A Simple Approximation of the Meta Distribution for Non-Poisson Cellular Networks

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***Abstract*—Recently a new fundamental performance metric, called the meta distribution of the signal-to-interference ratio (SIR), has been proposed for cellular networks. Compared to the standard success (coverage) probability, the meta distribution provides much more fine-grained information about the network performance. In this paper, we consider general (non-Poisson) network models. However, the exact analysis of non-Poisson network models is notoriously difficult, even in terms of the standard success probability, let alone the meta distribution. Hence we propose a simple approach to approximate the meta distribution for non-Poisson networks, which is based on the ASAPPP ("approximate SIR analysis based on the Poisson point process") method. For a stationary and ergodic point process model, we prove that the asymptotic horizontal gap G_0 between its standard success probability and that of the Poisson point process exactly characterizes the gap between the bth moment of the conditional success probability, as the SIR threshold goes to 0. Using detailed simulations, we confirm that the meta distribution of an arbitrary stationary and ergodic point process can be approximated by applying the horizontal shift of G_0 to the meta distribution of the Poisson point process.**

***Index Terms*—Cellular networks, interference, meta distribution, non-Poisson point process, SIR, stochastic geometry**

I. Introduction

A. Motivation and Objective

The accurate modeling of base station (BS) locations is important to characterize the performance of cellular networks and obtain useful design insights. Traditionally, in a cellular network, the BS locations were modeled in a deterministic (regular) manner using either triangular or square lattices. The lattice model has been extensively studied using simulations since it is usually analytically intractable. Furthermore, to meet an exponential growth in mobile traffic and improve the spatial reuse, the deployment of cellular networks has become more irregular. It is shown in [1] that the Poisson point process (PPP) may be used to model irregular and real-world BS deployments. The modeling of BS locations by the PPP has become popular due to its analytical tractability, which leads to crisp insights about the network performance.

In an actual cellular network, the BS deployment is neither completely random (as the PPP) nor completely regular (as the triangular and square lattices)—it lies somewhere in between. The BS deployment depends heavily on the topology and the type of the geographical area (urban or rural). As a result, a single point process model may not be applicable in all scenarios. For example, using the actual data from UK, it is shown in [1] that there exists repulsion among BSs, which can be modeled using hard-core point processes [2, Chapter 3]. On the other hand, in [3], the Poisson cluster process [2, Chapter 3] is shown to accurately model the BS deployment in many cities. Especially, at a larger geographical scale, the BS deployment appears to form a cluster point process due to the high density in urban regions and low density in rural regions. Hence it is important to investigate the performance of non-Poisson cellular networks.

The main impediment to the study of non-Poisson cellular networks is that, compared to the PPP, their analysis is much harder due to the dependence between the BS locations. Thus it would be convenient if the performance of non-Poisson cellular networks could be related (approximately) to that of Poisson cellular networks.

Recently, in [4], a new fundamental performance metric called the *meta distribution* of the signal-to-interference ratio (SIR) is introduced for cellular networks. The meta distribution, defined as the distribution of the conditional success probability given the point process, is an important metric as it answers the key question "How are the individual link success (or coverage) probabilities[1] distributed in a realization of the cellular network?" The answer directly leads to the performance of the "5% user," which corresponds to the performance of the top 95% of users, and it is an important design criterion for cellular operators. The meta distribution provides much more fine-grained information about the network than the standard success (coverage) probability; the latter provides just the average of individual link success probabilities in each realization of the network and thus yields limited information about the network. On the other hand, the meta distribution provides the distribution of the link success probability conditioned on the point process and thus characterizes the network at a finer level.

The goal of this paper is to study the meta distribution in non-Poisson cellular networks, for which we have to overcome the following two main difficulties:

1) The direct calculation of the meta distribution seems infeasible even for the Poisson cellular network [4]—one has to calculate the moments of the conditional success

[1]The success probability of a link is the probability that the SIR at the receiver of that link is above the target SIR threshold θ.

probability and then use the Gil-Pelaez theorem [5] to calculate the meta distribution.

