Stochastic Analysis of the Mean Interference for the RTS/CTS Mechanism

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***Abstract*—The RTS/CTS handshake mechanism in WLAN is studied using stochastic geometry. The effect of RTS/CTS is treated as a thinning procedure for a spatially point process that models the potential transceivers in a WLAN, and the resulting concurrent transmitter processes are described. Exact formulas for the intensity of the concurrent transmitter processes and the mean interference are established. The analysis yields useful results for understanding how the design parameters of RTS/CTS affect the interference in the network.**

I. Introduction

The universal deployment of WLAN during the recent years has been a major driving force for the proliferation of personal wireless network access. In IEEE 802.11 MAC [1] both physical carrier sensing and virtual carrier sensing are introduced to avoid collisions. In physical carrier sensing, a potential transmitter first detects whether there are any active transmitters within a detectable region and defers its transmission if so [2]. In virtual carrier sensing, a Request-to-Send/Clear-to-Send (RTS/CTS) handshake mechanism is introduced to solve the hidden terminal problem [3]. In wireless networks, a hidden node is a node which is visible from a receiver, but not from the transmitter communicating with said receiver. To avoid collisions caused by hidden nodes, the RTS/CTS handshake mechanism sets up a protection zone around a receiver, and thus the overall protection region of a transceiver pair is the union of the carrier sensing cleaned region and the RTS/CTS cleaned region (Figure 1).

Despite the existing significant body of literature for the performance analysis of 802.11 WLAN (see, e.g., [4]–[8]), the analysis of the RTS/CTS mechanism for a network is still elusive due to the randomness of spatially distributed nodes and the strong correlation between the active transmitters. Since the purpose of RTS/CTS is to mitigate the network interference while allowing spatial reuse, it is important to understand how its design parameters such as the carrier sensing ranges affect the network interference. For applications, a system engineer may be interested to know under which conditions the network interference is comparable with the thermal noise and thus essentially negligible.

In this paper, we develop a spatial distribution model for the RTS/CTS mechanism and evaluate the mean interference. The frequently used homogeneous Poisson point process (PPP) [9] is not accurate, because RTS/CTS causes a strong coupling

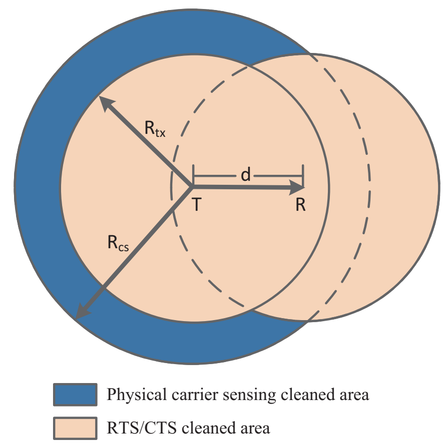

Fig. 1. Illustration of a transceiver pair with the RTS/CTS mechanism.

between neighboring transmitters. Additionally, former interference analyses mainly focus on the interference at either the typical transmitter or an arbitrary and fixed location, instead of the typical receiver. In our work, we propose a marked point process to characterize the distribution of both transmitters and their corresponding receivers under the RTS/CTS mechanism. In this marked point process, each point indicates the location of a transmitter with a mark denoting the relative location of the associated receiver, and the effect of RTS/CTS is similar to the effect that Matérn's hard core models [10] have on the underlying PPP. By employing the model, we follow a general method to derive an exact formula for the mean interference experienced by the typical receiver. Notice that the interference is averaged over all possible realizations of the PPP, and the location of the typical transceiver pair can be chosen arbitrarily because of the stationarity of the proposed marked point process. A key ingredient in our derivation is to relate the reduced Palm expectation with the second-order factorial measure and the second-order product density of the marked point process, which eventually admits a convenient geometric interpretation. Our results reveal how the performance of the RTS/CTS mechanism varies with the density of network nodes and the carrier sensing ranges.

Related works are as follows. In [11], an optimal power control strategy in RTS/CTS is proposed to maximize the probability that a typical link gets access to the medium by assuming a max-interference model; however, the analysis of interference statistic is ignored. In [12] and [13], the interference analysis is mainly based on Matérn's hard core model, which is not capable of precisely describing the behavior of the RTS/CTS mechanism. In [12], the outage probability of Matérn's hard core process is derived by approximating it with a PPP, which is really tractable but not that accurate. In [13], the accurate expressions of the mean interference in the Matérn's hard core model are derived. However, when the RTS/CTS mechanism is considered, deriving the mean interference becomes more difficult because the locations of the receivers are considered as the marks of the transmitters and play a crucial role in suppressing other transmissions. Therefore, using the reduced second moment measure and the Ripley K-function to derive the mean interference, which is the method introduced by [13], is no longer applicable. In our work, we extend the reduced second moment measure and the Ripley K-function to a more general form so that they can be applied to the marked point process; then based on the extension, we derive accurate expressions for the mean interference under the RTS/CTS mechanism.

