Delay Analysis of Spatio-Temporal Channel Access for Cognitive Networks

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***Abstract*—Most channel access schemes for cognitive radio only consider using the idle periods of the primary users. Such schemes are not using the spectrum efficiently, since transmission opportunities also arise when a primary transmitter is active but its corresponding primary receiver is far away from the cognitive user. By detecting the signal power of the acknowledgments, a cognitive user is able to estimate the distance and channel condition between the primary *receiver* and itself. Then if the primary receiver is far away, the cognitive user can transmit simultaneously with the primary transmitter without affecting the primary link. Based on this idea, the spatio-temporal channel access scheme, utilizing both idle periods and spatial reuse, can be applied. This paper provides fundamental analysis of the channel access delays and shows the advantage of using the spatio-temporal access scheme in the carrier sense multiple access (CSMA)-based network with bi-directional links.**

I. INTRODUCTION

Channel access for cognitive radio networks is an active research area due to the increasing demand for spectrum resources. Traditional cognitive radio channel access schemes use the idle periods of the primary users. In wireless networks, transmission opportunities arise not only in time and frequency but also in space [1]: As long as the transmitter is far enough from other receivers, spatial reuse is possible. This idea can be applied to the cognitive radio channel access in which the cognitive user transmits simultaneously with the primary transmitter when the primary receiver is far away (this is a form of the interference temperature model schemes [2]). In practical applications, the distances can be estimated by detecting the acknowledgments (ACK) from the primary receiver.

In this paper, the spatio-temporal (ST) channel access scheme, which utilizes both the idle duration and the idea of spatial reuse, is analyzed. The cognitive users are considered to be employed in a carrier sense multiple access (CSMA)-based primary network with bi-directional links, and our analysis is focused on the channel access delay (CAD). The cognitive user channel access delay (CUCAD) is defined as the duration between the time that the cognitive transmission is requested and the time that the next opportunity appears. The primary user channel access delay (PUCAD) discussed here is the delay due to the interference from cognitive transmissions. The channel access delay inherent in the CSMA-based primary network (without the cognitive users) is well studied and not considered here. The primary user channel access delay is thus defined as the duration between the time the primary user would transmit if the network only has primary users and the time that the channel is free of interference from the cognitive users.

Many papers had considered the cognitive radio channel access problem, but most of them assumed time-slotted systems (see [3] and the survey [4]) and very few of them had considered using the ACKs for improved spatial reuse. The paper by Lapiccirella *et al.* [5] uses the ACK information, but their model is different from ours in many ways: this paper considers bi-directional links (they assume one-directional) and the CSMA-based network (theirs is a time-slotted system). This CSMA model with bi-directional links is better suited to model today's wireless networks, and the one-directional link model is included as a special case. To our best knowledge, this is the first paper to consider the cognitive radio channel access in CSMA-based networks with bi-directional links.

II. SYSTEM MODEL

Let us consider a cognitive network consisting of two primary users (PU) (one primary user transmission pair), one is close to the cognitive user (denoted with the subscript n) and the other is far away (denoted with the subscript f); see Fig. 1. The primary user link is assumed bi-directional and works as follows. It starts with an idle duration and then either the near or the far primary user transmits. Let p_n (resp. p_f) denote the probability that the near (resp. far) primary user transmits given that one of the primary users transmits. The transmission is followed immediately by a *physical layer* ACK with the short interframe spacing (SIFS) in between. Right after the ACK is another idle duration, so the primary user transmissions are always separated by ACKs and idle durations. Let the random variables τ_n (with support $[\tau_{\mathrm{n,min}}, \tau_{\mathrm{n,max}}]$ and mean $\bar{\tau}_\mathrm{n}$) and τ_f (with support $[\tau_{\mathrm{f,min}}, \tau_{\mathrm{f,max}}]$ and mean $\bar{\tau}_\mathrm{f}$) denote the transmission length of the near and the far primary users. Define the distributions $\mathrm{F}_\mathrm{n}(x) = \mathbb{P}(\tau_\mathrm{n} \leq x)$ and $\mathrm{F}_\mathrm{f}(x) = \mathbb{P}(\tau_\mathrm{f} \leq x)$, and let $\mathrm{f}_\mathrm{n}(x)$ and $\mathrm{f}_\mathrm{f}(x)$ be the corresponding densities (PDFs). In order to simplify the notation, let the total length of the SIFS and the ACK signal be τ_a, which is a fixed value. The random variable τ_i (with support $[\tau_{\mathrm{i,min}}, \tau_{\mathrm{i,max}}]$ and mean $\bar{\tau}_\mathrm{i}$) is used to denote

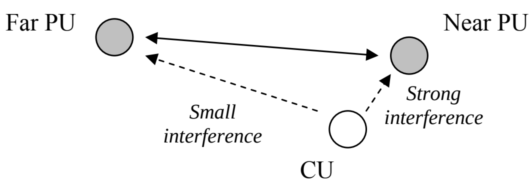


Figure 1. The relative locations of the primary users and the cognitive user under consideration.

