SIR Asymptotics in Poisson Cellular Networks without Fading and with Partial Fading

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***Abstract*—In this paper, we consider cellular networks with the base station locations modeled by a Poisson point process. However, unlike earlier works, we consider the cases of no fading and partial fading and analyze the asymptotic behavior of the distribution of the downlink signal-to-interference ratio (SIR) of a typical user. This non-fading case has been elusive since the standard Laplace trick cannot be used. We provide the asymptotics of the SIR distribution $F_{\mathsf{SIR}}(\theta) = \mathbb{P}(\mathsf{SIR} < \theta)$ as $\theta \to 0$ for the cases of no-fading and partial fading, where only the interfering base stations are subject to fading. We also introduce a new point process—the squared relative distance process—that facilitates the asymptotic analysis of the SIR and expedites simulations.**

***Index Terms*—Cellular networks, stochastic geometry, signal-to-interference ratio, Poisson point processes.**

I. Introduction

A. Motivation

Much work has been devoted in recent years to the analysis of the signal-to-interference ratio (SIR) in cellular networks. The SIR distribution is a key performance indicator for interference-limited wireless systems. The typically used network models include small-scale fading in the links from the serving base station (BS) and the interfering ones. However, if small-scale fading is mitigated by transmission schemes providing diversity or using beamforming, a no-fading model is more appropriate. Also, in mm-wave communication, it has been shown that the desired link exhibits less or no fading [1]. However, the no-fading case is harder to analyze, and closed-form results on the SIR distribution cannot be expected.

The partial fading case (interferer fading only) applies to the situation where the desired signal travels along a line-of-sight (LOS) path while the interfering ones do not, or to the situation where fading in the desired link is mitigated using power control or other mechanisms.

B. SIR distribution in cellular networks

We focus on general single-tier cellular networks where users are connected to the strongest (nearest) BS, and the BS locations form a stationary simple point process $\Phi \subset \mathbb{R}^2$. We denote by $x_0 \in \Phi$ the serving BS of the typical user at the origin, *i.e.*, define $x_0 \triangleq \arg\min\{x \in \Phi \colon \|x\|\}$. Assuming all BSs transmit at the same power level, the downlink SIR is given by

$$\mathsf{SIR} \triangleq \frac{S}{I} = \frac{h_{x_0}\ell(x_0)}{\sum_{x\in\Phi\setminus\{x_0\}} h_x \ell(x)}, \tag{1}$$

where (h_x) are iid random variables representing the fading and $\ell(x)$ is the path loss law. We assume $\mathbb{E}[h_x] = 1, \forall x \in \Phi$. The complementary cumulative distribution (ccdf) of the SIR is

$$\bar{F}_{\mathsf{SIR}}(\theta) \triangleq \mathbb{P}(\mathsf{SIR} > \theta). \tag{2}$$

Under the SIR threshold model for reception, the ccdf of the SIR can also be interpreted as the success probability of a transmission, *i.e.*, $p_{\mathrm{s}}(\theta) \equiv \bar{F}_{\mathsf{SIR}}(\theta)$.

The success probability can be expressed as

$$p_{\mathrm{s}}(\theta) = \mathbb{E}(\mathbb{P}(h\bar{S} > \theta I \mid I, x_0)) = \mathbb{E}\bar{F}_h(\theta\, \mathsf{I\bar{S}R}), \tag{3}$$

where $\bar{S} = \mathbb{E}_h(S) = \ell(x_0)$ is the signal power averaged over the fading, $\bar{F}_h$ is the ccdf of the fading random variables, and $\mathsf{I\bar{S}R}$ is the interference-to-(average-)signal ratio, defined as

$$\mathsf{I\bar{S}R} \triangleq \frac{I}{\mathbb{E}_h(S)} = \frac{I}{\bar{S}}.$$

The Rayleigh fading case is the easiest to analyze since $\bar{F}_h(x) = e^{-x}$ and thus $p_{\mathrm{s}}(\theta) = \mathbb{E}\exp(-\theta\, \mathsf{I\bar{S}R})$ is just the Laplace transform of the ISR evaluated at θ. This also holds if only the desired link is subject to Rayleigh fading, while the interfering links may fade differently (or not at all). In contrast, the cases where the desired link is not fading (and thus $\mathsf{I\bar{S}R} = \mathsf{ISR}$ since $S = \mathbb{E}_h(S)$) are harder to analyze. We focus on these two scenarios:

- The no-fading case, where $h_x \equiv 1$ for all $x \in \Phi$. In this case, $\bar{F}_h(x) = \mathbf{1}(x < 1)$ and $p_{\mathrm{s}}(\theta) = \mathbb{P}(\mathsf{ISR} < \theta^{-1})$.
- The partial fading case, where only the interfering links are subject to fading, *i.e.*, $h_{x_0} = 1$ and all other h_x are iid exponential with mean 1.

The goal is to find the asymptotic behavior of the SIR cdf $F_{\mathsf{SIR}}(\theta)$ (or, equivalently, the outage probability $1 - p_{\mathrm{s}}(\theta)$) as $\theta \to 0$.

C. Prior work

In the case where Φ forms a homogeneous Poisson point process (PPP), Rayleigh fading, and $\ell(x) = \|x\|^{-\alpha}$, the success probability can be expressed in terms of the Gaussian hypergeometric function ${}_2F_1$ as [2]

$$p_{\mathrm{s,PPP}}(\theta) = \frac{1}{{}_2F_1(1, -\delta; 1-\delta; -\theta)}, \tag{4}$$

where $\delta \triangleq 2/\alpha$. For all other cases, the success probability is intractable or can at best be expressed using combinations of infinite sums and integrals. In [3], it is shown that for general stationary (and simple) point process models where the link from the serving BS is subject to Rayleigh fading,

$$F_{\mathsf{SIR}}(\theta) \sim \mathsf{MISR}\,\theta, \quad \theta \to 0,$$

where $\mathsf{MISR} = \mathbb{E}(\bar{\mathsf{ISR}})$. For the PPP, $\mathsf{MISR} = 2/(\alpha-2)$. This holds independently of the type of fading in the interfering links. In [4], an expression for the MISR in general point processes is given and the higher moments $\mathbb{E}(\bar{\mathsf{ISR}}^m)$ are derived for PPPs, which determine the asymptotic distribution in the case of Nakagami-m fading in the desired link, where

$$F_{\mathsf{SIR}}(\theta) \sim \frac{m^{m-1}}{\Gamma(m)}\mathbb{E}(\bar{\mathsf{ISR}}^m)\theta^m, \quad \theta \to 0.$$

[4] also studied the tail of the distribution, and it is shown that

$$\bar{F}_{\mathsf{SIR}}(\theta) \sim \mathsf{EFIR}^\delta \theta^{-\delta},$$

where EFIR is the expected fading-to-interference ratio. For the PPP, $\mathsf{EFIR}^\delta = \operatorname{sinc}\delta$, for arbitrary fading in any of the links.

In the no-fading case, nearest-BS association is the same as instantaneously-strongest BS association (where fading is taken into account to determine the serving BS), for which the SIR distribution is known exactly for $\theta \geq 1$ [5, Cor. 2]. Hence it follows that in the no-fading case,

$$\bar{F}_{\mathsf{SIR}}(\theta) = \operatorname{sinc}(\delta)\theta^{-\delta}, \quad \theta \geq 1. \tag{5}$$

II. System Model

The base station locations are modeled as a stationary point process $\Phi \subset \mathbb{R}^2$. Without loss of generality, we assume that the typical user is located at the origin o. The path loss between the typical user and a BS at $x \in \Phi$ is given by $\ell(x) = \|x\|^{-\alpha}$, $\alpha > 2$. The link from the serving BS to the user at o is not fading, *i.e.*, we can denote the $\bar{\mathsf{ISR}}$ as ISR, while the interfering links may include fading.