2) As aforementioned, the analysis for non-Poisson cellular networks is significantly more difficult than that for Poisson cellular networks. In fact, obtaining an analytical expression of the (standard) coverage probability is extremely difficult in non-Poisson networks. Even for the arguably second-simplest model, the Ginibre point process, it can only be given in the form of an expression in which 3 integrals, an infinite product, and an infinite sum are nested [6, Theorem 2].

Consequently, analyzing the meta distribution in non-Poisson networks is very challenging. In this paper, we propose a simple and indirect approach to approximately calculate the meta distribution of an arbitrary stationary and ergodic point process by comparing its meta distribution to that of the PPP.

B. Related Work

In [1], it was observed that the coverage probability curve (SIR distribution) for the downlink of cellular networks modeled by different non-Poisson point processes can be closely approximated by simply applying a horizontal shift to the coverage probability curve of the PPP. The approximation becomes asymptotically exact as the SIR threshold $\theta \to 0$ [7]. The horizontal shift is termed the deployment gain in [7] since the shift is because of the deployment. This method of approximating the coverage probability of a non-Poisson point process by that of the PPP is called "Approximate SIR analysis based on the PPP" (ASAPPP) in [8]. In [9], it was shown that the deployment gain as $\theta \to 0$ can be expressed as the ratio of the mean interference-to-signal ratio (MISR) of two different point processes under consideration. Further in [10], it was proved that the deployment gain as $\theta \to \infty$ is determined by the expected fading-to-interference ratio (EFIR). A key observation from [10] is that the deployment gain as $\theta \to 0$, denoted by G_0, provides an excellent approximation to the entire SIR distribution.

The idea of the meta distribution of the SIR for the cellular network was proposed in [4], where the focus was on the downlink of the Poisson cellular network. Further the meta distribution of the SIR was calculated for both the downlink and the uplink of the Poisson cellular network with power control in [11], for the downlink Poisson cellular network underlaid with a device-to-device (D2D) network in [12], and with base station cooperation in [13].

C. Contributions

This paper makes the following contributions:

1) For cellular networks, we apply the idea of ASAPPP to the meta distribution and propose a simple and novel method, called AMAPPP which stands for "Approximate meta distribution analysis using the PPP," to obtain the meta distribution for an arbitrary stationary and ergodic point process from the meta distribution of the PPP.
2) We prove that, as $\theta \to 0$, the bth moment of the conditional success probability of a stationary and ergodic point process can be obtained exactly by shifting that of the PPP by the asymptotic deployment gain G_0.
3) For Rayleigh fading and an unbounded path loss model, we confirm by simulations that applying the horizontal shift by the gain G_0 to the meta distribution of the PPP closely approximates the meta distribution of the triangular lattice and the Matérn cluster process.

II. System Model

We consider a cellular network where the locations of base stations (BSs) are modeled by an arbitrary stationary and ergodic point process $\Phi \subset \mathbb{R}^2$. We focus on the cellular user situated at the typical location $o = (0, 0)$, henceforth called the typical user. All BSs are always active, and they transmit at unit power. We focus on the downlink with the nearest-BS association, where other BS transmissions cause interference. The signal propagation experiences fading as well as path loss. We assume independent and identically distributed (i.i.d.) Rayleigh fading where the channel power gains are exponentially distributed with mean 1. The path loss function is given by $\ell(x) = \|x\|^{-\alpha}$, where $\alpha > 2$ is the path loss exponent.

We focus on an interference-limited network where the received SIR determines the network performance. Let $x_0 \triangleq \arg\min\{x \in \Phi\colon \|x\|\}$ be the nearest BS to the typical user. The downlink SIR at the typical user is then given by

$$\mathsf{SIR} = \frac{h_{x_0}\ell(x_0)}{\sum_{x\in\Phi\setminus\{x_0\}} h_x \ell(x)}, \tag{1}$$

where h_x represents the i.i.d. exponential random variable corresponding to the channel between the BS at x and the typical user, and $\sum_{x\in\Phi\setminus\{x_0\}} h_x\ell(x)$ is the interference experienced by the typical user.