The remaining part of this paper is organized as follows. Section II describes the marked point process models for nodes under RTS/CTS. Section III then establishes the main analytical results of this paper, including formulas of the intensity of the marked point process and the mean interference experienced by a typical receiver. Section IV will present the numerical illustration.

II. Network Model

In this section, we introduce the statistical network model for pairs of transceivers, under the RTS/CTS mechanism.

A. Basic Assumptions

We consider a geographic region in which a large number of potential transmitters and their corresponding receivers reside. For simplicity, we set the transmit power of each potential transmitter to a common value P_t. Regarding signal propagation, we consider a general deterministic path-loss function $l(\cdot)$, so that the power received at a point of distance r to the transmitter, denoted by P_r, is given by $P_r = l(r)P_t$. For example, a simple model commonly used in analysis is $l(r) = Ar^{-\alpha}$ where $\alpha > 2$ is the so-called path-loss exponent.

B. Concurrent Transmitter Processes

To characterize the locations of transmitters and receivers, we begin with a Poisson bipolar model [14] where each point of a PPP is a potential transmitter and has its receiver at some fixed distance d and a random orientation of angle θ. For simplicity, we only consider the case where d is fixed, while the approach may be extended to the case where d is a random variable. By applying the RTS/CTS mechanism, a pair of transceivers sets up an exclusion zone in which other potential transmitters are prohibited to transmit. We assume that the physical carrier sensing cleaned region is a circular region centered at the transmitter with radius R_{cs}, and that the virtual carrier sensing cleaned region is the union of two circular regions, which are centered at the transmitter (i.e., RTS) and the receiver (i.e., CTS), respectively, with the same radius R_{tx} ($R_{\mathrm{tx}} < R_{\mathrm{cs}}$[1]) . Note that the the physical carrier sensing cleaned region and the virtual carrier sensing cleaned region are overlapping. See Figure 1 for illustration.

We denote the effect of RTS/CTS as *RTS/CTS thinning*, which selectively removes some points of the Poisson bipolar process of potential transceivers following the RTS/CTS rule. We consider two types of RTS/CTS thinning, similar to Matérn's hard core model. In type I thinning, a given transceiver pair is kept only when there is no other potential transmitter lying in the exclusion zone of said transceiver pair. In type II thinning, each transceiver pair is endowed with a random mark (time stamp), and a given transceiver pair is kept only when there is no other potential transmitter with a smaller mark lying in the exclusion zone of said transceiver pair. The type I thinning characterizes the scenario where the time is slotted, and the transceiver pairs compete in the previous time slot and terminate the transmissions in the current time slot if a collision is detected, while the type II thinning always retains the transceiver pairs that start earlier than other nearby transceiver pairs. In view of the actual RTS/CTS mechanism, type II thinning better captures the reality, while type I thinning is overly conservative suppressing too many potential transceivers.

C. Mathematical Description

1) Type I Concurrent Transmitter Process: We begin with $\widetilde{\Phi}_a = \{(x_i, \theta_i, e_i)\}$ which is a dependently marked PPP with ground process Φ_a of intensity λ_p on $\mathbb{R}^2$, where

- $\Phi_a = \{x_i\}$ denotes the locations of potential transmitters;
- θ_i denotes the i.i.d. orientation of the receiver for the transmitter located at x_i, uniformly distributed in $[0, 2\pi]$. Having assumed a constant distance of d between a transmitter and its receiver, the orientation θ_i together with x_i uniquely determines the location of the receiver.
- e_i is the medium access indicator.

For $(x_i, \theta_i, e_i) \in \widetilde{\Phi}_a$, let

$$\mathcal{N}(x_i, \theta_i, e_i) = \{(x_j, \theta_j, e_j) \in \widetilde{\Phi}_a : x_j \in B_{x_i}(R_{\mathrm{cs}}) \cup B_{x_i + (d\cos\theta_i, d\sin\theta_i)}(R_{\mathrm{tx}}), j \neq i\} \quad (1)$$

be the set of neighbors of node (x_i, θ_i, e_i), where $B_x(r)$ denotes the disk centered at x with radius r.

The medium access indicator e_i of node (x_i, θ_i, e_i) is a dependent mark defined as

$$e_i = \mathbb{1}\left(\#\mathcal{N}(x_i, \theta_i, e_i) = 0\right), \quad (2)$$

where $\#$ denotes the number of elements in its operand set.

