Optimal Base Station Density for Power Efficiency in Cellular Networks

Sanglap Sarkar, Radha Krishna Ganti
Dept. of Electrical Engineering
Indian Institute of Technology
Chennai, 600036, India
Email:{ee11s053, rganti}@ee.iitm.ac.in

Martin Haenggi
Dept. of Electrical Engineering
University of Notre Dame
Notre Dame, IN 46556, USA
Email: mhaenggi@nd.edu

***Abstract*—In cellular networks, cell size reduction is an important technique for improving the spectral reuse and achieving higher data rates. In addition, it results in power savings as it leads to a decrease in transmit power. However, it is not clear if the transmit power can be indefinitely decreased with the cell sizes. In this paper, we analyze the impact of transmit power reduction (cell size reduction) on the performance of the network. More precisely, we obtain a lower bound on the transmit power such that a minimum coverage and a minimum data rate can be guaranteed. We then analyze the *area power consumption* metric, which denotes the total power consumed per unit area. Under the constraints of target coverage and target data rate, the area power consumption is minimized and the optimal base station density is obtained. For a path loss exponent $\alpha > 4$, we observe the existence of a minimum cell size below which shrinking the cell would result in an overall increase of power. However, for $\alpha \leq 4$, there exists no such optimal cell-size, as the area power consumption *increases* with base station density.**

***Index Terms*—Small cells, green communication, cell size, quality of service, power consumption, power efficiency, optimal base station density.**

I. Introduction

Cell size reduction provides increased spectral reuse and increased data rates to mobile users. As the cell size decreases, the number of users per base station (BS) decreases leading to a greater bandwidth (or time share) per user. Cell size reduction can be achieved by increasing the density of the base stations, either by increasing the number of macro base stations or adding tiers of low powered base stations. There are two advantages of cell size reduction. Firstly, increased bandwidth per user. Secondly, lower transmit power since the mobile user is much closer to a base station.

In this paper, we investigate if the downlink transmit power can be decreased arbitrarily by increasing the density of base stations for a given target rate and coverage. It turns out that after a certain power threshold, noise plays a significant role on both coverage and rate.

For $\alpha > 4$, we obtain an expression for the optimum base station density which minimizes area power consumption and maximizes power efficiency[1] under target rate and coverage constraints. If the cell density exceeds an optimal threshold the total power consumption increases. The optimum density gives us a limit up to which cell size can be shrunk and power savings can be achieved. It also indicates when operator should stop further deployment of small cells in this regime. For $\alpha \leq 4$, we observe that increasing the base station density leads to an overall increase of power.

[1] Power efficiency is defined as inverse of the area power consumption. We call the network to be power efficient if the area power consumption decreases with increase of base station density.

II. System Model and problem formulation

A. System Model

In this paper, we assume a single tier of BSs and focus on the downlink. The locations of the base stations are modeled by a spatial Poisson point process (PPP) [1] Φ_b of density λ_b. We assume that users forms a stationary point process with density $\lambda_u \geq \lambda_b$ that is independent of the base station process. Since the users are uniformly distributed, the average number of users per cell is λ_u/λ_b. We assume a path loss function $l(x) = K\|x\|^{-\alpha}$, $\alpha > 2$, where K is a constant chosen[2] as 3.2×10^{-6}. Without loss of generality, we focus on a mobile user at the origin for computing coverage and rate. We assume that each mobile user is served by its closest BS. We also assume that all BSs transmit with the same power P.

The bandwidth (BW) per user is a random variable that depends on the cell size and the number of users in the cell. However, for ease of analysis we set it to be equal to the total available bandwidth (B) divided by the average number of users, *i.e.*, $B_u(\lambda_b) = B\frac{\lambda_b}{\lambda_u}$. Hence the average rate per user is

$$R = B_u(\lambda_b)\mathbb{E}[\ln(1 + \mathtt{SINR})].$$

Since the user BW depends on the densities, the noise power is $FkTB_u(\lambda_b)$, where F is the receiver noise figure, k is the Boltzmann constant, and T is the ambient temperature.