We assume nearest-BS association, wherein a user is served by the closest BS. Let x_0 denote the closest BS to the typical user at the origin, and define $R \triangleq \|x_0\|$ and $\Phi^! = \Phi \setminus \{x_0\}$. With the nearest BS association rule, the downlink SIR (1) of the typical user can be expressed as

$$\mathsf{SIR} = \frac{R^{-\alpha}}{\sum_{x\in\Phi^!} h_x \ell(x)}. \tag{6}$$

With no or partial fading, the ISR is given by

$$\mathsf{ISR} = R^\alpha \sum_{x\in\Phi^!} h_x \ell(x) = \sum_{x\in\Phi} h_x \left(\frac{\|x\|}{R}\right)^{-\alpha}.$$

So the SIR or ISR is determined only by the relative distances. To exploit this fact, we introduce a new point process, the *squared relative distance process*.

III. The Squared Relative Distance Process

In [4, Def. 2], we have introduced the relative distance process

$$\mathcal{R} \triangleq \{x \in \Phi \setminus \{x_0\} \colon \|x_0\|/\|x\|\},$$

which is a non-locally finite point process on $[0, 1]$. For this paper, it is advantageous to invert it and square it, hence the name squared (inverted) relative distance process (SRDP).[1]

A. Definition

Definition 1 (Squared relative distance process)**.** *The squared relative distance process of a point process Φ is defined as*

$$\Psi \triangleq \left\{x \in \Phi \setminus \{x_0\} \colon \frac{\|x\|^2}{\|x_0\|^2}\right\} = \{y \in \mathcal{R} \colon y^{-2}\}.$$

Since x_0 is the nearest point, $\Psi \subset [1, \infty)$. The ISR for a path loss exponent $\alpha = 2/\delta$ in the non-fading case is

$$\mathsf{ISR} = \sum_{x\in\Psi} x^{-1/\delta}.$$

B. Properties of the SRDP of a PPP

When the underlying point process is a stationary PPP of intensity λ, the SRDP has the following properties:

- It follows from the mapping theorem [6, Thm. 2.34] that, given the distance R to the nearest point, the SRDP is a Poisson process of intensity $\lambda\pi R^2$ on $[1, \infty)$. This makes it a Cox process [6, Sec. 3.3].
- Since R^2 is exponential with mean $1/(\lambda\pi)$, the intensity of the SRDP is $\lambda = 1$ on $[1, \infty)$.
- The mean gap between neighboring points is ∞, hence

$$\mathbb{E}|x - y| = \infty, \qquad x, y \in \Psi,\ x \neq y.$$

Next we give an expression for the probability generating functional (PGFL).

Lemma 1. *Let $f(x)$ be a function such that $1 + \int_1^\infty (1 - f(x))\mathrm{d}x > 0$. The PGFL of the SRDP is*

$$G_\Psi[f] = \frac{1}{1 + \int_1^\infty (1 - f(x))\mathrm{d}x}.$$

[1] Since this process is still a relative distance process and "inverted" only pertains to the original RDP in [4], we do not include "inverted" in the acronym.

Proof: The PGFL is given by

$$G_\Psi[f] = \mathbb{E}\prod_{x\in\Psi} f(x) = \mathbb{E}\prod_{x\in\Phi} f\left(\frac{\|x\|^2}{\|x_0\|^2}\right)$$
$$\stackrel{(a)}{=} 2\pi\lambda\int_0^\infty r e^{-\lambda\pi r^2 - \lambda 2\pi\int_r^\infty \eta(1-f(\eta^2/r^2))\mathrm{d}\eta}\mathrm{d}r$$
$$\stackrel{(b)}{=} 2\pi\lambda\int_0^\infty r e^{-\lambda\pi r^2(1+\int_1^\infty(1-f(y))\mathrm{d}y}\mathrm{d}r,$$

where (a) follows from the PGFL of the PPP and the distribution of the nearest neighbor distance in a PPP. We obtain (b) by the substitution $\eta^2/r^2 \to y$. The condition $1+\int_1^\infty(1-f(x))\mathrm{d}x > 0$ ensures that the integral with respect to the nearest-neighbor distance is finite. ∎