From (1), we can define the MISR as

$$\mathsf{MISR} \triangleq \mathbb{E}\left(\frac{\sum_{x\in\Phi\setminus\{x_0\}} h_x\ell(x)}{\ell(x_0)}\right) = \mathbb{E}\left(\frac{\sum_{x\in\Phi\setminus\{x_0\}} \ell(x)}{\ell(x_0)}\right). \tag{2}$$

III. Approximate Meta Distribution Analysis Using the PPP (AMAPPP)

A. The Meta Distribution

For an SIR threshold θ and the reliability threshold x, the meta distribution of the SIR is given by

$$\bar{F}(\theta, x) = \bar{F}_{P_\mathrm{s}}(\theta, x) \triangleq \mathbb{P}^0(P_\mathrm{s}(\theta) > x), \quad \theta \in \mathbb{R}^+, x \in [0, 1], \tag{3}$$

where $P_\mathrm{s}(\theta)$ is a random variable that represents the link success probability conditioned on the point process Φ, given by

$$P_\mathrm{s}(\theta) \triangleq \mathbb{P}(\mathsf{SIR} > \theta \mid \Phi). \tag{4}$$

Here the probability is taken with respect to the fading and the channel access scheme (if random). In (3), $\mathbb{P}^0(\cdot)$ is the reduced Palm probability, given that the link corresponding to the receiver is active, and the SIR is calculated at that receiver. The meta distribution is the complementary cumulative distribution function (ccdf) of the conditional link success probability $P_{\rm s}(\theta)$. Interpreted differently, for an ergodic point process, the meta distribution yields the fraction of cellular users that achieve an SIR of θ with reliability at least x.

The standard success (coverage) probability $p_{\rm s}(\theta)$ (the SIR distribution) can be directly obtained from the meta distribution as the mean of the conditional link success probability $P_{\rm s}(\theta)$, *i.e.*,

$$p_{\rm s}(\theta) = \mathbb{E}(P_{\rm s}(\theta)) = \int_0^1 \bar{F}(\theta, x) {\rm d}x. \tag{5}$$

Clearly the distribution of $P_{\rm s}(\theta)$ provides much more fine-grained information than merely its average $p_{\rm s}(\theta)$.

Finding the exact meta distribution directly seems infeasible, but if we can calculate or approximate the moments

$$M_b(\theta) \triangleq \mathbb{E}(P_{\rm s}(\theta)^b), \quad b \in \mathbb{C}, \tag{6}$$

as has been done in [4] for Poisson cellular networks, we can calculate the meta distribution in (3) using the Gil-Pelaez theorem [5] as

$$\bar{F}(\theta, x) = \frac{1}{2} + \frac{1}{\pi} \int\limits_0^\infty \frac{\Im(e^{-jt\log x} M_{jt})}{t} {\rm d}t, \tag{7}$$

where $\Im(u)$ is the imaginary part of $u \in \mathbb{C}$. Even though the expression of the meta distribution in (7) is exact, it is too complex to gain useful insights and not very convenient to evaluate numerically. The things become more difficult if we try to calculate the meta distribution in a non-Poisson cellular network. Hence it would be extremely useful to have a simple approximation to obtain the meta distribution in a non-Poisson cellular network. We propose to do so by combining the meta distribution with ASAPPP, discussed in the next subsection.

B. ASAPPP

As aforementioned, ASAPPP is the method that provides the approximation of the SIR distributions of non-Poisson networks by that of the Poisson network by applying a horizontal shift to the Poisson SIR distribution. This method asserts that if the network model under consideration and the Poisson model only differ in the type of the underlying point process, then the SIR ccdf for network model under consideration can be closely approximated using the SIR ccdf of the PPP by scaling the SIR threshold θ by some factor G_0 [10], *i.e.*,

$$p_{\rm s}(\theta) \approx p_{\rm s}^{\rm PPP}(\theta/G_0), \tag{8}$$

which corresponds to a horizontal shift by G_0 (in dB) if θ is plotted in dB. The subscript in G_0 correponds to $\theta \to 0$, *i.e.*, the shift is calculated for $\theta \to 0$. This asymptotic shift G_0 can also be interpreted as an SIR gain, similar to the notion of the coding gain in coding theory [14]. As shown in [10], the gain G_0 provides an excellent approximation to the entire SIR distribution. This approximation becomes exact as $\theta \to 0$, *i.e.*,

$$p_{\rm s}(\theta) \sim p_{\rm s}^{\rm PPP}(\theta/G_0), \quad \theta \to 0. \tag{9}$$