The small scale fading is denoted by $\mathtt{h}_x$ which is assumed to follow an exponential distribution (square of Rayleigh fading) with unit mean. The signal-to-interference-plus-noise ratio for

[2] A typical femtocell covers a range of 40 m with a transmit power of 0.2 W [2]. The received power at the cell boundary should be at least equal to the receiver sensitivity, which is -80 dBm. So $r = 40$, $\alpha = 3$, transmit power $P_t = 0.2$, received power $P_r = 10^{\frac{-80-30}{10}} = K \cdot P_t \cdot r^{-\alpha}$. Hence, $K = 3.2 \times 10^{-6}$.

the mobile user at the origin is

$$\mathtt{SINR} = \frac{\mathtt{h}_x \|x\|^{-\alpha}}{\frac{\sigma^2}{P} + \sum_{y \in \Phi_b \setminus \{x\}} \mathtt{h}_y \|y\|^{-\alpha}},$$

where $\sigma^2 = \lambda_b FkTB/(\lambda_u K)$, and $x \in \Phi_b$ is the BS closest to the origin. The coverage probability is defined as $\Delta(P) = \mathbb{P}(\mathtt{SINR} > \theta)$, and the spectral efficiency is defined as $\tau(P) = \mathbb{E}[\ln(1 + \mathtt{SINR})]$.

B. Problem Formulation

With BSs transmitting at power P, we define the area transmit power consumption as $\lambda_b P$. In addition to the transmission power, each BS requires a fixed amount of operational power which we denote by P_0. The power P_0 includes the fixed operational power of the hardware and also the transmit power of the pilot tones. Hence the area power consumption is $\lambda_b(P + P_0)$. When the transmit power is infinity, it is shown in [3] that the SINR distribution and hence the coverage probability and spectral efficiency does not depend on the BS density. The rate demanded by each user is denoted by R_T; it is independent of the base station density. The interference-limited spectral efficiency, corresponding to $P = \infty$, is $\tau(\infty)$. It is independent of the base station density and depends only on path loss exponent α. So, irrespective of the transmit power, the maximum achievable rate for a particular density λ_b is given by $\tau(\infty)B_u(\lambda_b)$. The user demand rate can be satisfied only if it is less than the maximum achievable rate. But if the user demand exceeds the maximum achievable rate, then the best the operator can do is to support a data rate of $\tau(\infty)B_u(\lambda_b)$. So, the operator must satisfy the target rate or maximum achievable rate, whichever is minimum, with a maximum rate outage δ.

The limiting coverage probability that can be achieved for a given SINR threshold θ and path loss exponent α is given by $\Delta(\infty)$. So, the operator must provide $\Delta(\infty)$ coverage with a maximum outage of ϵ. Our goal is to obtain the optimal parameters λ_b^* and transmit power P so that the area power consumption $\lambda_b(P + P_0)$ is minimal while maintaining the QoS constraints of rate and coverage. Formally,

$$\begin{aligned} &\lambda_b^* = \arg\min_{\lambda_b} \{\lambda_b(P + P_0)\} \\ &\text{s.t.} \qquad \Delta(P) \geq (1-\epsilon)\Delta(\infty), && (1) \\ &\qquad\qquad R(P) \geq (1-\delta)\min\{R_T, \tau(\infty)B_u(\lambda_b)\}. && (2) \end{aligned}$$

We consider both the rate and coverage constraints for the following two reasons:

- Even though $B_u(\lambda_b)\ln(1 + \mathtt{SINR})$ might be high, $\mathtt{SINR}$ might be low. This might be because of a large BW B.
- Because of the inclusion of R_T, the lower bound on transmit power obtained from the two constraints, may scale differently with the BS density as can be seen in Lemma 1 and Lemma 2 and Fig. 3 and Fig. 6.

C. Prior Work

Energy efficiency in HetNets has received increasing attention recently. Mostly only interference-limited networks have been considered [4], [5]. Different energy saving techniques in communication networks, have been presented in [6]. Energy efficiency in cellular networks has also been analyzed in [7],[8]. However, the authors have focused on a hexagonal grid model with fixed downlink transmit power. In [4], an optimal base-station density for both homogeneous and heterogeneous cellular networks has been obtained under the constraint of target rate. However, noise has been ignored. Coverage constraints have not been considered, either.Energy efficiency in HetNets has been studied in [5], which provides an optimal macro-pico density ratio that maximizes the overall energy efficiency under fixed noise assumption (*i.e.*, noise power does not vary with the allocated bandwidth).

In this paper, we consider noise and both target coverage constraints and target rate constraints. We also show how the transmit power has to be scaled down with increase of base station density.