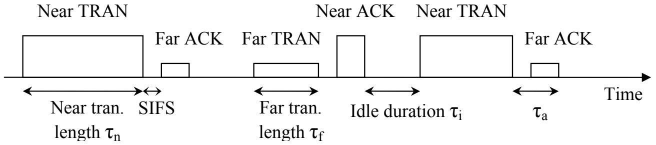


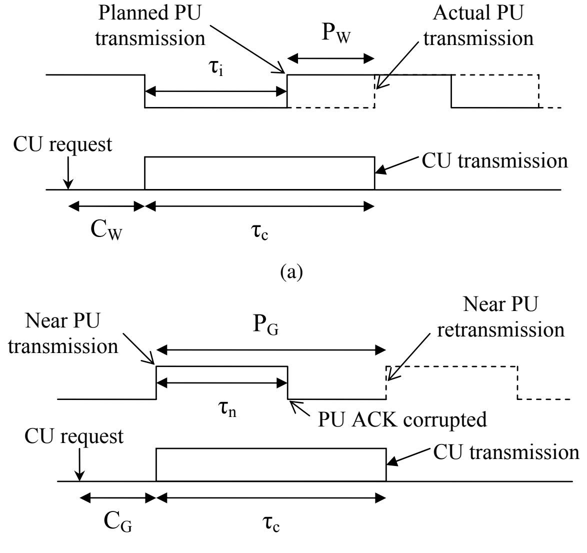
Figure 2. An example showing the transmissions of primary users.

the idle duration. Define the distribution $\mathrm{F_i}(x) = \mathbb{P}(\tau_\mathrm{i} \leq x)$ and let $\mathrm{f_i}(x)$ be the corresponding density.

For any time t, the cognitive user observes the primary users to be in one of the states $s \in${IDLE, NEAR_TRAN, FAR_TRAN, ACK}, which represents idle, the transmission from the near PU, the transmission from the far PU, and the ACK, respectively. The ACK state is a combination of the SIFS and the ACK signal since an SIFS always precedes the ACK. Also, the SIFS+ACK duration is assumed to be too short for cognitive user transmission. The states NEAR_ACK and FAR_ACK (ACKs from the near and the far user) are combined into one single state ACK since for the purpose of analyzing the channel access schemes, there is no need to distinguish which primary user the ACK signal is from. The probabilities that the primary users are in each state are p_i, $p_\mathrm{n,t}$, $p_\mathrm{f,t}$, and p_a, respectively. Let $\mu_\mathrm{o} = \bar{\tau}_\mathrm{i} + p_\mathrm{n}\bar{\tau}_\mathrm{n} + p_\mathrm{f}\bar{\tau}_\mathrm{f} + \tau_\mathrm{a}$, then $p_\mathrm{i} = \frac{\bar{\tau}_\mathrm{i}}{\mu_\mathrm{o}}$, $p_\mathrm{n,t} = \frac{p_\mathrm{n}\bar{\tau}_\mathrm{n}}{\mu_\mathrm{o}}$, $p_\mathrm{f,t} = \frac{p_\mathrm{f}\bar{\tau}_\mathrm{f}}{\mu_\mathrm{o}}$, and $p_\mathrm{a} = \frac{\tau_\mathrm{a}}{\mu_\mathrm{o}}$. Note that $p_\mathrm{n} = \frac{p_\mathrm{n,t}}{p_\mathrm{n,t}+p_\mathrm{f,t}}$ and $p_\mathrm{f} = \frac{p_\mathrm{f,t}}{p_\mathrm{n,t}+p_\mathrm{f,t}}$. Suppose the above statistics are collected by the cognitive users during the sensing phase, and assume the statistics (PDFs) and the locations of the primary users do not change over time. Let us also assume the cognitive users perform perfect spectrum sensing before taking actions, so the primary user state is known to the cognitive users. In addition, assume the cognitive transmission request is equally likely to happen at any time instance t, and let the cognitive user transmission length (including the length of ACK) be a random variable τ_c with support $[\tau_\mathrm{c,min}, \tau_\mathrm{c,max}]$. Although only the case $\tau_\mathrm{n,min} \leq \tau_\mathrm{c,min} < \tau_\mathrm{c,max} \leq \tau_\mathrm{n,max}$ is considered in this paper, other scenarios follow similar steps. Define the distribution $\mathrm{F_c}(x) = \mathbb{P}(\tau_\mathrm{c} \leq x)$ and let $\mathrm{f_c}(x)$ be the corresponding density (PDF).