Moreover, the gain G_0 shows little sensitivity to the path loss exponent or the fading model [10]; it is a robust constant that captures the difference in the network topologies due to the underlying point process models. The gain G_0 can be expressed using the MISRs of the point process under consideration and the PPP as

$$G_0 = \frac{\mathsf{MISR}_{\rm PPP}}{\mathsf{MISR}}, \tag{10}$$

$$= \frac{2}{\alpha - 2} \frac{1}{\mathsf{MISR}}, \tag{11}$$

where $\mathsf{MISR}_{\rm PPP}$ and MISR denote the MISRs of the PPP and the network model under consideration, respectively. From (11), we can see that the calculation of G_0 numerically is quite easy, which motivates us to investigate whether a horizontal shift of G_0 to the meta distribution of the Poisson cellular network approximates the meta distribution of a non-Poisson cellular network. We call this approach "Approximate meta distribution analysis using the PPP" (AMAPPP).

C. Combining ASAPPP and the Meta Distribution to AMAPPP

The goal here is an approximation of the form

$$\bar{F}(\theta, x) \approx \bar{F}^{\rm PPP}(\theta/G_0, x). \tag{12}$$

For the downlink Poisson cellular network, the bth moment $M_b^{\rm PPP}(\theta)$ of the conditional success probability is given by [4]

$$M_b^{\rm PPP}(\theta) = \frac{1}{{}_2F_1(b, \delta; 1-\delta; -\theta)}, \quad b \in \mathbb{C}, \tag{13}$$

where ${}_2F_1(\cdot, \cdot; \cdot; \cdot)$ denotes the Gaussian hypergeometric function and $\delta \triangleq 2/\alpha$. As shown in (7), the meta distribution can be exactly calculated using the moments $M_b(\theta)$. Hence an interesting question is how the bth moments $M_b(\theta)$ of an arbitrary stationary and ergodic point process model and $M_b^{\rm PPP}(\theta)$ are related to each other, as $\theta \to 0$. The following theorem answers it.

Theorem 1. *For any stationary and ergodic point process and $b \in \mathbb{C}$,*

$$M_b(\theta) \sim M_b^{\rm PPP}(\theta/G_0), \quad \theta \to 0. \tag{14}$$

Proof: From [4, (22)], for any stationary and ergodic point process model, we have

$$M_b(\theta) = \mathbb{E} \prod_{x \in \Phi \setminus \{x_0\}} \frac{1}{(1 + \theta(\|x_0\|/\|x\|)^\alpha)^b}, \quad b \in \mathbb{C} \tag{15}$$

$$\overset{\rm (a)}{\sim} \mathbb{E} \prod_{y \in \mathcal{R}} (1 - b\theta y^\alpha), \quad \theta \to 0 \tag{16}$$

$$\sim 1 - b\theta \left(\mathbb{E} \sum_{y \in \mathcal{R}} y^\alpha \right), \quad \theta \to 0 \tag{17}$$

$$\overset{\rm (b)}{=} 1 - b\theta\, \mathsf{MISR}, \tag{18}$$

where $\mathcal{R} \triangleq \{x \in \Phi \setminus \{x_0\} \colon \|x_0\|/\|x\|\}$ is the relative distance process (RDP) [10, Def. 2], (a) follows by letting

$y = \|x_0\|/\|x\|$ and using Taylor series expansion, and (b) follows from the definition of the MISR for the RDP [10]. Using (10) and (18), we reach the desired result. ■

Note that even though all moments are shifted by the same amount G_0 asymptotically, this does not imply that the meta distribution is also shifted by that amount. However, we can expect the shifted meta distribution of the PPP to provide a good approximation. In the next section, we explore by simulation whether this is the case.