III. Optimal power for target coverage and rate

A. Minimum transmit power for coverage

As the BS density increases, the transmit power of the base stations may be decreased because of the decreasing cell size. However, reducing the transmit power, decreases the coverage probability because of the noise. See Fig. 1. In the next lemma we evaluate the transmit power required to achieve $(1-\epsilon)\Delta(\infty)$ coverage.

Lemma 1. *The minimum downlink transmit power for which a minimum* $\mathtt{SINR}$ *of* θ *can be guaranteed for a fraction* $1-\epsilon$ *of the users is given by*

$$P_c^* \approx \frac{k_1}{\lambda_b^{\alpha/2-1}}, \tag{3}$$

where $k_1 = \frac{\theta\sigma^2\Gamma(\alpha/2+1)}{\epsilon\pi^{\alpha/2}[1+\rho(\theta,\alpha)]^{\alpha/2}}$.

Proof: The coverage probability without noise (or infinite power) is [3]

$$\Delta(\infty) = (1 + \rho(\theta, \alpha))^{-1}, \tag{4}$$

where $\rho(\theta,\alpha) = \theta^{2/\alpha}\int_0^\infty \frac{1}{1+x^{\alpha/2}}dx$. The coverage probability with noise is [3]

$$\Delta(P) = \pi\lambda_b \int_0^\infty e^{-\pi\lambda_b(1+\rho(\theta,\alpha))v - \theta\sigma^2\lambda_b v^{\alpha/2}P^{-1}} dv. \tag{5}$$

Now, using the substitution $\theta\sigma^2P^{-1} \to s$ in (5) and the approximation $e^{-sv^{\alpha/2}} \approx (1 - sv^{\alpha/2})$ for small s (or, equivalently, small σ^2), we get

$$\Delta(P) \approx \Delta(\infty)\left(1 - \frac{\theta\sigma^2\Gamma(\alpha/2+1)}{P\lambda_b^{\alpha/2-1}[\pi(\rho(\theta,\alpha)+1)]^{\alpha/2}}\right), \tag{6}$$

where $\Gamma(x) = \int_0^\infty t^{x-1}e^{-t}dt$ is the standard gamma function. This approximation (linearization of the exponential term) is

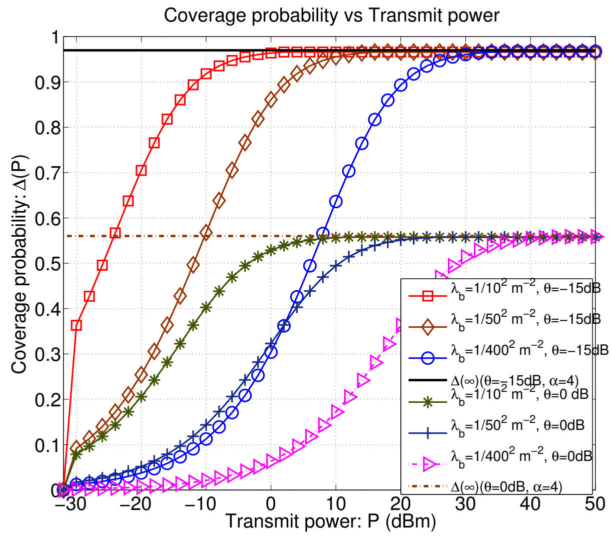

Fig. 1: Effect of transmit power on coverage probability: User density $\lambda_u = 0.01$ m^{-2}, $\alpha = 4$ and $\sigma^2 = \lambda_b \times 1.035 \times 10^{-5}$ W.

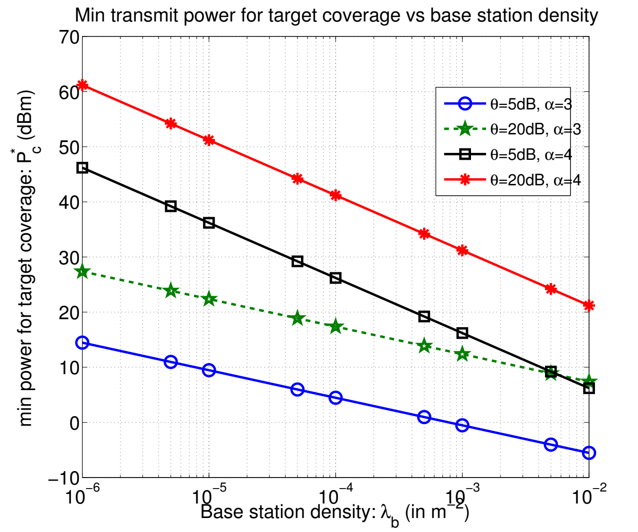

Fig. 3: Minimum transmit power for target coverage: User density $\lambda_u = 0.01$ m^{-2}, Outage probability $\epsilon = 0.25$ and $\sigma^2 = \lambda_b \times 1.035 \times 10^{-5}$ W.