The factorial moment measures provide an alternative characterization of the point process. They are related to the PGFL as [7, p. 116]

$$\alpha^{(n)}(t_1,\ldots,t_n) \equiv$$
$$(-1)^n\frac{\partial}{\partial s_1}\cdots\frac{\partial}{\partial s_n}G_\Psi[1-s_1\mathbf{1}_{(1,t_1)}-\ldots-s_n\mathbf{1}_{(1,t_n)}],\quad t_i>1. \tag{7}$$

Using Lemma 1, we obtain

$$\alpha^{(n)}(t_1,\ldots,t_n) = n!\prod_{i=1}^n(t_i-1),\quad t_i>1,$$

and the moment densities follow as

$$\rho^{(n)}(t_1,\ldots,t_n) = n!,\quad t_i>0.$$

Hence the SRDP is a clustered point process, which is consistent with the fact that it is a Cox process by construction.

IV. Asymptotics of the SIR Distribution for the PPP without Fading

The behavior of $F_{\mathsf{SIR}}(\theta)$ for $\theta\to\infty$ is discussed in [4]. In this paper, we are interested in the regime near 0, *i.e.*, $\theta\to 0$. Since $\mathbb{P}(\mathsf{SIR}<\theta)=\mathbb{P}(\mathsf{ISR}>\theta^{-1})$, we may instead analyze the tail behavior (at ∞) of the random variable ISR. To do so, we use the Laplace transform $\mathcal{L}_{\mathsf{ISR}}(s)$ of the ISR. The basic idea is as follows: We compute the smallest s^* for which the Laplace transform $\mathcal{L}_{\mathsf{ISR}}(s^*)<\infty$. Since $\mathcal{L}_{\mathsf{ISR}}(0)=1$, such s^* is surely negative. Hence

$$\mathcal{L}_{\mathsf{ISR}}(s^*) = \int_0^\infty e^{|s^*|x} f_{\mathsf{ISR}}(x)\mathrm{d}x < \infty,$$

while for any $s<s^*$, $\mathcal{L}_{\mathsf{ISR}}(s)=\infty$. This implies that the pdf $f_{\mathsf{ISR}}(x)$ of ISR decays exponentially with rate $|s^*|$, *i.e.*, $f_{\mathsf{ISR}}(x)=\Theta(e^{s^*x})$, $x\to\infty$. We begin with computing the Laplace transform of ISR.

A. The Laplace transform of the ISR

With Lemma 1, we can calculate the Laplace transform of the ISR. We use ${}_1F_1$ to denote the confluent hypergeometric function. Let $\Re(z)$ denote the real part of z.

Lemma 2. *Let $s^*\in\mathbb{R}$ be the (unique) solution to*

$${}_1F_1(-\delta, 1-\delta, -s^*) = 0.$$

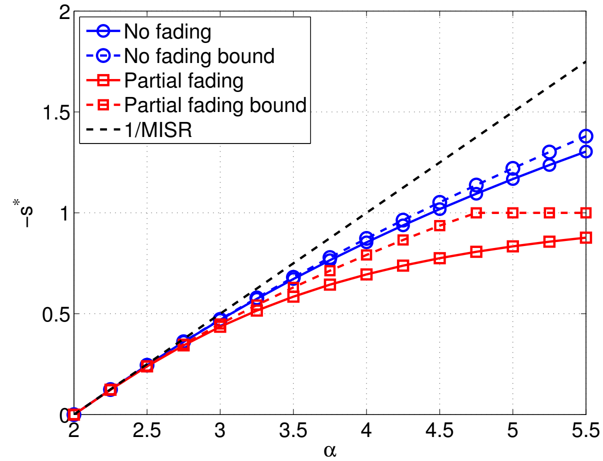

Figure 1. The value of s^* that determines the ROC for the Laplace transform of the ISR for the no fading and partial fading cases and their bounds. The three bounds are (from top to bottom) obtained from (10), (11), and (13).