[1]This inequality holds because of the fact that the sensing distance is always larger than the transmitting distance and that the RTS and CTS signals are too short to be sensed at the sensing moment of the transmitters.

Thus, the type I RTS/CTS thinning transforms $\widetilde{\Phi}_a$ into

$$\widetilde{\Phi} := \{(x, \theta) : (x, \theta, e) \in \widetilde{\Phi}_a, e = 1\}, \tag{3}$$

which defines the set of transceivers retained by the thinning procedure. Finally, let Φ consist of the retained transmitters:

$$\Phi := \{x : (x, \theta) \in \widetilde{\Phi}\}. \tag{4}$$

2) Type II Concurrent Transmitter Process: Since transmission attempts happen asynchronously among nodes, the RTS/CTS mechanism operates in chronological order. We assume $\widetilde{\Phi}_a = \{(x_i, \theta_i, m_i, e_i)\}$ to be a dependently marked PPP with ground process Φ_a of intensity λ_p on $\mathbb{R}^2$, where

- $\Phi_a = \{x_i\}$ denotes the locations of potential transmitters;
- θ_i denotes the i.i.d. orientation of the receiver for node x_i, uniformly distributed in $[0, 2\pi]$;
- $\{m_i\}$ are i.i.d. time marks uniformly distributed in $[0, 1]$.
- e_i is the medium access indicator.

For $(x_i, \theta_i, m_i, e_i) \in \widetilde{\Phi}_a$, let

$$\mathcal{N}(x_i, \theta_i, m_i, e_i) = \{(x_j, \theta_j, m_j, e_j) \in \widetilde{\Phi}_a : x_j \in B_{x_i}(R_{\text{cs}}) \cup B_{x_i+(d\cos\theta_i, d\sin\theta_i)}(R_{\text{tx}}), j \neq i\} \tag{5}$$

be the set of neighbors of node $(x_i, \theta_i, m_i, e_i)$.

The medium access indicator e_i of node $(x_i, \theta_i, m_i, e_i)$ is a dependent mark defined as follows:

$$e_i = \mathbb{1}\left(\forall (x_j, \theta_j, m_j, e_j) \in \mathcal{N}(x_i, \theta_i, m_i, e_i), m_i < m_j\right). \tag{6}$$

Thus, the type II RTS/CTS thinning transforms $\widetilde{\Phi}_a$ into

$$\widetilde{\Phi} := \{(x, \theta) : (x, \theta, m, e) \in \widetilde{\Phi}_a, e = 1\}. \tag{7}$$

Finally, let Φ be defined as follows:

$$\Phi := \{x : (x, \theta) \in \widetilde{\Phi}\}. \tag{8}$$

III. Intensity and Mean Interference

In this section, we establish the node intensity and the mean interference experienced by a typical receiver.

Before presenting our results, it is convenient to introduce the following quantities:

- V_o: the area of the exclusion zone of a transceiver pair, given by (see Figure 1)
$$V_o = (\pi - \xi_1) R_{\text{cs}}^2 + (\pi - \xi_2) R_{\text{tx}}^2 + d R_{\text{cs}} \sin \xi_1, \tag{9}$$
where $\xi_1 = \arccos\left(\frac{d^2+R_{\text{cs}}^2-R_{\text{tx}}^2}{2dR_{\text{cs}}}\right)$ and $\xi_2 = \arccos\left(\frac{d^2+R_{\text{tx}}^2-R_{\text{cs}}^2}{2dR_{\text{tx}}}\right)$.
- $S_1 = \{(r, \beta, \theta) : r \leq R_{\text{cs}}\}$, $S_2 = \{(r, \beta, \theta) : r^2 - 2rd\cos\beta + d^2 \leq R_{\text{tx}}^2\}$, $S_3 = \{(r, \beta, \theta) : r^2 + 2rd\cos(\beta - \theta) + d^2 \leq R_{\text{tx}}^2\}$. Consider two transmitters of distance r apart with phase angle difference β, and orientation marks 0 and θ, respectively; see Figure 2. Then, when the triplet (r, β, θ) is taken from S_1, the two transmitters will be within the physical sensing cleaned region of each other, and when the triplet (r, β, θ) is in the set S_2 or S_3, at least one of the two transmitters will be within the RTS/CTS cleaned region of the other.