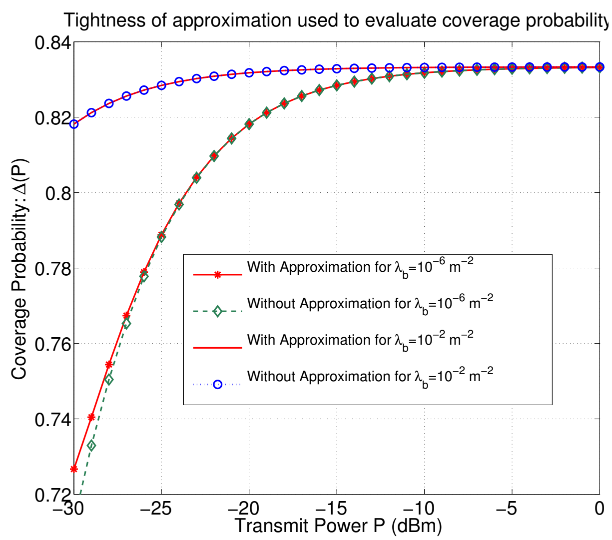

Fig. 2: Tightness of numerical approximation $e^{-(\theta\sigma^2 P^{-1})v^{\alpha/2}} \approx (1 - \theta\sigma^2 P^{-1} v^{\alpha/2})$ used to evaluate coverage probability: SINR threshold θ = -30dB, User density $\lambda_u = 0.01$ m^{-2}, $\alpha = 3$ and $\sigma^2 = \lambda_b \times 1.035 \times 10^{-5}$ W.

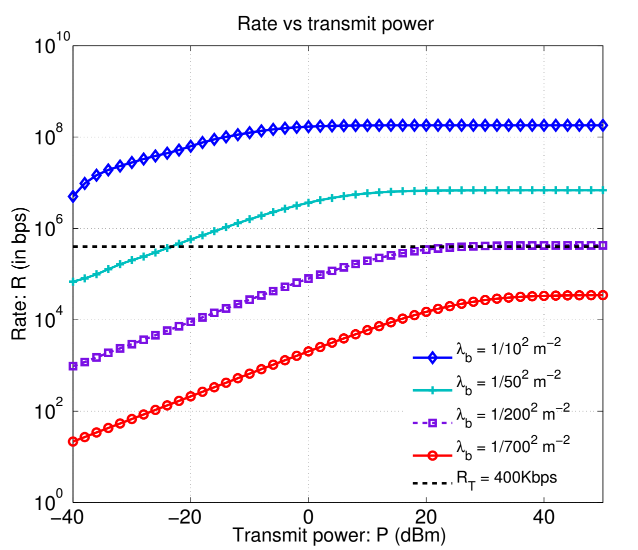

Fig. 4: Effect of noise on rate: User density $\lambda_u = 0.01$ m^{-2}, $\alpha = 4.5$ and $\sigma^2 = \lambda_b \times 1.035 \times 10^{-5}$ W.

very tight, as can be seen from Fig. 2. The lower bound P_c^* on transmit power P can be obtained by combining (4) and (6) with the coverage constraint (1). ∎

With increasing base station density, the operator can scale down the transmit power at a rate proportional to $\lambda_b^{1-\alpha/2}$ (as shown in Fig. 3), while guaranteeing a certain minimum coverage.

B. Minimum transmit power for target data rate

The effect of noise on rate is shown in Fig. 4. The transmit power can decrease with increasing BS density. However, a minimum transmit power is required to combat the noise and provide average rate to satisfy the user demand or the maximum achievable rate, whichever is minimum. This constraint imposes a limit on the downlink transmit power, which is provided in the next lemma.