The Laplace transform of the ISR *is given by*

$$\mathcal{L}_{\mathsf{ISR}}(s) = \frac{1}{{}_1F_1(-\delta,1-\delta,-s)},\quad \Re(s)>\Re(s^*). \tag{8}$$

Proof:

$$\mathcal{L}_{\mathsf{ISR}}(s) = \mathbb{E}(e^{-s\,\mathsf{ISR}})$$
$$= \mathbb{E}\prod_{x\in\Psi} e^{-sx^{-1/\delta}}$$
$$= \frac{1}{1+\int_1^\infty(1-e^{-sx^{-1/\delta}})\mathrm{d}x}$$
$$= \frac{1}{1+\delta\int_0^1\frac{1-e^{-st}}{t^{1+\delta}}\mathrm{d}t} \tag{9}$$
$$= \frac{1}{{}_1F_1(-\delta,1-\delta,-s)}$$

s^* is the value where the denominator of (9) is 0. ∎

Remarks:

- From (9), it is easily seen that the denominator of the Laplace transform $f(s)={}_1F_1(-\delta,1-\delta,-s)$ is monotonically increasing in s and that $f(0)=1$; thus $f(s)$ has a single (real) root at some $s<0$, which is s^*.
- The region of convergence (ROC) of the Laplace transform is $\Re(s)>\Re(s^*)$.
- The Laplace transform can also be expressed as

$$\mathcal{L}_{\mathsf{ISR}}(s) = \frac{e^s}{{}_1F_1(1,1-\delta,s)}$$

or as

$$\mathcal{L}_{\mathsf{ISR}}(s) = \frac{1}{\delta s^\delta(\Gamma(-\delta,s)-\Gamma(-\delta))}.$$

- $\mathcal{L}_{\mathsf{ISR}}(\theta)$ is the SIR ccdf without fading at the interferers but Rayleigh fading in the desired link. This is the complementary partial fading case to the one we are interested in in this paper. As pointed out before, this case is easier since we obtain the entire ccdf from the Laplace transform.

- By the property of the Laplace transform,

$$\frac{\mathrm{d}f(s)}{\mathrm{d}s}\Big|_{s=0} = \mathsf{MISR},$$

and

$$\frac{\mathrm{d}^2}{\mathrm{d}s^2}\left[\frac{1}{f(s)}\right]\Big|_{s=0} = \mathbb{E}(\mathsf{ISR}^2) = 2\,\mathsf{MISR}^2 + \frac{\delta}{2-\delta},$$

in agreement with [4, Thm. 2].

Since the hypergeometric function is not an elementary function, we next provide some simple bounds for s^*.

Lemma 3. *We have the following bounds and asymptotic results for s^*: Let $\hat{s} = -1/\mathsf{MISR} = 1-\alpha/2 = 1-\delta^{-1}$. Then*

$$s^* > \hat{s}\,; \qquad \lim_{\delta\to 1} s^* = \hat{s}. \tag{10}$$

A tighter bound is

$$s^* > 1 + \frac{\alpha - \sqrt{2(\alpha-1)(\alpha^2-2\alpha+2)}}{\alpha-2}. \tag{11}$$

Proof: The Taylor series of order n for $f(s) = 1/\mathcal{L}_{\mathsf{ISR}}(s)$ at 0, which follows from the series representation of the confluent hypergeometric function or can be calculated in a straightforward manner from (9), is

$$f^{(n)}(s) = \sum_{k=0}^{n} \frac{(-1)^{k+1}}{k!}\frac{\delta}{k-\delta}s^k, \quad n \in \mathbb{N}_0. \tag{12}$$

For $s<0$, the error term $f(s) - f^{(n)}(s) < 0$ for all n, hence solving $f^{(n)}(s) = 0$ yields a lower bound for s^* that gets increasingly tight as n increases. Solving $f^{(1)}(s) = 0$ yields (10), while solving $f^{(2)}(s) = 0$ yields (11). The limit $s^* \to \hat{s}$ follows since the linear term becomes dominant in the Taylor expansion as $\delta \to 1$, *i.e.*, $f(s) \to f^{(1)}(s)$ as $\delta \to 1$. ■

Fig. 1 shows the exact value of $-s^*$ together with the bounds (10) (dashed, no markers) and (11) (dashed, with ○ markers).

B. The asymptotic behavior of the SIR distribution

With the Laplace transform, we are equipped to state the result on the asymptotic behavior of the tail of the ISR, or, equivalently, of the SIR distribution near 0.

