Interference and Outage in Doubly Poisson Cognitive Networks

Chia-han Lee and Martin Haenggi
{clee14,mhaenggi}@nd.edu
Department of Electrical Engineering
University of Notre Dame
Notre Dame, IN 46556, USA

***Abstract*— We consider a cognitive radio network with two types of users: primary users (PUs) and cognitive users (CUs), whose locations follow two independent Poisson point processes. The cognitive users follow the policy that a cognitive transmitter is active only when it is outside the primary user exclusion regions. Due to the interaction between the primary users and the cognitive users through exclusion regions, an exact calculation of the interference and the outage probability seems unfeasible. Instead, we derive bounds for the interference (in the form of Laplace transform) and the outage probability. Two network models, the bipolar and the nearest-neighbor model, are considered.**

Keywords- Cognitive Radio, Cognitive Network, Interference Modeling, Poisson Point Process, Poisson Cluster Process, Stochastic Geometry.

I. Introduction

The proposal of cognitive radio stems from the inefficiency of the spectrum usage. The idea is that a cognitive (secondary) user can share the spectrum with a primary user as long as the interference caused is below a threshold [1]. In wireless networks, a cognitive user can take advantage of either the time (when a primary user is not transmitting), the frequency (when a primary user is transmitting at a different frequency band), or the space (when a primary user is far away). The last is a form of spatial reuse, thus the geometry plays a key role in this type of (cognitive) network that is considered in this paper. A cognitive user may transmit when the neighboring primary users are idle, but the signals could still interfere with farther primary users. As a result, there is a need to characterize the *aggregate* interference in order to satisfy the interference temperature metric [1].

Although there is already a vast body of research on cognitive networks, relatively little research has focused on the *aggregate interference* caused by multiple secondary users, together with the interference that the primary users cause among themselves. Hong *et al.* [2] and Ghasemi and Sousa [3] modeled the aggregate interference from the cognitive users, but both papers did not consider the interference from other primary users. Yin *et al.* [4] derived the maximum primary and secondary transmitter densities given outage constraints for the overlaid network under AWGN channel, without considering the exclusion regions and the fading channel.

In this paper, we consider a cognitive radio network with two types of users: primary users (PUs) and cognitive users (CUs). Primary users are licensed users while cognitive users are allowed to transmit only if the performance of the primary network is hardly affected. The cognitive users employ the following "cognition" in order to control their interference: a cognitive user will transmit only when it is outside the *primary exclusion regions* of all primary users. Although through this policy a cognitive user will limit its interference to the primary receivers, the aggregate interference might still harm the primary receivers. We are, therefore, interested in calculating the aggregate interference and the outage probability of the cognitive radio network. Due to the interaction between the primary users and the cognitive users, an exact calculation seems unfeasible. Instead, the interference and outage for two different Poisson cognitive network models, *bipolar* (BP) and *nearest-neighbor* (NN), are analyzed and bounded.

Our main contribution is that this paper, to our best knowledge, is the first to analyze *all four types of aggregate interference* between primary and cognitive users in spectrum sensing cognitive networks (including the auto-interference between primary users among themselves and secondary users among themselves as well as the cross-interference from secondary to primary users and vice versa). Furthermore, this is the first paper to analyze the interference and outage for the Poisson-type cognitive network with the nearest-neighbor model. The nearest-neighbor model, where each node is communicating with its nearest neighbor, is more challenging to analyze than the so-called bipolar model, where each transmitter is assumed to have a receiver at a fixed distance. See Fig. 1 for an illustration of both models.