Lemma 2. *The minimum downlink transmit power that achieves a fraction $1 - \delta$ of the minimum of the target rate R_T and maximum supported rate $\tau(\infty)B_u(\lambda_b)$ is given by*

$$P_r^* \approx \frac{\min\{h(\lambda_b), k_2\}}{\lambda_b^{\alpha/2-1}}, \tag{7}$$

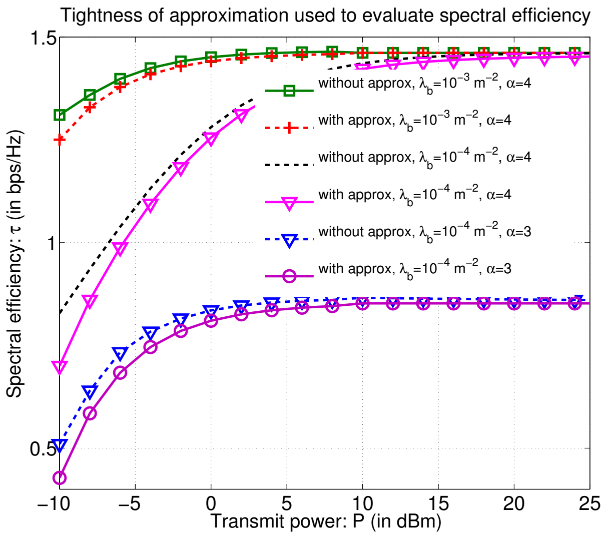

Fig. 5: Tightness of numerical approximation $e^{-\frac{\sigma^2}{P}r^{\alpha}(e^t-1)} \approx [1-\frac{\sigma^2}{P}r^{\alpha}(e^t-1)]$ used to evaluate spectral efficiency: User density $\lambda_u = 0.01\text{m}^{-2}$, noise power $\sigma^2 = \lambda_b \times 1.035 \times 10^{-5}$ W.

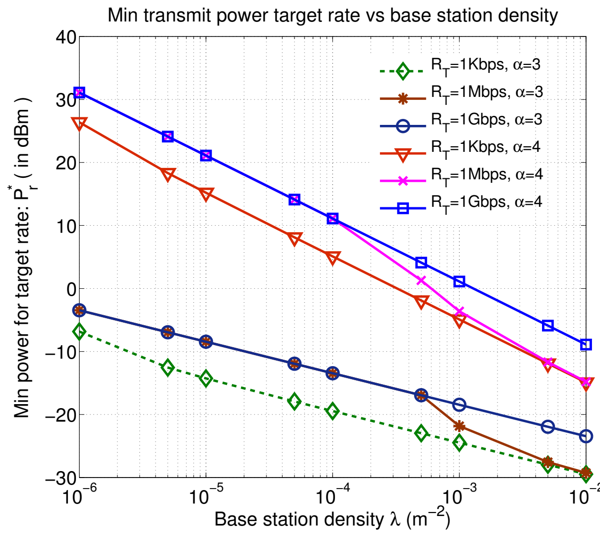

Fig. 6: Minimum transmit power for target rate: User density $\lambda_u = 0.01$ m^{-2}, total bandwidth $B = 20$ MHz, Outage probability $\delta = 0.25$ and noise power $\sigma^2 = \lambda_b \times 1.035 \times 10^{-5}$ W.

where $k_2 = \frac{\sigma^2 g(\alpha)}{\delta\tau(\infty)}$, $h(\lambda_b) = \frac{\sigma^2 g(\alpha)}{[\tau(\infty)-(1-\delta)\{\frac{R_T\lambda_u}{B\lambda_b}\}]}$, *and*

$$g(\alpha) = \pi^{-\alpha/2}\int_{z>0} e^{-z}z^{\alpha/2}\cdot \int_{t>0}(e^t-1)e^{-z\{(e^t-1)^{2/\alpha}\int_{(e^t-1)^{-2/\alpha}}^{\infty}\frac{1}{1+x^{\alpha/2}}\mathrm{d}x\}}\mathrm{d}t\mathrm{d}z.$$

Proof: The spectral efficiency $\tau(P)$ has been derived in [3] as

$$\tau(P) \triangleq \mathbb{E}[\ln(1+\mathtt{SINR})] = \int_{r>0} e^{-\pi\lambda_b r^2}\int_{t>0} e^{-\frac{\sigma^2\lambda_b}{P}r^{\alpha}(e^t-1)}\mathcal{L}_{I_r}(r^{\alpha}(e^t-1))\mathrm{d}t2\pi\lambda_b r\mathrm{d}r, \quad (8)$$

where $\mathcal{L}_{I_r}(r^{\alpha}(e^t-1))$ is the Laplace transform of the interference I_r evaluated at $r^{\alpha}(e^t-1)$, which is given by

$$\exp\left(-\pi\lambda_b r^2(e^t-1)^{2/\alpha}\int_{(e^t-1)^{-2/\alpha}}^{\infty}\frac{1}{1+x^{\alpha/2}}\mathrm{d}x\right).$$