IV. Simulation Results

In this section, using simulations, we verify the accuracy of approximating the meta distribution of stationary and ergodic point processes by shifting the meta distribution of the PPP by G_0.

We focus on two types of stationary and ergodic non-Poisson processes: 1) The triangular lattice (the most regular stationary point process) and 2) The Matérn cluster process (MCP) [2, Chapter 3]. The BS deployment using the PPP is completely random, *i.e.*, it is between a regular deployment (as in the triangular lattice) and a clustered deployment (as in the MCP).

Matérn cluster process (MCP): The MCP is a doubly Poisson cluster process, where the parent point process $\Phi_{\rm p}$ is a PPP with intensity $\lambda_{\rm p}$ and the daughter points are uniformly distributed within a ball of radius $r_{\rm c}$ with each parent point $x_{\rm p} \in \Phi_{\rm p}$ as its center. The intensity of the daughter point process of parent $x_{\rm p}$ is given by

$$\lambda_{\rm d}(x) = \frac{\bar{c}}{\pi r_{\rm c}^2} \mathbf{1}_{B(x_{\rm p}, r_{\rm c})}(x), \tag{19}$$

where $B_{(x_{\rm p}, r_{\rm c})}(x) \triangleq \{x \in \mathbb{R}^2 \colon \|x - x_{\rm p}\| \leq r_{\rm c}\}$, $\bar{c}$ is the average number of daughter points in a cluster, and $\mathbf{1}(\cdot)$ is the indicator function. The intensity of the MCP is $\lambda = \lambda_{\rm p}\bar{c}$.

Simulation setup: We perform simulations over a square region with side length 200 for two values of the path loss exponent, $\alpha = 3$ and $\alpha = 4$. For the MCP, we assume the following simulation parameters: $\lambda_{\rm p} = 0.01$, $\bar{c} = 10$, and $r_{\rm c} = 5$. We average over 10000 fading realizations and 10000 realizations of the point process.

Fig. 1 plots the meta distribution values for different SIR thresholds θ for the PPP, the MCP, and the triangular lattice with $\alpha = 3$. We observe that ASAPPP can indeed be used to approximately calculate the meta distribution of a non-Poisson network, *i.e.*, the meta distribution of the MCP and the triangular lattice can be obtained approximately by simply applying a horizontal shift of G_0 (in dB) to the meta distribution of the PPP.

For the reliability thresholds $x = 0.95$ and $x = 0.70$, Figs. 2 and 3, respectively, validate the approximation obtained by applying the shift of G_0 to the meta distribution value of the PPP for $\alpha = 4$. For the MCP, the approximation is accurate for all values of θ, while for the triangular lattice, the approximation is quite close for the meta distribution values of practical interest, *i.e.*, for $\bar{F}(\theta, x) > 0.6$.

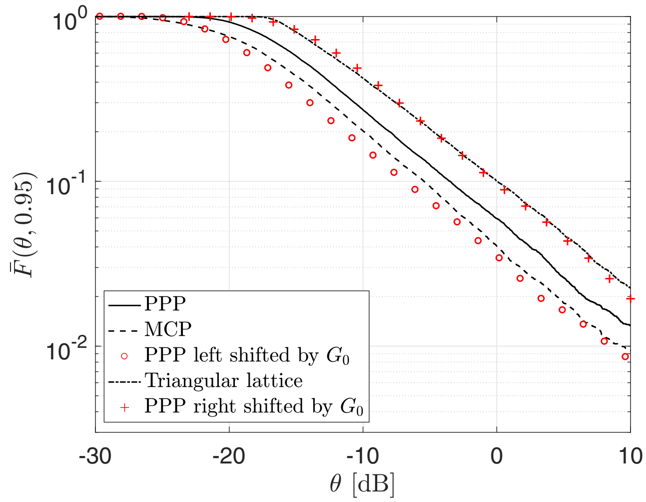

Fig. 1. The meta distribution $\bar{F}(\theta, x)$ of the PPP, the MCP, and the triangular lattice against the SIR threshold θ for the path loss exponent $\alpha = 3$ and the reliability threshold $x = 0.95$. The asymptotic gain G_0 is -3.34 dB for the MCP and 3.32 dB for the triangular lattice.