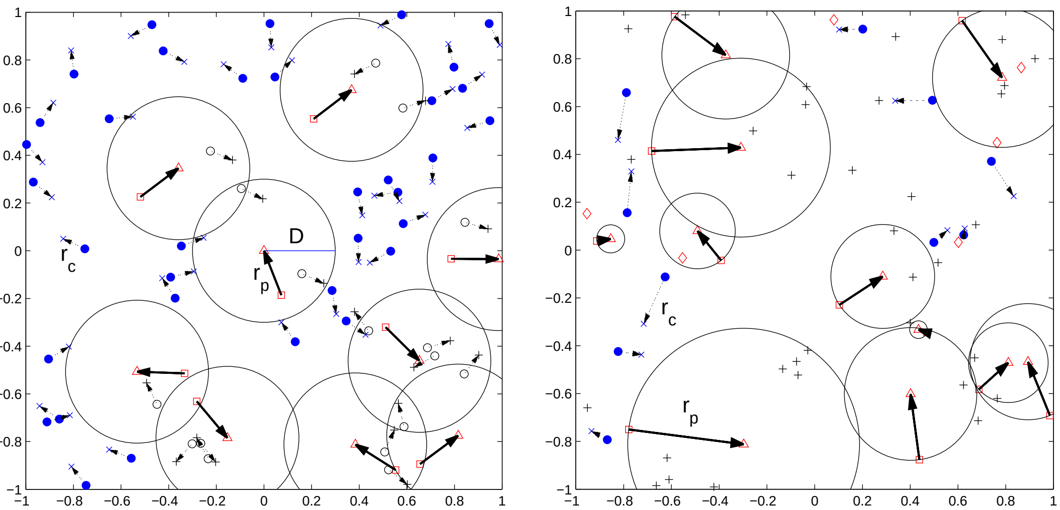
II. Network Model

We consider two different network models, the bipolar and the nearest-neighbor model. Although the bipolar model might not be as realistic as the nearest-neighbor model, it is easier to manipulate and can be viewed as an approximation to more complex models.

A. The Bipolar Model

The bipolar (BP) model is shown in Fig. 1(a). The locations of the primary transmitters follow a homogeneous Poisson point process (PPP)[1] $\Phi_p = \{x_1, x_2, \ldots\} \subset \mathbb{R}^2$ of density λ_p, and the locations of the potential cognitive transmitters follow another, independent, homogeneous Poisson point process $\Phi_c = \{y_1, y_2, \ldots\} \subset \mathbb{R}^2$ of density λ_c. We assume that all the primary transmitters use the same transmission

[1] The advantages and validity of using PPP for modeling the locations of the wireless devices have been stated in many articles. Readers may refer to [5] and [6] for more information.



(a) (b)

Fig. 1. (a) The bipolar model. The squares are the primary transmitters and the triangles are the primary receivers, and the transmitter-receiver pairs are represented by thick lines with the arrows pointing to the receivers. The distance between a primary transmitter-receiver pair is r_p. The big circles are the exclusion regions with radius D. The filled circles are the cognitive transmitters and the x's are the cognitive receivers. The hollow circles and the +'s are the cognitive transmitters and receivers that are inactive due to the exclusion regions. The cognitive transmitter-receiver pairs are represented by thin lines with the arrows pointing to the receivers, and the distance between a cognitive transmitter-receiver pair is r_c. (b) The nearest-neighbor model. The squares, triangles, filled circles, and x's represent primary transmitters, primary receivers, active cognitive transmitters, and active cognitive receivers, respectively, as in the bipolar model. The diamonds and the +'s are, respectively, the primary users and the cognitive users that are inactive. The big circles are the exclusion regions with radius equal to r_p, which is a random variable. The distance r_c between a cognitive transmitter-receiver pair is also a random variable.

power μ_p, and all the primary receivers are at a distance r_p away from the corresponding primary transmitters in a random direction. Similarly, all the cognitive transmitters use the same transmission power μ_c, and all the cognitive receivers are at a distance r_c away from the corresponding cognitive transmitters. Under this setup, the locations of the primary and the cognitive receivers are also PPPs with density λ_p and λ_c, respectively. We assume that r_p is small relative to the mean nearest-neighbor distance of Φ_c ($r_c \ll \lambda_c^{-\frac{1}{2}}$) since the transmission power and the range of the cognitive users are usually small. The activation of the cognitive users depends on the exclusion region setup of the primary users. The exclusion region is a circular region with radius D whose purpose is to guarantee that cognitive transmitters will on average not cause an aggregate interference larger than the signal-to-interference ratio (SIR)[2] threshold θ_p that a primary user can tolerate. An outage happens when the instantaneous SIR is lower than θ_p. Similarly, the SIR threshold for the cognitive users is denoted as θ_c.

Definition 1: The radius D of the exclusion region in the bipolar model is chosen as

$$D = r_p \left[\theta_p \left(\frac{\beta\mu_c}{\mu_p}\right)\right]^{\frac{1}{\alpha}}, \tag{1}$$

where α is the path loss exponent and β is a design factor such that β cognitive transmitters will on average not cause an aggregate interference that would result in an SIR below the threshold.
We assume D is larger than $r_p + r_c$, ensuring the primary transmitters are inside the exclusion regions such that a cognitive receiver and a primary transmitter cannot be arbitrarily close.

[2]Throughout the paper, we neglect the noise since interference is what causes the interaction between primary and cognitive users. Hence we focus on the SIR instead of the SINR.