Since σ^2 is very small, we use the approximation $e^{-\frac{\sigma^2}{P}r^{\alpha}(e^t-1)} \approx (1-\frac{\sigma^2}{P}r^{\alpha}(e^t-1))$ in (8). This approximation is tight as shown in Fig. 5. Substituting $\pi\lambda_b r^2 \to z$ in (8), we obtain

$$\tau(P) = \tau(\infty) - \sigma^2 g(\alpha)P^{-1}\lambda_b^{-\alpha/2+1} + o(\sigma^2), \quad \sigma^2 \to 0, \quad (9)$$

where $\tau(\infty)$ is the spectral efficiency at zero noise or high power, which is independent of the BS density [3]. A fraction $1-\delta$ of the users must achieve a data rate at least equal to the target rate or maximum achievable rate, whichever is

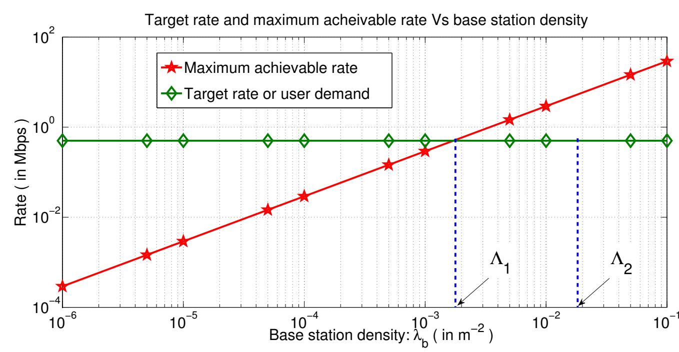

Fig. 7: Maximum achievable rate and target rate (user demand): User density $\lambda_u = 0.1$ m^{-2}, $\alpha = 4$, total bandwidth $B = 20$ MHz and target rate (user demand) $R_T = 500$ kbps.

minimum. This, in turn, imposes a constraint on the spectral efficiency $\tau(P)$:

$$\tau(P) \geq (1-\delta)\min\left\{\frac{R_T}{B_u(\lambda_b)}, \tau(\infty)\right\}. \quad (10)$$

Combining (10) and (9) we get a lower bound P_r^* on the transmit power P. ■

Depending on the target data rate, the transmit power of the base stations can be scaled down (under target data rate constraint) with the increase of small cell density λ_b, at a rate proportional to $\lambda_b^{1-\frac{\alpha}{2}}$ or faster than that as shown in Fig. 6.

IV. Optimum Base Station Density

To ensure QoS (both coverage and user demand rate), the minimum downlink transmit power should be P_m^*=max(P_c^*, P_r^*). So, the total power consumption by a base station is $P_m^* + P_0$. The area power consumption follows as

$P_A(\lambda_b) = \lambda_b(P_m^* + P_0) = \lambda_b(\max(P_c^*, P_r^*) + P_0)$. Using (3) and (7), we get

$$P_A(\lambda_b) = \frac{\max\{k_1, \min\{h(\lambda_b), k_2\}\}}{\lambda_b^{\alpha/2-2}} + P_0\lambda_b. \qquad (11)$$

The optimal λ_b^* is then obtained by setting $\frac{\mathrm{d}P_A}{\mathrm{d}\lambda_b} = 0$.

Theorem 1. *When $\alpha > 4$, the optimum base station density λ_b^* that minimizes the area power consumption under coverage and rate constraints is given by*

$$\lambda_b^* = \begin{cases} \left[\dfrac{(\alpha-4)\max\{k_1,k_2\}}{2P_0}\right]^{\frac{2}{\alpha-2}} & R_T \geq R_2, \\ \left[\dfrac{(\alpha-4)k_1}{2P_0}\right]^{\frac{2}{\alpha-2}} & R_T \leq R_1, \end{cases}$$

where

$$R_1 = \left[\frac{(\alpha-4)(B\tau(\infty))^{\alpha/2-1}k_1}{2P_0\lambda_u^{\alpha/2-1}}\right]^{\frac{2}{\alpha-2}},$$

and

$$R_2 = \left[\frac{(\alpha-4)(B\tau(\infty))^{\alpha/2-1}\max\{k_1,k_2\}}{2P_0\lambda_u^{\alpha/2-1}}\right]^{\frac{2}{\alpha-2}}.$$