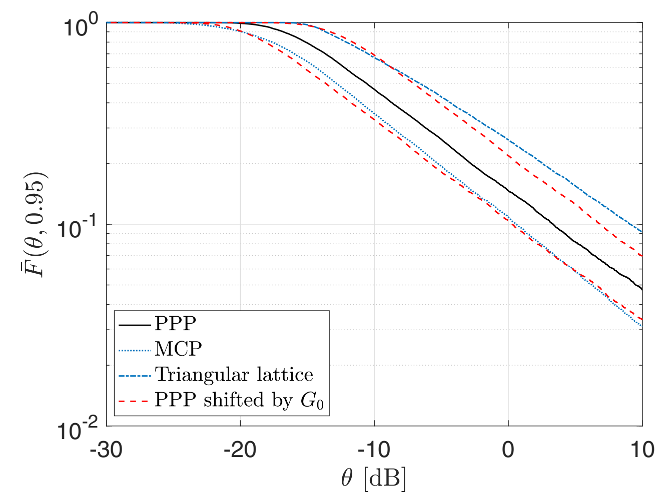

Fig. 2. The meta distribution $\bar{F}(\theta, x)$ of the PPP, the MCP, and the triangular lattice against the SIR threshold θ for the path loss exponent $\alpha = 4$ and the reliability threshold $x = 0.95$. The asymptotic gain G_0 is -3.04 dB for the MCP and 3.51 dB for the triangular lattice.

In Fig. 4, we take a closer look at the regime where θ is very small (the high-reliability regime). For the MCP, we observe that though there is a gap between the actual curve and the approximation (the shifted PPP), the gap is constant, *i.e.*, the slope of both curves is the same. For the triangular lattice, we have an interesting case; at $\theta = -16.90$ dB, the value of $1 - \bar{F}(\theta, x)$ drops to zero. This happens because the fraction of users that have a reliability smaller than $x = 0.95$ is zero for $\theta < -16.90$ dB. For all x, such a threshold can be calculated by shifting the lattice such that the user sits at a Voronoi vertex for the triangular lattice, which is the worst-case scenario for the triangular lattice since there are 3 nearest base stations to

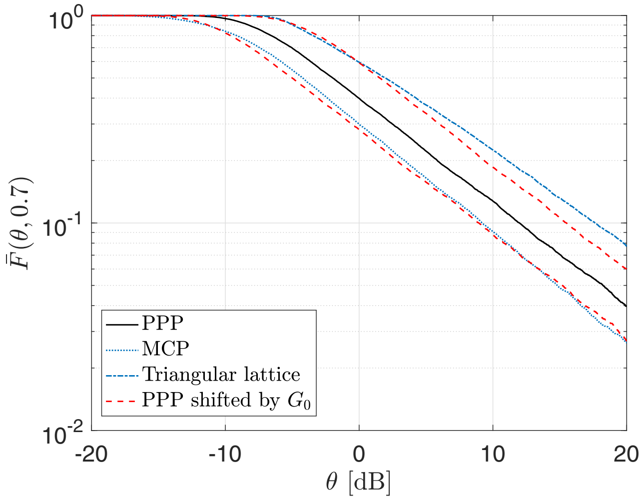

Fig. 3. The meta distribution $\bar{F}(\theta, x)$ of the PPP, the MCP, and the triangular lattice against the SIR threshold θ for the path loss exponent $\alpha = 4$ and the reliability threshold $x = 0.70$. The asymptotic gain G_0 is -3.04 dB for the MCP and 3.51 dB for the triangular lattice.