B. The Nearest-Neighbor Model

The second network model is the nearest-neighbor (NN) model (Fig. 1(b)). We assume that the locations of the primary users (including both the transmitters and receivers) follow a *single* homogeneous Poisson point process (PPP) $\Phi_p = \{x_1, x_2, \dots\} \subset \mathbb{R}^2$ of density λ_p, and the locations of the cognitive users (including both the transmitters and receivers) follow another *single*, independent, homogeneous Poisson point process $\Phi_c = \{y_1, y_2, \dots\} \subset \mathbb{R}^2$ of density λ_c. The primary users begin with identifying their nearest neighbors[3]. If *every* primary user talks to its nearest neighbor (we call this the *unconditional nearest-neighbor* (UNN) case), full-duplex communication is required, which is impractical. A more practical way is to first have primary nodes identify their nearest neighbors and then form suitable transmitter-receiver pairs. One way to achieve this is the following. Suppose the nearest neighbor of a node $\mathcal{A}$ is node $\mathcal{B}$ (which is unique almost surely in the Poisson case). A *mutual nearest-neighbor* (MNN) pair $\{\mathcal{A}, \mathcal{B}\}$ is formed when the nearest neighbor of node $\mathcal{B}$ is node $\mathcal{A}$. For each pair, we then randomly pick one of the nodes as the transmitter, resulting in the pairs $\mathcal{P}_p = \{(T_{p,i}, R_{p,i}) : i \in 1, 2, \dots\}$, where $T_{p,i}$'s are primary transmitters and $R_{p,i}$'s are primary receivers. Forming the MNN pairs can be viewed as a thinning of the transmitters in the UNN case. At the end of this pairing process, about 38% of the primary users will be left unpaired and forced inactive (see Lemma 9). In order to further lower the interference, the primary users may apply the ALOHA protocol, i.e., each transmitter (every node in the UNN case or every transmitter

[3]This is achieved by broadcasting neighborhood discovery messages.

$T_{p,i}$ in the MNN case) randomly (with probability p_p) decides whether to transmit or not. Note that ALOHA can only be applied to the transmitters after the determination of the nearest neighbor, otherwise the distance profile would be different. The range of the exclusion regions in the NN model is defined as follows.

Definition 2: The radius D of the exclusion region in the NN model is r_p, the distance between a primary transmitter and its receiver. The r_p's are i.i.d. random variables with probability density function (pdf) $f_{r_p}(r) = 2\pi r\lambda_p \exp\left(-\lambda_p \pi r^2\right)$ for the UNN case and

$$f_{r_p}(r) = \frac{e^{-\lambda_p A_M r^2}}{P_M} 2\pi r\lambda_p e^{-\lambda_p \pi r^2} \tag{2}$$

for the MNN case, where $A_M = \pi/3 + \sqrt{3}/2$ and P_M is the probability of a node having a mutual nearest neighbor (see Lemma 9).

Assume that power control is applied such that the transmission power of the primary users is set to $\mu_p = r_p^\alpha$. The received power is thus equal to 1 ($\mu_p r_p^{-\alpha} = 1$). The cognitive users first determine their nearest neighbors and perform the transmitter-receiver pairing to obtain the cognitive transmitter-receiver pairs $\mathcal{P}_c = \{(T_{c,i}, R_{c,i}) : i \in 1, 2, \ldots\}$, where $T_{c,i}$'s are cognitive transmitters and $R_{c,i}$'s are cognitive receivers. The distance r_c between a cognitive transmitter-receiver pair is also a random variable. Then, the cognitive transmitters outside the exclusion regions are tagged with $\tau_i = 1$, while the ones inside the exclusion regions are tagged $\tau_i = 0$. The receivers $R_{c,i}$ are allowed in the exclusion regions. The transmission power of the cognitive users is set to $\mu_c = r_c^\alpha$, so the received power is always 1. After that, ALOHA with parameter p_c is applied to the cognitive transmitters $T_{c,i}$ with tag $\tau_i = 1$.

C. Notation

For both models, we assume that the cognitive users can sense the spectrum perfectly with respect to the primary receivers[4] so there are no active cognitive users within the exclusion regions. We define $I(y) = \sum_{x\in\Phi} \mu_x h_x g(y-x)$ as the total interference from x to y, where $g(x) = \|x\|^{-\alpha}$ is the large-scale path loss model, and assume the fading h is i.i.d. exponential (Rayleigh fading) with $\mathbb{E}[h] = 1$. In the bipolar model, μ_x is either μ_p or μ_c (thus a fixed value), while in the nearest-neighbor model, μ_x is a random variable which depends on the distance to the nearest neighbor. The Laplace transform of $I(y)$ is $\mathbb{E}[\exp(-sI)]$ and denoted as $\mathcal{L}_I(s)$.