III. ANALYSIS OF CHANNEL ACCESS SCHEMES

Consider the example in Fig. 2. It is obvious that two different channel access opportunities are available to a cognitive user: utilizing the idle duration or transmitting along with the *near* primary user. The *white space* channel access opportunity, using the idle duration of the primary transmissions, is the cognitive radio channel access scheme that previous work considered. The *gray space* opportunity, which is based on the fundamental fact that when studying the effect of interference, it is the receiver that matters, not the transmitter. This exposed terminal problem tells us that transmitting simultaneously with the primary users causes no harm as long as the receiver is far away. In order to use the gray space, the distance to the far primary user needs to be estimated. By detecting the signal power of the physical layer ACK signal from the far PU, the distance information is available. The *spatio-temporal* channel access scheme is to take advantage of *both* the idle duration and the spatial reuse, i.e., in this scheme, the cognitive user uses the next available opportunity, no matter whether it is white or gray space.



(b)

Figure 3. An example showing the channel access delay for the cognitive user and the primary users when using (a) the white space and (b) the gray space. Note that only the case with non-zero cognitive user channel access delay is shown.

In this section, we will derive the channel access delays for the spatio-temporal channel access scheme. The cognitive user channel access delay is the time the cognitive user must wait before the channel is available. Since the idle duration and the length of the primary and cognitive transmissions are random, it is unavoidable that the cognitive transmissions cause interference to the primary users, resulting in backoff or even retransmission. The primary user channel access delay thus reflects the primary user's waiting or retransmission time.

A. White space channel access opportunity

The white space scheme is to utilize the next available idle duration for cognitive transmissions. The cognitive user channel access delay, the time to wait for the next white space opportunity to show up, can be calculated as the following. If the current state is IDLE, the cognitive user can transmit immediately (the channel access delay is zero); if the current state is NEAR_TRAN or FAR_TRAN, the cognitive user needs to wait until the end of the primary transmission and the ACK signal; if the current state is ACK, the cognitive user needs to wait until the ACK signal is over. The expected

$$\begin{aligned}
\bar{C}_{\mathrm{G}} &= p_{\mathrm{i}}\left\{\sum_{k=1}^{\infty} p_{\mathrm{f}}^{k-1} p_{\mathrm{n}}\left[\mathbb{E}\left[\tilde{\tau}_{\mathrm{i}}\right]+(k-1)\left(\bar{\tau}_{\mathrm{i}}+\bar{\tau}_{\mathrm{f}}+\tau_{\mathrm{a}}\right)\right]\right\}+p_{\mathrm{f,t}}\left\{\sum_{k=1}^{\infty} p_{\mathrm{f}}^{k-1} p_{\mathrm{n}}\Bigg[\mathbb{E}\left[\tilde{\tau}_{\mathrm{f}}\right]\right. \\
&\quad \left.+k\left(\bar{\tau}_{\mathrm{i}}+\tau_{\mathrm{a}}\right)+(k-1)\bar{\tau}_{\mathrm{f}}\Bigg]\right\}+p_{\mathrm{a}}\left\{\sum_{k=1}^{\infty} p_{\mathrm{f}}^{k-1} p_{\mathrm{n}}\left[k\bar{\tau}_{\mathrm{i}}+(k-1)\bar{\tau}_{\mathrm{f}}+\left(k-\frac{1}{2}\right)\tau_{\mathrm{a}}\right]\right\} && (1) \\
&= \frac{p_{\mathrm{n}}}{\left(1-p_{\mathrm{f}}\right)^{2}}\Bigg\{p_{\mathrm{i}}\left(1-p_{\mathrm{f}}\right)\mathbb{E}\left[\tilde{\tau}_{\mathrm{i}}\right]+p_{\mathrm{f,t}}\left(1-p_{\mathrm{f}}\right)\mathbb{E}\left[\tilde{\tau}_{\mathrm{f}}\right]+\left(p_{\mathrm{i}}p_{\mathrm{f}}+p_{\mathrm{f,t}}+p_{\mathrm{a}}\right)\bar{\tau}_{\mathrm{i}}+\left(p_{\mathrm{i}}p_{\mathrm{f}}+p_{\mathrm{f,t}}p_{\mathrm{f}}+p_{\mathrm{a}}p_{\mathrm{f}}\right)\bar{\tau}_{\mathrm{f}} \\
&\quad +\left[p_{\mathrm{i}}p_{\mathrm{f}}+p_{\mathrm{f,t}}+\frac{p_{\mathrm{a}}}{2}\left(1+p_{\mathrm{f}}\right)\right]\tau_{\mathrm{a}}\Bigg\}. && (2)
\end{aligned}$$