Theorem 1. *The tail of* ISR *is given by*

$$\mathbb{P}(\mathsf{ISR} > x) \sim e^{s^* x}, \quad x \to \infty,$$

with s^* given in Lemma 2.

Proof: The result follows from Lemma 2 and the Tauberian theorem in [8, Theorem 3]. ■

Fig. 2 shows the asymptotic outage for the PPP together with the simulated curves for $\alpha = 3$ and $\alpha = 4$. For $\alpha = 3$ the computed $s^* = -0.470$ and $s^* = -0.854$ for $\alpha = 4$ (see also Fig. 1). It is apparent that the asymptotic expression is quite accurate also for non-vanishing values of θ. In particular, for smaller values of α, it is tight up to 0 dB. For $\theta \geq 0$ dB, the exact expression is given in (5).

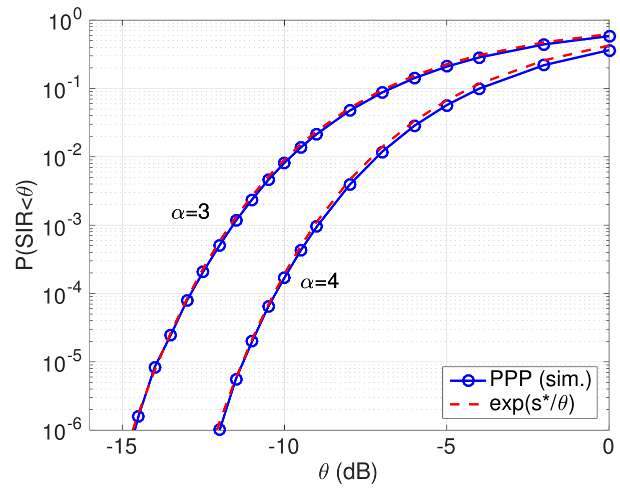

Figure 2. The asymptotic SIR cdf for $\alpha = 3$ and $\alpha = 4$ for the PPP in the no-fading case and the corresponding simulation curves. The corresponding values of s^* are -0.470 ($\alpha = 3$) and -0.854 ($\alpha = 4$).

C. Diversity analysis

Lett $F_{\mathsf{SIR}}(\theta)$ denote the SIR cdf for the no fading case and $F^{\mathrm{rf}}_{\mathsf{SIR}}(\theta)$ the SIR cdf for the Rayleigh fading case (full fading). We obtain from Theorem 1 that

$$F_{\mathsf{SIR}}(\theta) = \mathbb{P}(\mathsf{SIR} < \theta) \sim e^{s^*/\theta}, \quad \theta \to 0.$$

Also, we have from (4) that

$$F^{\mathrm{rf}}_{\mathsf{SIR}}(\theta) = \frac{1}{{}_2F_1(1,-\delta;1-\delta;-\theta)} \sim \mathsf{MISR}\,\theta, \;\; \theta \to 0.$$

Hence

$$\lim_{\theta\to 0}\frac{-\log(-\log F_{\mathsf{SIR}}(\theta))}{\log\theta} = \lim_{\theta\to 0}\frac{\log F^{\mathrm{rf}}_{\mathsf{SIR}}(\theta)}{\log\theta} = 1$$

and approximately

$$-\log(-\log F_{\mathsf{SIR}}(\theta)) \approx \log F^{\mathrm{rf}}_{\mathsf{SIR}}(\theta), \quad \theta \to 0,$$

with equality if $s^* = -1/\mathsf{MISR}$. So the no fading case provides exponential diversity compared to the fading case, as expected. From Figure 1, we see that $-s^*$, the rate of exponential diversity, increases with α, which is intuitive.