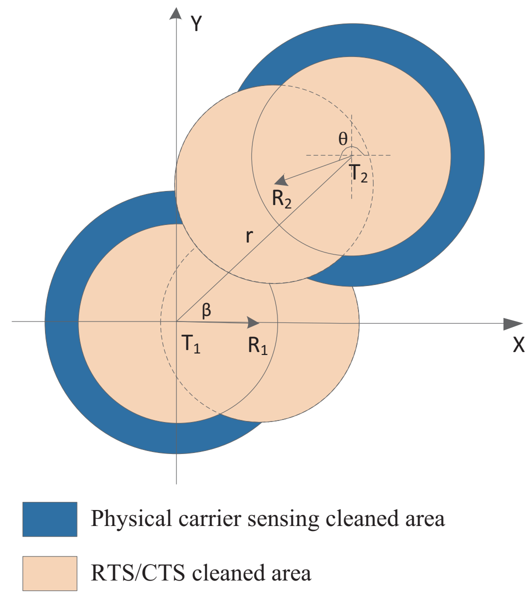

Fig. 2. Illustration of two transceiver pairs.

- $V(r, \beta, \theta)$: the area of the union of the protection zone (both the physical carrier sensing cleaned region and the RTS/CTS cleaned region) of two transceiver pairs, whose transmitters are at distance r with relative phase difference β, and whose receivers are at orientations 0 and θ respectively; see Figure 2.

A. Type I Concurrent Transmitter Process

1) Node Intensity: To characterize the node intensity and the mean interference, we apply the notion of Palm measure. Consider a stationary point process Ψ with finite non-zero intensity λ. Note that Ψ is a random variable taking values in the measurable space $(\mathbb{M}, \mathcal{M})$, where $\mathbb{M}$ is the set of all possible simple point patterns, $\mathcal{M}$ is a $\sigma-$algebra defined on $\mathbb{M}$. If the measurable space is endowed with a probability measure P, the Palm distribution of a point process is

$$P_o(Y) = \frac{1}{\lambda v_d(B)} \int_{\mathbb{M}} \sum_{x \in \psi \bigcap B} \mathbb{1}_Y(\psi_{-x}) P(\mathrm{d}\psi), \quad Y \in \mathcal{M} \tag{10}$$

where v_d is the Lebesgue measure, B is an arbitrary Borel set of positive Lebesgue measure, and $\psi_{-x} = \{y \in \psi : y - x\}$; see [15] for the general theory.

Considering the type I concurrent transmitter process, the retained Φ is a dependent thinning of the original PPP Φ_a. Let $P_o^{(\Phi_a)}$ be the Palm measure of Φ_a. Since the intensity of a stationary point process is defined as the ratio of the expected number of points in a given region to the Lebesgue measure of that region, by combining the definition of Palm measure given by (10), we get the intensity of the type I concurrent transmitter process as

$$\lambda = \lambda_p P_o^{(\Phi_a)}(o \in \Phi) = \lambda_p e^{-\lambda_p V_o}. \tag{11}$$

Intuitively, the intensity of the type I concurrent transmitter process is equal to the intensity of the original PPP of transmitters λ_p multiplied by the void probability of the original PPP in the RTS/CTS exclusion zone V_o. From (11), we notice that if λ_p is small, λ increases with λ_p, and when λ_p is large, λ starts to decrease. In the extreme case as $\lambda_p \to \infty$, $\lambda \to 0$. This shows that the type I RTS/CTS thinning is overly conservative suppressing too many potential transceivers. The maximum of λ occurs when $\lambda_p = 1/V_o$.

2) Mean Interference: In this subsection, we derive the mean interference of a typical receiver in the type I concurrent transmitter process. Without loss of generality, we focus on the typical transceiver pair with the transmitter at the origin o and the receiver at $z_o = (d\cos\theta_o, d\sin\theta_o)$, i.e., we condition on $(o,\theta_o) \in \widetilde{\Phi}$. When calculating the interference I_{z_o}, the typical point (o,θ_o) is not included because the signal from this pair of transceiver does not constitute interference. This means that the probability measure used in the analysis of interference is the *reduced* Palm measure $P^!_{(o,\theta_o)}$ of the marked point process $\widetilde{\Phi}$. The corresponding expectation is denoted as $E^!_{(o,\theta_o)}$. In fact, the mean interference does not rely on the value of θ_o, which means that $E^!_{(o,\theta_o)}(I_{z_o}) = E^!_{(o,0)}(I_{z_o})$. We condition on θ_o because it is necessary for the derivation and also will bring in more general intermediate results.