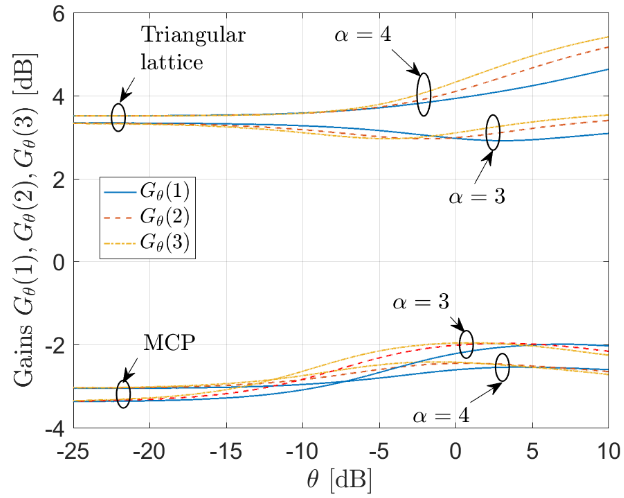

Fig. 5. The gains $G_\theta(1)$, $G_\theta(2)$, and $G_\theta(3)$ corresponding to the moments $M_1(\theta)$, $M_2(\theta)$, and $M_3(\theta)$, respectively, for the reliability threshold $x = 0.95$.

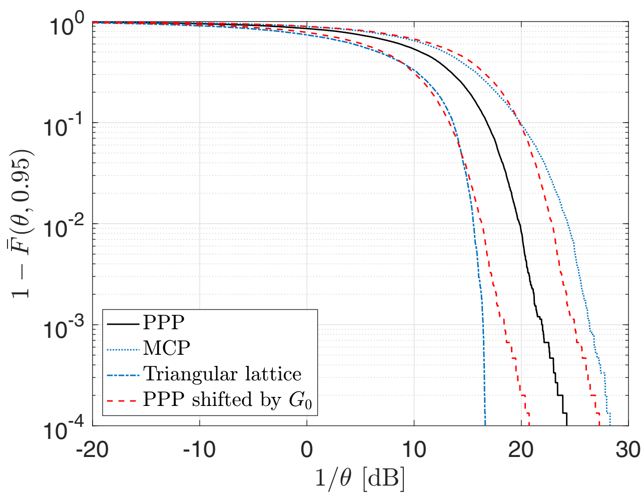

Fig. 4. The meta distribution $1 - \bar{F}(\theta, x)$ of the PPP, the MCP, and the triangular lattice against $1/\theta$ for the path loss exponent $\alpha = 4$ and the reliability threshold $x = 0.95$. The asymptotic gain G_0 is -3.04 dB for the MCP and 3.51 dB for the triangular lattice.

the user. For all lattices, there exists such a vertical asymptote, and thus the approximation by shifting G_0 breaks down as θ approaches that threshold. But for values of θ for which the 5% user achieves 95% reliability, the approximation is tight, and for smaller values, it provides a lower bound.

Given that ASAPPP aims at approximating the first moment $M_1(\theta)$ of the conditional link success probability $P_{\rm s}(\theta)$ and the meta distribution can be obtained using the moments (see (7)), we calculate the deployment gain $G_\theta(b)$ with respect to bth moment $M_b(\theta)$ for an arbitrary θ. The gain $G_\theta(b)$ is the ratio (gap if measured in dB) θ'/θ, where θ' is given by $M_b(\theta') = M_b^{\rm PPP}(\theta)$. The moment $M_b^{\rm PPP}(\theta)$ is given by (13).

Recall that the asymptotic deployment gain $G_0 = G_0(1)$ corresponds to $M_1(\theta)$ as $\theta \to 0$. For different values of θ, Fig. 5 plots the gains $G_\theta(1), G_\theta(2)$, and $G_\theta(3)$ for the triangular lattice and the MCP with $\alpha = 3, 4$. We observe from Fig. 5 that for both the triangular lattice and the MCP, the asymptotic gain $G_0(1)$ is a good approximation of $G_\theta(1)$, $G_\theta(2)$, and $G_\theta(3)$ for all values of θ and that the asymptotic value is essentially reached at $\theta = -15$ dB. Moreover, there is little sensitivity to the value of the path loss exponent α.