When $R_1 < R_T < R_2$, the optimal base station density λ_b^* is the solution to the following equation

$$\lambda_b^{*\alpha/2-2}(2P_0c_2^2) - \lambda_b^{*\alpha/2-1}(4P_0\tau(\infty)c_2) + 2\lambda_b^{*\alpha/2}P_0(\tau(\infty))^2 \cdot \\ -\lambda_b^*((\alpha-4)c_1\tau(\infty)) + ((\alpha-6)c_1c_2) = 0,$$

where $c_1 = \sigma^2 g(\alpha)$, $c_2 = (1-\delta)\frac{R_T\lambda_u}{B}$. There exists no optimum density for $\alpha \leq 4$.

Proof: We begin the proof by considering 3 different ranges of λ_b. We define $\Lambda_1 = \frac{R_T\lambda_u}{B\tau(\infty)}$ and $\Lambda_2 = \frac{k_1c_2}{k_1\tau(\infty)-c_1}$. We also re-write $h(\lambda_b)$ as $h(\lambda_b) = \frac{\lambda_b c_1}{\lambda_b\tau(\infty)-c_2}$. The demand rate R_T is at least equal to the maximum achievable rate $B_u(\lambda_b)\tau(\infty)$ only when $\lambda_b \in (0, \Lambda_1]$, as can be seen in Fig. 7. Re-arranging the expressions of $h(\lambda_b)$, Λ_1, Λ_2 and k_2 we get

$$\min\{h(\lambda_b), k_2\} = \begin{cases} k_2 & \forall\ \lambda_b \in (0, \Lambda_1], \\ h(\lambda_b) & \forall\ \lambda_b \in (\Lambda_1, \Lambda_2), \\ h(\lambda_b) & \forall\ \lambda_b \in [\Lambda_2, \infty). \end{cases}$$

Re-arranging the expressions of Λ_2, k_1 and $h(\lambda_b)$, it can be shown that, $k_1 \geq h(\lambda_b)$ only for $\lambda_b \in [\Lambda_2, \infty)$. Therefore, the area power consumption given in (11) can be re-written as

$$P_A = \begin{cases} \dfrac{\max\{k_1,k_2\}}{\lambda_b^{\alpha/2-2}} + P_0\lambda_b & \forall\ \lambda_b \in (0, \Lambda_1], \\ \dfrac{h(\lambda_b)}{\lambda_b^{\alpha/2-2}} + P_0\lambda_b & \forall\ \lambda_b \in (\Lambda_1, \Lambda_2), \\ \dfrac{k_1}{\lambda_b^{\alpha/2-2}} + P_0\lambda_b & \forall\ \lambda_b \in [\Lambda_2, \infty). \end{cases} \qquad (12)$$

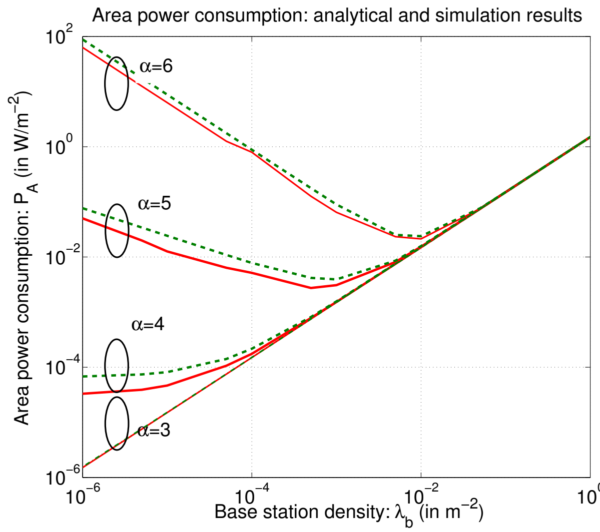

Fig. 8: Area power consumption: Target rate $R_T = 500$Mbps, SINR threshold $\theta = 12$ dB, User density $\lambda_u = 0.01$ m^{-2}, Outage probability $\epsilon = 0.25$, Rate outage $\delta = 0.25$, $P_0 = 1.5$ W and noise power $\sigma^2 = \lambda_b \times 1.035 \times 10^{-5}$ W. The dashed lines correspond to our analysis while the bold lines are obtained by Monte Carlo simulation.