Theorem 1. *The mean interference experienced by a typical receiver in type I concurrent transmitter process is*

$$E^!_{(o,0)}(I_{z_o}) = \frac{\lambda_p^2 P_t}{2\pi\lambda}\int_0^\infty\int_0^{2\pi}\int_0^{2\pi} l(\sqrt{r^2-2rd\cos\beta+d^2})k(r,\beta,0,\theta)r\mathrm{d}\theta\mathrm{d}\beta\mathrm{d}r \quad (12)$$

where $k(r,\beta,0,\theta)$ is given by

$$k(r,\beta,0,\theta) = \begin{cases} 0 & (r,\beta,\theta)\in S_1\bigcup S_2\bigcup S_3 \\ \exp(-\lambda_p V(r,\beta,\theta)) & \text{otherwise.}\end{cases} \quad (13)$$

Proof. Let $\widetilde{\mathbb{M}}$ be the set of all marked point processes. The interference experienced by the receiver located at z_o is

$$\begin{aligned} E^!_{(o,\theta_o)}(I_{z_o}) &= E^!_{(o,\theta_o)}\Big(\sum_{(x,\theta)\in\widetilde{\Phi}} P_t l(|x-z_o|)\Big) \\ &= \int_{\widetilde{\mathbb{M}}}\Big(\int_{\mathbb{R}^2\times[0,2\pi]} P_t l(|x-z_o|)\widetilde{\varphi}(\mathrm{d}(x,\theta))\Big)P^!_{(o,\theta_o)}(\mathrm{d}\widetilde{\varphi}) \\ &= \int_{\mathbb{R}^2\times[0,2\pi]} P_t l(|x-z_o|)\int_{\widetilde{\mathbb{M}}}\Big(\widetilde{\varphi}(\mathrm{d}(x,\theta))P^!_{(o,\theta_o)}(\mathrm{d}\widetilde{\varphi})\Big) \\ &= \lambda P_t\int_{\mathbb{R}^2\times[0,2\pi]} l(|x-z_o|)\mathcal{K}_{\theta_o}(\mathrm{d}(x,\theta)). \end{aligned} \quad (14)$$

Similar to the case of unmarked point processes, we define $\mathcal{K}_{\theta_o}(B\times L), B\subset\mathbb{R}^2, L\subset[0,2\pi]$, as the reduced second-order factorial measure of the marked point process of $B\times L$. Intuitively, $\lambda\mathcal{K}_{\theta_o}(B\times L)$ is the expected number of points located in B with marks taking values in L under the condition that $(o,\theta_o)\in\widetilde{\Phi}$, and its mathematical description is given by

$$\mathcal{K}_{\theta_o}(B\times L) = \frac{1}{\lambda}\int_{\widetilde{\mathbb{M}}}\widetilde{\varphi}(B\times L)P^!_{(o,\theta_o)}(\mathrm{d}\widetilde{\varphi}). \quad (15)$$

Next we focus on the evaluation of $\mathcal{K}_{\theta_o}(B\times L)$.

The second-order factorial moment measure of the marked point process $\widetilde{\Phi}$ with probability measure P is [16, p. 114]

$$\begin{aligned} &\alpha^{(2)}(B_1\times B_2\times L_1\times L_2) \\ &= E\Bigg(\sum^{\neq}_{(x_1,\theta_1),(x_2,\theta_2)\in\widetilde{\Phi}}\mathbb{1}_{B_1}(x_1)\mathbb{1}_{B_2}(x_2)\mathbb{1}_{L_1}(\theta_1)\mathbb{1}_{L_2}(\theta_2)\Bigg) \\ &= \int_{\widetilde{\mathbb{M}}}\sum_{(x,\theta)\in\widetilde{\varphi}}\mathbb{1}_{B_1}(x)\mathbb{1}_{L_1}(\theta)\widetilde{\varphi}(B_2\times L_2\setminus\{(x,\theta)\})P(\mathrm{d}\widetilde{\varphi}), \end{aligned} \quad (16)$$

where $\sum^{\neq}$ means that the summation is taken over all pairs of points $(x_1,\theta_1)\in\widetilde{\Phi}, (x_2,\theta_2)\in\widetilde{\Phi}$ with $(x_1,\theta_1)\neq(x_2,\theta_2)$.

Letting $h(x,\theta,\widetilde{\varphi}) = \mathbb{1}_{B_1}(x)\mathbb{1}_{L_1}(\theta)\widetilde{\varphi}(B_2\times L_2)$ and plugging into (16), we get