V. Asymptotics of the SIR Distribution for the PPP with Partial Fading

If only the interferers are subject to fading, we can reinterpret (4) as the Laplace transform of the ISR:

$$\mathcal{L}_{\mathsf{ISR}}(s) = \frac{1}{{}_2F_1(1,-\delta;1-\delta;-s)}, \quad \Re(s) > \Re(s^*_{\mathrm{pf}}),$$

where $s^*_{\mathrm{pf}} \in \mathbb{R}$ is the value where the denominator, again denoted as $f(s)$, is zero. Hence, in analogy with Theorem 1, we have the following result:

Theorem 2. *In the partial fading case, the tail of* ISR *is given by*

$$\mathbb{P}(\mathsf{ISR} > x) \sim e^{s^*_{\mathrm{pf}} x}, \quad x \to \infty,$$

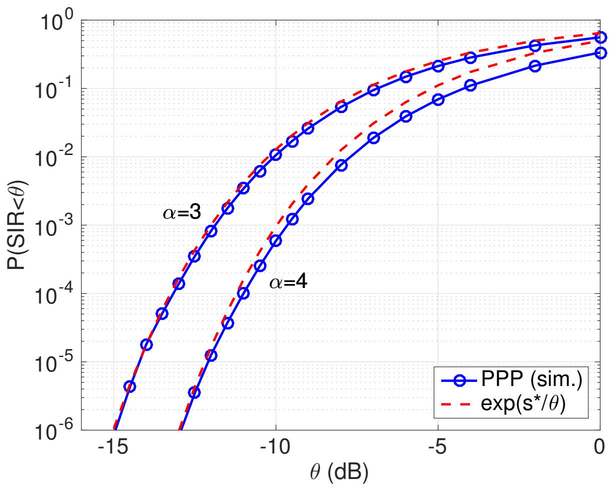

Figure 3. The asymptotic SIR cdf for $\alpha = 3$ and $\alpha = 4$ for the PPP in the partial fading case and the corresponding simulation curves. The corresponding values of $s^*_{\rm pf}$ are -0.435 ($\alpha = 3$) and -0.694 ($\alpha = 4$).

where $s^*_{\rm pf}$ is given by

$$ {}_2F_1(1, -\delta; 1-\delta; -s^*_{\rm pf}) = 0. $$

Proof: Same as for Theorem 1. ■

As in the no fading case, we can obtain tight bounds on s^* from a Taylor series expansion of the Gauss hypergeometric function, which is

$$ f^{(n)}(s) = \sum_{k=0}^{n} (-1)^{k+1} \frac{\delta}{k-\delta} s^k, \quad n \in \mathbb{N}_0. $$

This is very similar to (12) for the confluent hypergeometric function, except for the missing factor $1/k!$. Since this series diverges for $\Re(s) \leq -1$, we have the lower bound $s^*_{\rm pf} > -1$, for all δ.

From the linear expansion $f^{(1)}$ we obtain the same bound as for the no fading case, while for $n = 2$, we have

$$ s^*_{\rm pf} > \max\left\{ -1, 1 + \frac{1 - \sqrt{(\alpha-1)(\alpha^2 - 3\alpha + 3)}}{\alpha - 2} \right\}. \quad (13) $$

Fig. 1 also shows this bound (dashed curve, marked by □), and Fig. 3 shows the asymptotic outage for the partial fading case together with the simulation results. The values of $s^*_{\rm pf}$ for $\alpha = 3$ and $\alpha = 4$ are $s^*_{\rm pf} = -0.435$ and $s^*_{\rm pf} = -0.694$, respectively.

VI. Conclusions

In this paper, we analyzed the cdf of SIR at $\theta \to 0$ in a Poisson cellular network with nearest-base station connectivity and limited fading. The asymptotic behavior of the SIR cdf as $\theta \to 0$ (and thus the ccdf of the ISR as $\theta \to \infty$) in the no fading and partial fading cases is governed by a certain root of a confluent and a Gauss hypergeometric function, respectively. When fading is absent on the desired link, the cdf decays exponentially with θ^{-1} as $\theta \to 0$. In contrast, the cdf exhibits a polynomial decay θ^m as $\theta \to 0$ with Nakagami-m fading for the desired link [4]. This confirms that even interference-limited cellular networks provide exponential diversity if there is no fading in the link from the serving base station.

We also introduced a new type of point process, the squared relative distance process, which has nice properties. It proves useful also for simulations, since only a one-dimensional point process needs to be simulated, no matter how many dimensions the underlying base station process has.

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