Re-arranging the expressions of R_1, R_2, Λ_1, Λ_2 and using the derivative of (12), for $\alpha > 4$, we obtain

$$\begin{cases} \dfrac{\mathrm{d}P_A(\Lambda_1)}{\mathrm{d}\lambda_b} \geq 0 & R_T \geq R_2, \\ \dfrac{\mathrm{d}P_A(\Lambda_1)}{\mathrm{d}\lambda_b} < 0, \dfrac{\mathrm{d}P_A(\Lambda_2)}{\mathrm{d}\lambda_b} > 0 & R_1 < R_T < R_2, \\ \dfrac{\mathrm{d}P_A(\Lambda_2)}{\mathrm{d}\lambda_b} \leq 0 & R_T \leq R_1. \end{cases}$$

Therefore for $\alpha > 4$,

$$\lambda_b^* \in \begin{cases} (0, \Lambda_1] & R_T \geq R_2, \\ (\Lambda_1, \Lambda_2) & R_1 < R_T < R_2, \\ [\Lambda_2, \infty) & R_T \leq R_1. \end{cases} \qquad (13)$$

Now, for $\alpha > 4$, optimal base station density λ_b^* can be obtained by setting the derivative of (12) to zero.

When $\alpha \leq 4$, the total area transmit power consumption $\lambda_b P_m^*$ as well as the total area fixed power consumption $\lambda_b P_0$ increase with base station density, for $\lambda_b \in (0, \Lambda_1]$ and $\lambda_b \in [\Lambda_2, \infty)$, see (12). Hence, in this case, there exists no optimal density. ■

The area power consumption always increases with base station density for $\alpha = 3$ and $\alpha = 4$ as can be seen in Fig. 8. The simulation curves are obtained using the Monte Carlo method. This result indicates that cell size shrinking or dense small cell deployment is not power-efficient for these values of the path loss exponent. Fig. 8 also shows the area power consumption and the optimal base station densities for $\alpha = 5$ and $\alpha = 6$. This result indicates that small cell deployment can be power efficient in urban areas (where it is possible that $\alpha > 4$).

V. CONCLUSION

In this paper, we have derived a lower bound on downlink transmit power in a homogeneous Poisson networks under target coverage and target data rate constraints. For a path loss exponent $\alpha > 4$, we have proven the existence of an optimal deployment density after which further reduction of the cell size is not efficient from an energy viewpoint. Our results indicate that, for $\alpha \leq 4$, counter-intuitively, *smaller* BS densities lead to improved area power consumption. In other words, increasing the BS density is not "green" for $\alpha \leq 4$.

REFERENCES

[1] M. Haenggi, *Stochastic geometry for wireless networks*. Cambridge University Press, 2012.

[2] V. Chandrasekhar, J. Andrews, and A. Gatherer, "Femtocell networks: a survey," *Communications Magazine, IEEE*, vol. 46, no. 9, pp. 59–67, 2008.

[3] J. G. Andrews, F. Baccelli, and R. K. Ganti, "A tractable approach to coverage and rate in cellular networks," *IEEE Trans. on Communications*, vol. 59, no. 11, pp. 3122–3134, Nov. 2011.

[4] D. Cao, S. Zhou, and Z. Niu, "Optimal base station density for energy-efficient heterogeneous cellular networks," *International Conference on Communications (ICC), IEEE*, pp. 4379–4383, 2012.

[5] T. Q. Quek, W. C. Cheung, and M. Kountouris, "Energy efficiency analysis of two-tier heterogeneous networks," in *11th European Wireless Conference 2011-Sustainable Wireless Technologies (European Wireless)*. VDE, 2011, pp. 1–5.

[6] X. Wang, A. V. Vasilakos, M. Chen, Y. Liu, and T. T. Kwon, "A survey of green mobile networks: Opportunities and challenges," *Mobile Networks and Applications*, vol. 17, no. 1, pp. 4–20, 2012.

[7] F. Richter and G. Fettweis, "Cellular mobile network densification utilizing micro base stations," in *International Conference on Communications (ICC)*. IEEE, 2010, pp. 1–6.

[8] F. Richter, A. J. Fehske, P. Marsch, and G. P. Fettweis, "Traffic demand and energy efficiency in heterogeneous cellular mobile radio networks," in *71st Vehicular Technology Conference (VTC)*. IEEE, 2010-Spring, pp. 1–6.