$$\begin{aligned} &\alpha^{(2)}(B_1\times B_2\times L_1\times L_2) \\ &= \int_{\widetilde{\mathbb{M}}}\sum_{(x,\theta)\in\widetilde{\varphi}} h\big(x,\theta,\widetilde{\varphi}\setminus\{(x,\theta)\}\big)P(\mathrm{d}\widetilde{\varphi}) \\ &\overset{(a)}{=} \frac{\lambda}{2\pi}\int_{\mathbb{R}^2\times[0,2\pi]}\int_{\widetilde{\mathbb{M}}} h\big(x,\theta,\widetilde{\varphi}\big)P^!_{(x,\theta)}(\mathrm{d}\widetilde{\varphi})\mathrm{d}(x,\theta) \\ &= \frac{\lambda}{2\pi}\int_{\mathbb{R}^2\times[0,2\pi]}\int_{\widetilde{\mathbb{M}}}\mathbb{1}_{B_1}(x)\mathbb{1}_{L_1}(\theta) \\ &\qquad \widetilde{\varphi}(B_2\times L_2)P^!_{(x,\theta)}(\mathrm{d}\widetilde{\varphi})\mathrm{d}(x,\theta) \\ &= \frac{\lambda}{2\pi}\int_{\mathbb{R}^2\times[0,2\pi]}\int_{\widetilde{\mathbb{M}}}\mathbb{1}_{B_1}(x)\mathbb{1}_{L_1}(\theta) \\ &\qquad \widetilde{\varphi}((B_2-x)\times L_2)P^!_{(o,\theta)}(\mathrm{d}\widetilde{\varphi})\mathrm{d}(x,\theta) \\ &= \frac{\lambda^2}{2\pi}\int_{B_1\times L_1}\mathcal{K}_\theta((B_2-x)\times L_2)\mathrm{d}(x,\theta). \end{aligned} \quad (17)$$

where (a) follows from the refined Campbell theorem.

Consider the second-order product density $\varrho^{(2)}$ defined as follows [16, p. 111]

$$\begin{aligned} &\alpha^{(2)}(B_1\times B_2\times L_1\times L_2) \\ &= \int_{B_1\times B_2\times L_1\times L_2}\varrho^{(2)}(x_1,x_2,\theta_1,\theta_2)\mathrm{d}(x_1,x_2,\theta_1,\theta_2) \\ &\overset{(b)}{=} \int_{B_1\times L_1}\Bigg(\int_{(B_2-x_1)\times L_2}\varrho^{(2)}(x_2,\theta_1,\theta_2)\mathrm{d}(x_2,\theta_2)\Bigg)\mathrm{d}(x_1,\theta_1) \end{aligned} \quad (18)$$

where (b) follows from the fact that for a stationary point process, $\varrho^{(2)}(x_1,x_2,\theta_1,\theta_2)$ depends only on x_2-x_1, θ_1 and θ_2. By comparing (17) and (18), we get

$$\mathcal{K}_{\theta_o}(B\times L) = \frac{2\pi}{\lambda^2}\int_{B\times L}\varrho^{(2)}(x,\theta_o,\theta)\mathrm{d}(x,\theta), \quad (19)$$

whose differential form is

$$\mathcal{K}_{\theta_o}(\mathrm{d}(x,\theta)) = \frac{2\pi}{\lambda^2}\varrho^{(2)}(x,\theta_o,\theta)\mathrm{d}(x,\theta). \quad (20)$$

Plugging (20) into (14), we get

$$E^!_{(o,\theta_o)}(I_{z_o}) = \frac{2\pi P_t}{\lambda}\int_{\mathbb{R}^2\times[0,2\pi]} l(|x-z_o|)\varrho^{(2)}(x,\theta_o,\theta)\mathrm{d}(x,\theta). \quad (21)$$

At this point, without loss of generality, we assume $\theta_o = 0$; that is, the typical receiver is located at $z_o = (d, 0)$. The interference experienced at z_o is given by

$$E^!_{(o,0)}(I_{z_o}) = \frac{\lambda_p^2 P_t}{2\pi\lambda} \int_{\mathbb{R}^2\times[0,2\pi]} l(|x - z_o|)k(x, 0, \theta)\mathrm{d}(x, \theta), \tag{22}$$

which, when written in polar form, is

$$E^!_{(o,0)}(I_{z_o}) = \frac{\lambda_p^2 P_t}{2\pi\lambda} \int_0^\infty \int_0^{2\pi} \int_0^{2\pi} l(\sqrt{r^2 - 2rd\cos\beta + d^2})k(r, \beta, 0, \theta)r\mathrm{d}\theta\mathrm{d}\beta\mathrm{d}r. \tag{23}$$

The function $k(r, \beta, 0, \theta)$ denotes the probability that two transmitters of distance r apart and with phase angle difference β, the orientation marks of which are 0 and θ, respectively, are both retained (see Figure 2). The value of $k(r, \beta, 0, \theta)$ is then obtained from their geometric relationship and is given by (13): when the triplet (r, β, θ) is in any one set of S_1, S_2 or S_3, the two transceiver pairs are within their guard zone of the RTS/CTS mechanism and thus neither of them is retained; otherwise, these two pairs are retained if and only if there is no other potential transmitter within the exclusion region of area $V(r, \beta, \theta)$, which occurs with probability $\exp(-\lambda_p V(r, \beta, \theta))$ since Φ_a is a homogenous PPP with intensity λ_p. □

