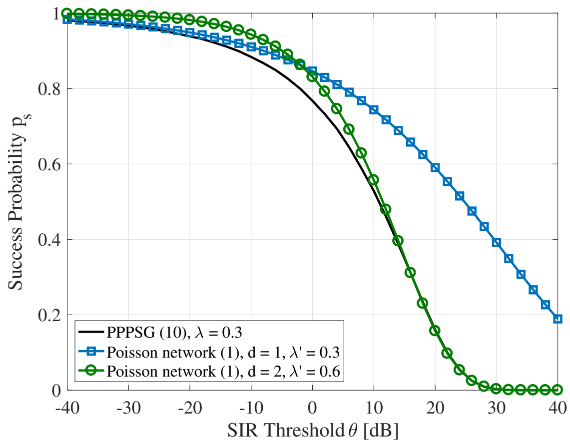

Figure 6. Success probability of the typical general user in the PPPSG vs. the success probability of the typical user in a Poisson network.

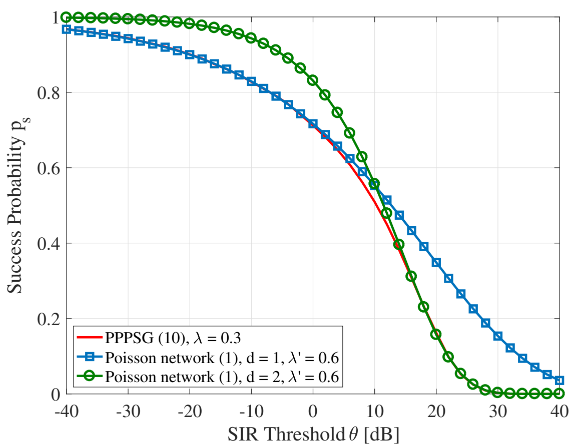

Figure 7. Success probability of the typical intersection user in the PPPSG vs. the success probability of the typical user in a Poisson network.

that the nearest interferer is within a distance r is higher for the intersection user than the general user. This results in lower success probability for the typical intersection user. Asymptotically, as $\theta \to \infty$, the success probabilities of the typical general and intersection users match in accordance with our low-reliability analysis.

Figs. 6 and 7 compare the success probabilities of the typical general and intersection users to that of the typical user in 1-D and 2-D PPPs. At low SIR threshold θ, the success probabilities of the typical general and intersection users match that of the 1-D Poisson bipolar network with intensity $\lambda' = \lambda \times m = 0.3m$, for $m = 1$ and 2, respectively. At high SIR threshold, the PPPSG with intensity $\lambda = 0.3$ matches 2-D Poisson bipolar network with intensity $\lambda' = 0.6 = 2\lambda$ for both types of users, consistent with our asymptotic analysis. Hence the upper bound on the success probability of the typical user is tight at the asymptotic regimes. The bound is least tight when the success probabilities of the 1-D and 2-D Poisson networks are the same, which happens when $\theta = -2$ dB in Fig. 6 and $\theta = 10$ dB in Fig. 7.

VI. Conclusions

This paper analyzed an orthogonal street system with transmitting and receiving vehicles on each street forming a Poisson bipolar network. Using tools from stochastic geometry, we have derived exact analytical expressions for the success probabilities of the typical general user, which characterizes the average performance of all the users, and of the typical intersection user, which characterizes the performance of the users at intersections. Using the expressions, for a given success probability, one would be able the find the maximum density of vehicles such that the typical general/intersection user satisfies a certain reliability constraint.

Our asymptotic analysis reveals that in the high-reliability regime, the interferers from the same streets as that of the typical receiver dominate the interference and the orthogonal street system behaves like a 1-D Poisson bipolar network. On the other hand, in the low-reliability regime, the orthogonal street system behaves like a 2-D Poisson bipolar network. Also, it is shown that the success probability of the typical general/intersection user is upper bounded by the minimum of the success probabilities of the typical user in the 1-D and 2-D Poisson networks, and that the bound gets tight in the asymptotic regimes. Hence the vehicles on the streets cannot be accurately modeled as 2-D Poisson point processes.

Acknowledgment

The partial support of the U.S. National Science Foundation through grant CCF 15259204 is gratefully acknowledged.

References

[1] W. Knight, "Car-to-car communication," *MIT Technology Review*, March 2015. [Online]. Available: https://www.technologyreview.com/s/534981/car-to-car-communication/

[2] F. Voss, C. Gloaguen, F. Fleischer, and V. Schmidt, "Distributional properties of Euclidean distances in wireless networks involving road systems," *IEEE Journal on Selected Areas in Communications*, vol. 27, no. 7, pp. 1047–1055, September 2009.

[3] Y. Jeong, J. W. Chong, H. Shin, and M. Z. Win, "Intervehicle communication: Cox-Fox modeling," *IEEE Journal on Selected Areas in Communications*, vol. 31, no. 9, pp. 418–433, September 2013.

[4] C. Jiang, H. Zhang, Z. Han, Y. Ren, V. C. M. Leung, and L. Hanzo, "Information-sharing outage-probability analysis of vehicular networks," *IEEE Transactions on Vehicular Technology*, vol. 65, no. 12, pp. 9479–9492, December 2016.

[5] B. Błaszczyszyn, P. Mühlethaler, and Y. Toor, "Stochastic analysis of ALOHA in vehicular ad hoc networks," *Annals of Telecommunications*, vol. 68, no. 1, pp. 95–106, 2013.

[6] A. Al-Hourani, R. J. Evans, S. Kandeepan, B. Moran, and H. Eltom, "Stochastic geometry methods for modeling automotive RADAR interference," *IEEE Transactions on Intelligent Transportation Systems, Accepted.*

[7] Q. Cui, N. Wang, and M. Haenggi, "Spatial point distribution analysis of real data of taxi GPS record information," Beijing University of Posts and Telecommunication and University of Notre Dame, Tech. Rep., 2016. [Online]. Available: http://www.nd.edu/~mhaenggi/pubs/techreport_taxi_analysis.pdf

[8] E. Steinmetz, M. Wildemeersch, T. Q. S. Quek, and H. Wymeersch, "A stochastic geometry model for vehicular communication near intersections," in *2015 IEEE Globecom Workshops*, December 2015, pp. 1–6.

[9] D. H. Thuen, H. V. Cuu, and H. N. Do, "VANET modelling and statistical properties of connectivity in urban environment," *REV Journal on Electronics and Communications*, vol. 3, no. 1-2, June 2013.

[10] M. Haenggi, *Stochastic Geometry for Wireless Networks*. Cambridge University Press, 2012.