channel access delay $\bar{C}_{\mathrm{W}}$ for the cognitive user using the white space alone is then given by

$$\bar{C}_{\mathrm{W}}=p_{\mathrm{n,t}}\left(\mathbb{E}\left[\tilde{\tau}_{\mathrm{n}}\right]+\tau_{\mathrm{a}}\right)+p_{\mathrm{f,t}}\left(\mathbb{E}\left[\tilde{\tau}_{\mathrm{f}}\right]+\tau_{\mathrm{a}}\right)+p_{\mathrm{a}}\frac{\tau_{\mathrm{a}}}{2}, \quad (3)$$

where the random variables $\tilde{\tau}_{\mathrm{n}}$ and $\tilde{\tau}_{\mathrm{f}}$ are the residual transmission time of the near and the far primary users given that the cognitive transmission request happens during the state NEAR_TRAN and FAR_TRAN, respectively. Consider the event $\mathcal{E}_{\mathrm{n}}$={The cognitive transmission request happens during the state NEAR_TRAN}, and then define the distribution $\tilde{\mathrm{F}}_{\mathrm{n}}(x)=\mathbb{P}(\tau_{\mathrm{n}}\leq x|\mathcal{E}_{\mathrm{n}})$ and let $\tilde{\mathrm{f}}_{\mathrm{n}}(x)$ be the corresponding density (PDF). It is found that

$$\mathbb{E}\left[\tilde{\tau}_{\mathrm{n}}\right]=\int_{\tau_{\mathrm{n,min}}}^{\tau_{\mathrm{n,max}}}\left[\int_{0}^{y}\frac{y-x}{y}\mathrm{d}x\right]\tilde{\mathrm{f}}_{\mathrm{n}}\left(y\right)\mathrm{d}y, \quad (4)$$

where $\tilde{\mathrm{f}}_{\mathrm{n}}\left(y\right)$ instead of $\mathrm{f}_{\mathrm{n}}\left(y\right)$ is used since the longer the transmission is, the more likely it is that the cognitive transmission request happens during this primary transmission. Thus $\tilde{\mathrm{f}}_{\mathrm{n}}\left(y\right)=\frac{2y}{\tau_{\mathrm{n,max}}^{2}-\tau_{\mathrm{n,min}}^{2}}$ and we get

$$\mathbb{E}\left[\tilde{\tau}_{\mathrm{n}}\right]=\frac{\tau_{\mathrm{n,max}}^{3}-\tau_{\mathrm{n,min}}^{3}}{3\left(\tau_{\mathrm{n,max}}^{2}-\tau_{\mathrm{n,min}}^{2}\right)}. \quad (5)$$

$\mathbb{E}\left[\tilde{\tau}_{\mathrm{f}}\right]$ can be obtained in a similar way.

When using the white space, if the cognitive user transmission is longer than the current idle duration, it will cause delay to the next primary user transmission since CSMA is applied (see Fig. 3(a)). $\bar{P}_{\mathrm{W}}$, the expected channel access delay of the primary users (due to the presence of cognitive users), is given by

$$\bar{P}_{\mathrm{W}}=\int_{\tau_{\mathrm{c,min}}}^{\tau_{\mathrm{c,max}}}\left[\int_{\tau_{\mathrm{i,min}}}^{y}\left(y-x\right)\mathrm{f}_{\mathrm{i}}\left(x\right)\mathrm{d}x\right]\mathrm{f}_{\mathrm{c}}\left(y\right)\mathrm{d}y. \quad (6)$$