B. Type II Concurrent Transmitter Process

1) Node Intensity: In the type II concurrent transmitter process, a transceiver pair with time stamp mark t leads to the removal of those transmitters lying in its induced guard zone and having time stamp mark larger than t. Conditioned upon t, the transceiver pairs with time stamp mark larger than t form an independently thinned version of the original PPP Φ_a. According to (11), the probability of retaining a given transceiver pair with given time stamp mark t is $e^{-\lambda_p t V_o}$. By averaging over t, we obtain the intensity of type II concurrent transmitter process:

$$\lambda = \lambda_p \int_0^1 e^{-\lambda_p t V_o}\mathrm{d}t = \frac{1}{V_o}(1 - e^{-\lambda_p V_o}), \tag{24}$$

where V_o is given by (9). From (24) we notice that λ monotonically increasing with λ_p, and as $\lambda_p \to \infty$, λ tends toward a constant limit $1/V_o$.

2) Mean Interference: In parallel with Theorem 1, the following theorem gives the mean interference for type II concurrent transmitter process.

Theorem 2. *The mean interference experienced by a typical receiver in type II concurrent transmitter process is*

$$E^!_{(o,0)}(I_{z_o}) = \frac{\lambda_p^2 P_t}{2\pi\lambda} \int_0^\infty \int_0^{2\pi} \int_0^{2\pi} l(\sqrt{r^2 - 2rd\cos\beta + d^2})k(r, \beta, 0, \theta)r\mathrm{d}\theta\mathrm{d}\beta\mathrm{d}r \tag{25}$$

where $k(r, \beta, 0, \theta)$ is given by

$$k(r, \beta, 0, \theta) = \begin{cases} 0 & (r, \beta, \theta) \in S_1 \bigcup (S_2 \bigcap S_3) \\ 2\eta(V) & (r, \beta, \theta) \in \overline{S_1} \bigcap \overline{S_2} \bigcap \overline{S_3} \\ \eta(V) & \text{otherwise.} \end{cases} \tag{26}$$

in which V is the abbreviation of $V(r, \beta, \theta)$, and $\eta(V)$ is

$$\eta(V) = \frac{V_o e^{-\lambda_p V} - V e^{-\lambda_p V_o} + V - V_o}{\lambda_p^2 (V - V_o) V V_o}. \tag{27}$$

Proof. The derivation of (25) follows the same line as that of the type I concurrent transmitter process in Theorem 1 except for the difference in the probability that both of the transceiver pairs are retained, namely, $k(r, \beta, 0, \theta)$, which is the probability that two transceiver pairs are both retained, is different from the type I concurrent transmitter process. since the time stamp marks are taken into consideration. We should consider not only the geometric relationship (see Figure 2), but also the relationship between the time stamp marks of the two transceiver pairs. When $(r, \beta, \theta) \in S_1 \bigcup (S_2 \bigcap S_3)$, at least one of the considered two transceiver pairs has to be removed after comparing their time stamp marks and thus $k(r, \beta, 0, \theta)$ is zero. When $(r, \beta, \theta) \in \overline{S_1} \bigcap \overline{S_2} \bigcap \overline{S_3}$, there is no direct comparison between the time stamp marks of the considered two transceiver pairs. Let their time stamp marks be t_1 and t_2 respectively. So by separately considering the cases of $t_1 \geq t_2$ and $t_1 < t_2$, we get

$$\begin{aligned} k(r, \beta, 0, \theta) &= \int_0^1 e^{-\lambda_p t_1 V_o} \left[\int_0^{t_1} e^{-\lambda_p t_2 (V - V_o)} \mathrm{d}t_2 \right] \mathrm{d}t_1 \\ &+ \int_0^1 e^{-\lambda_p t_2 V_o} \left[\int_0^{t_2} e^{-\lambda_p t_1 (V - V_o)} \mathrm{d}t_1 \right] \mathrm{d}t_2 \\ &= 2 \int_0^1 e^{-\lambda_p t_1 V_o} \left[\int_0^{t_1} e^{-\lambda_p t_2 (V - V_o)} \mathrm{d}t_2 \right] \mathrm{d}t_1, \end{aligned} \tag{28}$$

which can be evaluated as $2\eta(V)$. Here, $\int_0^{t_1} e^{-\lambda_p t_2 (V - V_o)} \mathrm{d}t_2$ is the conditional probability that, when the first transceiver pair is marked by t_1, there is no other transceiver pair of mark smaller than $t_2 < t_1$ lying in the region covered by the second transceiver pair only; $\int_0^{t_2} e^{-\lambda_p t_1 (V - V_o)} \mathrm{d}t_1$ is analogously interpreted. When either $(r, \beta, \theta) \in \overline{S_1} \bigcap S_2 \bigcap \overline{S_3}$ or $(r, \beta, \theta) \in \overline{S_1} \bigcap \overline{S_2} \bigcap S_3$, exactly one of the considered transmitter is within the RTS/CTS cleaned region of the other, and hence only one of the terms in (28) should be taken into account, resulting into $k(r, \beta, 0, \theta) = \eta(V)$. □