V. Conclusions

In this paper, we have proposed AMAPPP, a simple and novel approach to approximately obtain the meta distribution of an arbitrary stationary and ergodic point process from that of the PPP. For the bth moment $M_b(\theta)$ of the conditional success probability of any stationary and ergodic point process, we proved that $M_b(\theta) \sim 1 - b\theta\,\mathsf{MISR}$, as $\theta \to 0$. Through detailed simulations for the triangular lattice and the Matérn cluster process, we have shown that the asymptotic deployment gain G_0 of the standard success (coverage) probability can be used to relate the meta distribution to that of the PPP. We have observed that the approximation of the meta distribution of the triangular lattice by that of the PPP becomes pessimistic in the worst-case scenario, *i.e.*, when the typical cellular user is located such that it has three nearest base stations. Overall, given the generality of the model and the fine-grained nature of the meta distribution, AMAPPP works surprisingly well.

Acknowledgment

We gratefully acknowledge the partial support of the U.S. National Science Foundation through grant CCF 1525904.

References

[1] A. Guo and M. Haenggi, "Spatial stochastic models and metrics for the structure of base stations in cellular networks," *IEEE Transactions on Wireless Communications*, vol. 12, pp. 5800–5812, November 2013.

[2] M. Haenggi, *Stochastic Geometry for Wireless Networks*. Cambridge, U.K.: Cambridge University Press, 2012.

[3] C.-H. Lee, C.-Y. Shih, and Y.-S. Chen, "Stochastic geometry based models for modeling cellular networks in urban areas," *Wireless Networks*, vol. 19, pp. 1063–1072, August 2013.

[4] M. Haenggi, "The meta distribution of the SIR in Poisson bipolar and cellular networks," *IEEE Transactions on Wireless Communications*, vol. 15, pp. 2577–2589, April 2016.

[5] J. Gil-Pelaez, "Note on the inversion theorem," *Biometrika*, vol. 38, pp. 481–482, December 1951.

[6] N. Deng, W. Zhou, and M. Haenggi, "The Ginibre point process as a model for wireless networks with repulsion," *IEEE Transactions on Wireless Communications*, vol. 14, pp. 107–121, January 2015.

[7] A. Guo and M. Haenggi, "Asymptotic deployment gain: A simple approach to characterize the SINR distribution in general cellular networks," *IEEE Transactions on Communications*, vol. 63, pp. 962–976, March 2015.

[8] M. Haenggi, "ASAPPP: A simple approximate analysis framework for heterogeneous cellular networks," December 2014. Keynote presentation at the 2014 Workshop on Heterogeneous ans Small Cell Networks (HetSNets'14). Available at https://www3.nd.edu/~mhaenggi/talks/hetsnets14.pdf.

[9] M. Haenggi, "The mean interference-to-signal ratio and its key role in cellular and amorphous networks," *IEEE Wireless Communications Letters*, vol. 3, pp. 597–600, December 2014.

[10] R. K. Ganti and M. Haenggi, "Asymptotics and approximation of the SIR distribution in general cellular networks," *IEEE Transactions on Wireless Communications*, vol. 15, pp. 2130–2143, March 2016.

[11] Y. Wang, M. Haenggi, and Z. Tan, "The meta distribution of the SIR for cellular networks with power control," *IEEE Transactions on Communications*. Accepted.

[12] M. Salehi, A. Mohammadi, and M. Haenggi, "Analysis of D2D underlaid cellular networks: SIR meta distribution and mean local delay," *IEEE Transactions on Communications*, vol. 65, pp. 2904–2916, July 2017.

[13] Q. Cui, X. Yu, Y. Wang, and M. Haenggi, "The SIR meta distribution in Poisson cellular networks with base station cooperation," *IEEE Transactions on Communications*. Accepted.

[14] S. Lin and D. J. Costello, *Error Control Coding*. Englewood Cliffs, NJ: Prentice-Hall, 2nd ed., 2004.