If τ_{c} is uniformly distributed and τ_{i} is exponentially distributed with mean $1/\lambda$, then $\tau_{\mathrm{i,min}}=0$, $\mathrm{f}_{\mathrm{i}}\left(x\right)=\lambda e^{-\lambda x}$, and $\mathrm{f}_{\mathrm{c}}\left(y\right)=\frac{1}{\tau_{\mathrm{c,max}}-\tau_{\mathrm{c,min}}}$. A simple calculation leads to

$$\begin{aligned}
\bar{P}_{\mathrm{W}}=\frac{1}{\tau_{\mathrm{c,max}}-\tau_{\mathrm{c,min}}}&\left[\frac{\tau_{\mathrm{c,max}}^{2}-\tau_{\mathrm{c,min}}^{2}}{2}\right. \\
&\left.-\frac{e^{-\lambda\tau_{\mathrm{c,max}}}-e^{-\lambda\tau_{\mathrm{c,min}}}}{\lambda^{2}}-\frac{\tau_{\mathrm{c,max}}-\tau_{\mathrm{c,min}}}{\lambda}\right]. \quad (7)
\end{aligned}$$

B. Gray space channel access opportunity

For the gray space scheme, the cognitive user transmits along with the next transmission of the near primary user. The ACK signals are too short for practical use so only transmission periods are utilized. Now let us calculate the expected channel access delay $\bar{C}_{\mathrm{G}}$ for the cognitive user. If the current state is NEAR_TRAN, the cognitive transmission can start immediately (the channel access delay is zero); if the current state is IDLE, the next state could be either NEAR_TRAN or FAR_TRAN; if the current state is FAR_TRAN, the next state must be IDLE, but after that, the state could be NEAR_TRAN or FAR_TRAN. It is obvious that a cognitive user might observe several FAR_TRAN and IDLE states before eventually observing the NEAR_TRAN state. Therefore, $\bar{C}_{\mathrm{G}}$ is given by (2), where the random variable $\tilde{\tau}_{\mathrm{i}}$ is the residual idle duration given the cognitive transmission request happens during the idle state. The calculation of $\mathbb{E}\left[\tilde{\tau}_{\mathrm{i}}\right]$ is similar to the derivation of $\mathbb{E}\left[\tilde{\tau}_{\mathrm{n}}\right]$.

If $\tau_{\mathrm{c}}>\tilde{\tau}_{\mathrm{n}}$, the ACK signal from the far primary user will be corrupted[1], causing retransmission. Due to CSMA, the retransmission can only happen after the cognitive transmission is finished (see Fig. 3(b)), so the interference not only causes delay but also wastes energy for retransmission. On the other hand, if $\tau_{\mathrm{c}}<\tilde{\tau}_{\mathrm{n}}$, the interference level is determined by the distance to the far primary user and the channel condition, which is a function of the detected ACK signal power of the far primary user, assuming the channel condition is symmetric.

First let us calculate the expected channel access delay for the primary users $\bar{P}_{\mathrm{G}}$ due to the interference from the cognitive user. $\bar{P}_{\mathrm{G}}$ consists of two parts: when the current state is NEAR_TRAN, the cognitive transmission starts immediately from the middle of the near primary transmission; otherwise, the cognitive transmission starts from the beginning of the next near primary transmission. Therefore,

$$\bar{P}_{\mathrm{G}}=\left(1-p_{\mathrm{n,t}}\right)\bar{P}_{\mathrm{G,begin}}+p_{\mathrm{n,t}}\bar{P}_{\mathrm{G,mid}}. \quad (10)$$

$\bar{P}_{\mathrm{G,begin}}$ is given by

$$\bar{P}_{\mathrm{G,begin}}=\int_{\tau_{\mathrm{c,min}}}^{\tau_{\mathrm{c,max}}}y\left[\int_{\tau_{\mathrm{n,min}}}^{y}\mathrm{f}_{\mathrm{n}}\left(x\right)\mathrm{d}x\right]\mathrm{f}_{\mathrm{c}}\left(y\right)\mathrm{d}y. \quad (11)$$

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[1] The cognitive user signal is weakly detected by the far primary user, so, based on the CSMA rule, the far primary user assumes it is safe to send ACK.