IV. Numerical Results

The numerical results are obtained from the analytical results we have derived. The default configurations of system model are as follows. Unless otherwise specified, the intensity of potential transmitters is set as $\lambda_p = 8 \times 10^{-7}\mathrm{m}^{-2}$ and the virtual carrier sensing radius is $R_{\mathrm{tx}} = 100\mathrm{m}$. The physical carrier sensing radius is always set as $R_{\mathrm{cs}} = 1.2R_{\mathrm{tx}}$ and the distance between transmitter and receiver is set as $d = 0.8R_{\mathrm{tx}}$. The transmit power is set as $P_t = 15$ dBm, and the path loss model is $l(r) = Ar^{-\alpha}$ with $\alpha = 4$ and $A = 0.0001$ (-40 dB).

Figure 3 displays the mean interference of the type I and II concurrent transmitter processes as a function of the virtual sensing radius R_{tx} when the intensity of the original PPP is fixed as $\lambda_p = 8 \times 10^{-7}\mathrm{m}^{-2}$. From the curves, we observe that at the same R_{tx}, the mean interference of the type I process is always lower than that of the type II process because under the same λ_p, the intensity of the type I process is always smaller than that of the type II process. In view of the actual RTS/CTS mechanism, type II thinning better captures the reality, while type I thinning is overly conservative suppressing too many potential transceivers.

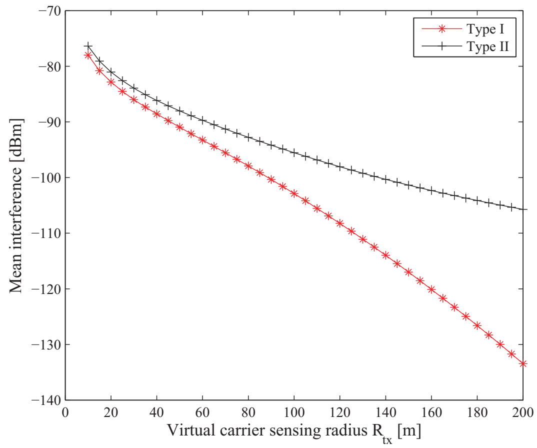

Fig. 3. The mean interference as a function of the virtual sensing radius R_{tx}.

Figure 4 displays the mean interference of the type I and II concurrent transmitter processes as a function of the intensity of the original PPP λ_p. From the curves, we observe that the mean interference of the type I process first increases with λ_p and then decreases after the mean interference reaches its maximum. To interpret this behavior, we note that as the intensity λ_p begins to increase, more concurrent transmissions occur and cause more interference; however as λ_p exceeds a certain threshold, the carrier sensing mechanism starts to play a significant role in diminishing the number of active transmissions. In the extreme case, when $\lambda_p \to \infty$ the number of active transmissions goes to zero. The curve of the type II concurrent transmitter process shows that the mean interference increases initially and then tends to be stable, because the network becomes saturated when there are a large number of potential transmitters.

V. Conclusion

In this paper, we proposed marked point process models to characterize the spatial distribution of transceivers under the RTS/CTS handshake mechanism in WLAN, focusing on the evaluation of the mean interference experienced by a typical receiver. Our analysis revealed how the mean interference under RTS/CTS varies with system parameters such as the carrier ranges and the density of transceiver nodes. The method employed in our work may be applicable to wireless networks modeled by more sophisticated kinds of access mechanisms.

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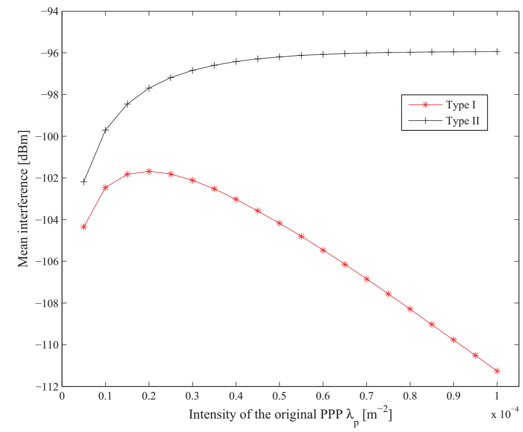

Fig. 4. The mean interference as a function of the intensity of the original PPP λ_p.

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