$$\bar{P}_{\mathrm{G,mid}} = \int_{\tau_{\mathrm{c,min}}}^{\tau_{\mathrm{c,max}}} \left\{ \int_{z}^{\tau_{\mathrm{n,max}}} \left[\int_{y-z}^{y} \frac{x+z}{y} \mathrm{d}x \right] \tilde{\mathrm{f}}_{\mathrm{n}}(y)\,\mathrm{d}y + \int_{\tau_{\mathrm{n,min}}}^{z} \left[\int_{0}^{y} \frac{x+z}{y} \mathrm{d}x \right] \tilde{\mathrm{f}}_{\mathrm{n}}(y)\,\mathrm{d}y \right\} \mathrm{f}_{\mathrm{c}}(z)\,\mathrm{d}z. \tag{8}$$

$$\bar{P}_{\mathrm{G,mid}} = \frac{1}{\tau_{\mathrm{n,max}}^2 - \tau_{\mathrm{n,min}}^2} \left\{ \frac{\left(\tau_{\mathrm{n,max}}^2 - \tau_{\mathrm{n,min}}^2\right)\left(\tau_{\mathrm{c,max}} + \tau_{\mathrm{c,min}}\right)}{2} + \frac{\tau_{\mathrm{n,max}}\left(\tau_{\mathrm{c,max}}^2 + \tau_{\mathrm{c,max}}\tau_{\mathrm{c,min}} + \tau_{\mathrm{c,min}}^2\right)}{3} - \frac{\left(\tau_{\mathrm{c,max}}^2 + \tau_{\mathrm{c,min}}^2\right)\left(\tau_{\mathrm{c,max}} + \tau_{\mathrm{c,min}}\right)}{6} - \frac{\tau_{\mathrm{n,min}}^3}{3} \right\}. \tag{9}$$

Let τ_{o} be the offset of the starting time of the cognitive transmission to the starting time of the near primary transmission. If $\tau_{\mathrm{c}} < \tau_{\mathrm{n}}$, the interference can happen only when $\tau_{\mathrm{n}} - \tau_{\mathrm{c}} < \tau_{\mathrm{o}} < \tau_{\mathrm{n}}$; if $\tau_{\mathrm{c}} > \tau_{\mathrm{n}}$, the interference happens no matter what τ_{o} is (but note that $0 < \tau_{\mathrm{o}} < \tau_{\mathrm{n}}$). Therefore, $\bar{P}_{\mathrm{G,mid}}$ is given by (8).

If both the transmission length of the primary and the cognitive users are uniformly distributed,

$$\bar{P}_{\mathrm{G,begin}} = \frac{2\left(\tau_{\mathrm{c,max}}^3 - \tau_{\mathrm{c,min}}^3\right) - 3\tau_{\mathrm{n,min}}\left(\tau_{\mathrm{c,max}}^2 - \tau_{\mathrm{c,min}}^2\right)}{6\left(\tau_{\mathrm{c,max}} - \tau_{\mathrm{c,min}}\right)\left(\tau_{\mathrm{n,max}} - \tau_{\mathrm{n,min}}\right)},$$

and $\bar{P}_{\mathrm{G,mid}}$ is given by (9).

$\bar{I}_{\mathrm{G}}$, the expected interference superimposed on the primary transmission when no retransmission is caused, is a function of the cognitive user transmission power η_{c} and the factor h that describes the channel condition (including the large scale and the small scale path loss). h can be estimated by observing the signal power of the ACK from the far primary user. Thus,

$$\bar{I}_{\mathrm{G}} = \eta_{\mathrm{c}} h \int_{\tau_{\mathrm{c,min}}}^{\tau_{\mathrm{c,max}}} y \left[\int_{y}^{\tau_{\mathrm{n,max}}} \mathrm{f}_{\mathrm{n}}(x)\,\mathrm{d}x \right] \mathrm{f}_{\mathrm{c}}(y)\,\mathrm{d}y. \tag{12}$$

When both the length of the primary and the cognitive transmissions are uniformly distributed, it is easy to obtain

$$\bar{I}_{\mathrm{G}} = \eta_{\mathrm{c}} h \cdot \frac{3\tau_{\mathrm{n,max}}\left(\tau_{\mathrm{c,max}}^2 - \tau_{\mathrm{c,min}}^2\right) - 2\left(\tau_{\mathrm{c,max}}^3 - \tau_{\mathrm{c,min}}^3\right)}{6\left(\tau_{\mathrm{n,max}} - \tau_{\mathrm{n,min}}\right)\left(\tau_{\mathrm{c,max}} - \tau_{\mathrm{c,min}}\right)}. \tag{13}$$

Finally, the expected retransmission length $\bar{R}_{\mathrm{G}}$ is simply $\bar{\tau}_{\mathrm{n}}$.

C. Spatio-temporal channel access scheme

The spatio-temporal access scheme utilizes both the white space and the gray space opportunities, i.e., whenever an idle duration or a near primary transmission appears, the cognitive user can transmit. If the current state is either IDLE or NEAR_TRAN, the cognitive user channel access delay is zero; if the current state is FAR_TRAN, the cognitive user will wait until the next available opportunity, which is the white space, so the delay is $\mathbb{E}\left[\tilde{\tau}_{\mathrm{f}}\right] + \tau_{\mathrm{a}}$; if the current state is ACK, the average channel access delay is $\frac{\tau_{\mathrm{a}}}{2}$. The expected channel access delay for the cognitive user using the spatio-temporal scheme is thus given by

$$\bar{C}_{\mathrm{ST}} = p_{\mathrm{f,t}}\left(\mathbb{E}\left[\tilde{\tau}_{\mathrm{f}}\right] + \tau_{\mathrm{a}}\right) + p_{\mathrm{a}}\frac{\tau_{\mathrm{a}}}{2}. \tag{14}$$

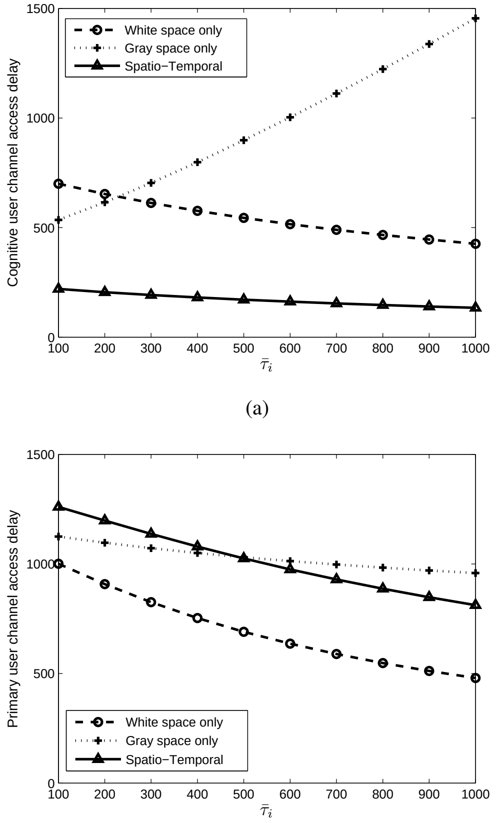
Since only when the current state is NEAR_TRAN would the gray space be used, the expected retransmission length $\bar{R}_{\mathrm{ST}}$ is simply $p_{\mathrm{n,t}}\bar{R}_{\mathrm{G}}$ and the expected interference $\bar{I}_{\mathrm{ST}}$ is $p_{\mathrm{n,t}}\bar{I}_{\mathrm{G}}$. The expected channel access delay for the primary users $\bar{P}_{\mathrm{ST}}$ is given by

$$\bar{P}_{\mathrm{ST}} = p_{\mathrm{n,t}}\bar{P}_{\mathrm{G,mid}} + \left(1 - p_{\mathrm{n,t}}\right)\bar{P}_{\mathrm{W}}, \tag{15}$$

where $\bar{P}_{\mathrm{G,mid}}$ instead of $\bar{P}_{\mathrm{G}}$ is used because if the gray space is utilized in the spatio-temporal scheme, it must start somewhere in the middle of the near primary transmission.

IV. NUMERICAL EXAMPLES

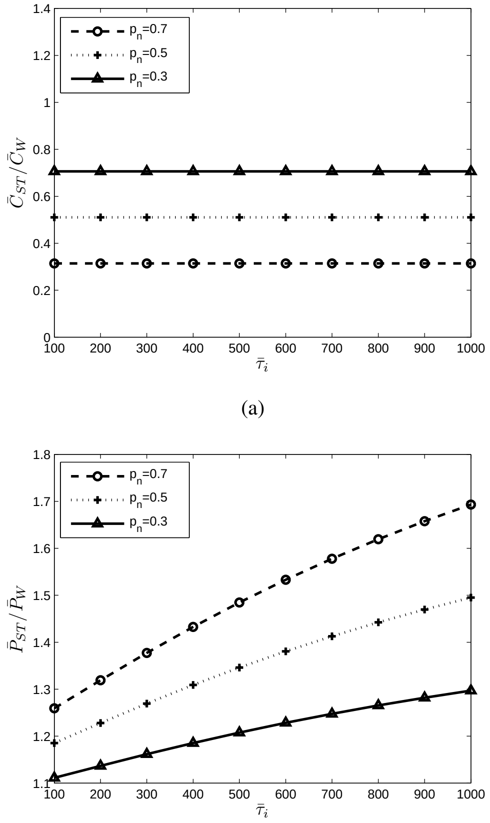
Now let us evaluate the performance of the spatio-temporal channel access scheme by showing some numerical examples. If $\eta_{\mathrm{c}} h$ is small due to the distant far primary user, the large path loss, or the small cognitive transmission power, then $\bar{I}_{\mathrm{G}}$ is negligible. Fig. 4 compares the channel access delays for the cognitive user and for the primary user. Schemes utilizing white space alone, gray space alone, and spatio-temporal channel access are compared under different mean idle durations, using the equations derived in the previous section. The idle duration is assumed exponentially distributed. Assume the length of the near primary transmissions is uniformly distributed with $\tau_{\mathrm{n,min}} = 200$ and $\tau_{\mathrm{n,max}} = 2000$, the length of the cognitive transmissions is uniformly distributed with $\tau_{\mathrm{c,min}} = 200$ and $\tau_{\mathrm{c,max}} = 2000$, and $\tau_{a} = 200$. The cognitive transmission requests are assumed to occur equally likely at any time t. For the bi-directional links, assume 70% of the transmissions are from the near primary user, i.e., $p_{\mathrm{n}} = 0.7$ and $p_{\mathrm{f}} = 0.3$. Fig. 4(a) shows that when $\bar{\tau}_{\mathrm{i}}$ becomes larger, the cognitive user channel access delay using the gray space opportunity increases whereas the cognitive user channel access delay using the other two schemes decreases. This is intuitive since increasing $\bar{\tau}_{\mathrm{i}}$ will increase p_{i} and decrease $p_{\mathrm{n,t}}$, making the interval between consecutive near primary transmissions larger. Notice that the average cognitive user channel access delay using the spatio-temporal scheme is the smallest, and as expected the average cognitive user delay when using the gray space alone is the largest due to the uncertainty of the link direction of the next primary transmission. However, when $\bar{\tau}_{\mathrm{i}}$ is smaller than 200, the average CUCAD using the gray space is smaller than using the white space. This is easy to understand since the primary users have higher probability to be in the NEAR_TRAN state when $\bar{\tau}_{\mathrm{i}}$ is smaller. The smaller average CUCAD of the spatio-temporal scheme is obtained at



(a)

(b)

Figure 4. Channel access delay for (a) the cognitive user and (b) the primary user using the white space, the gray space, and the spatio-temporal access when $p_{\rm n} = 0.7$.



(a)

(b)

Figure 5. The ratio of the channel access delay using the spatio-temporal access scheme to the delay using only the white space for (a) the cognitive user and (b) the primary user.

the cost of causing larger average PUCAD compared to using the white space alone, as shown in Fig. 4(b).

In order to understand the effect of $p_{\rm n}$, Fig. 5(a) shows the ratio of the average CUCAD using the spatio-temporal access scheme to the average CUCAD using only the white space and Fig. 5(b) shows the ratio of the average PUCAD using the spatio-temporal access scheme to the average PUCAD using only the white space under different $p_{\rm n}$. The average CUCAD using the spatio-temporal scheme is only about 70%, 50%, and 30% of the average CUCAD using the white space when $p_{\rm n} = 0.3$, $p_{\rm n} = 0.5$, and $p_{\rm n} = 0.7$, respectively. When $p_{\rm n}$ becomes larger, the spatio-temporal scheme is more efficient compared to using the white space alone, and this improvement does not change with different $\bar{\tau}_{\rm i}$ settings. However, the PUCAD becomes larger with the increase in $p_{\rm n}$, and the difference becomes more significant with larger mean idle duration.

V. CONCLUSION

The opportunities for the cognitive user arise not only in time but also in space. This paper analyzed the channel access delays of the spatio-temporal channel access scheme which utilizes both the white space and the gray space. Compared to using the white space alone, we have shown that the spatio-temporal scheme has much shorter cognitive user channel access delay with slightly larger primary user channel access delay. When more transmissions are from the near primary user, the spatio-temporal scheme is significantly more efficient than using the white space alone.

Our future work includes: (1) extension to the scenario with multiple cognitive users; (2) performance analysis when the primary user signals cannot be perfectly detected due to, for example, fading or shadowing; (3) system analysis with other metrics, such as throughput and scalability, and comparison with the results from network simulators.

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