The interference to the primary users and the interference to the cognitive users are considered separately. For each case, the interference is comprised of contributions by both primary users and cognitive users, so there are four types of interference: the interference from the primary transmitters to the primary receivers I_{pp}, the interference from the primary transmitters to the cognitive receivers I_{pc}, the interference from the cognitive transmitters to the primary receivers I_{cp}, and the interference from the cognitive transmitters to the cognitive receivers I_{cc}. To calculate the interference to the primary users, we consider having a primary receiver at the origin, the *typical receiver*, which yields the Palm distribution for the primary transmitters. By Slivnyak's theorem, this conditional distribution is the same as the original one for the rest of the primary network (but might be different for the secondary network). For the secondary network, however, conditioning on a typical cognitive receiver generally changes the distance distribution since the activation of the cognitive transmitters is determined by the locations of the primary users. This is the reason why we can only obtain bounds for any interference involving the cognitive users.

[4] How to perform sensing of the primary users is outside the scope of this paper, and many schemes have been proposed. If the primary receivers are passive, detecting the power leakage of local oscillator (LO) is a possible way. See [7] for a survey.

III. Analysis of the Bipolar Model

In this section, we consider the bipolar model with the exclusion regions around the primary receivers.

Lemma 1: Let $I(y) = \sum_{x\in\Phi} \eta h \|x-y\|^{-\alpha}$ where Φ is PPP with density ν and h is i.i.d. exponential with $\mathrm{E}\{h\} = 1$, $\delta \triangleq 2/\alpha$, and

$$\mathcal{L}_0(\nu, \eta, s) \quad \triangleq \quad \exp\left\{-\nu \frac{\pi^2\delta}{\sin(\pi\delta)} \eta^\delta s^\delta\right\}. \tag{3}$$

Then the Laplace transform of the interference I is $\mathcal{L}_0(\nu, \eta, s)$.

Proof: Follows from Eq. (3.21) in [8]. ∎

Lemma 2: Let

$$\mathcal{L}_1(\nu, \eta, \rho, s) \triangleq \exp\Bigg\{-\nu\pi\Bigg(\eta^\delta s^\delta \mathbb{E}_h\left[h^\delta \gamma\left(1-\delta, s\eta h\rho^{-\alpha}\right)\right] - \frac{s\eta\rho^{2-\alpha}}{1+s\eta\rho^{-\alpha}}\Bigg)\Bigg\}, \tag{4}$$

where $\gamma(a,z) = \int_0^z \exp(-t) t^{a-1} \mathrm{d}t$ is the *lower* incomplete gamma function. Following the setup in Lemma 1, when a CSMA-type MAC is applied with a carrier sensing range ρ, the Laplace transform of the interference I is $\mathcal{L}_1(\nu, \eta, \rho, s)$.

Proof: Follows from Eq. (3.46) in [8]. ∎

A. Interference to Primary Users

Lemma 3: The Laplace transform of the interference to a primary receiver from other primary transmitters is

$$\mathcal{L}_{I_{pp}}(s) = \mathcal{L}_0(\lambda_p, \mu_p, s). \tag{5}$$

Proof: Since the fading is Rayleigh and the primary transmitters are distributed as a PPP, the Laplace transform of the interference I_{pp} is obtained from Lemma 1 with density λ_p and transmission power μ_p. ∎

Definition 3: A stochastically dominates B if the cumulative distribution functions satisfy $F_A(x) \geq F_B(x)$ for all x.

Lemma 4: The interference I_{cp} to a primary receiver from the cognitive transmitters is stochastically dominated by the random variable $\hat{I}_{cp}$, denoted as $I_{cp} \overset{s}{<} \hat{I}_{cp}$, with Laplace transform

$$\mathcal{L}_{\hat{I}_{cp}}(s) = \mathcal{L}_1(\lambda_c, \mu_c, D, s). \tag{6}$$

Proof: Let Φ_a and $\Phi_{a'}$ be the partition of Φ_c into active and inactive nodes depending on whether the cognitive transmitters are outside or inside the exclusion regions. Let Φ_D

include all the points in Φ_c, except the points that are within the exclusion region of the typical primary receiver. Since $\Phi_a \subset \Phi_D$, the interference I_{cp} caused by the active cognitive transmitters is stochastically dominated by the interference $\hat{I}_{cp}$ caused by Φ_D. Since the cognitive transmitter is at least at distance D, the Laplace transform of $\hat{I}_{cp}$ is exactly (4) with density λ_c and transmission power μ_c. ■

Theorem 1: The outage probability of the primary users ϵ_p is upper-bounded as

$$\epsilon_p < 1 - \exp\left\{ -\theta_p^\delta r_p^2 \left[\lambda_p \frac{\pi^2 \delta}{\sin(\pi\delta)} + \lambda_c \pi \left(\frac{\mu_c}{\mu_p}\right)^\delta \times \left(\mathbb{E}_h \left[h^\delta \gamma \left(1-\delta, \frac{h}{\beta}\right)\right] - \frac{\beta^\delta}{1+\beta} \right) \right] \right\}. \quad (7)$$

Proof: With Rayleigh fading, the transmission success probability of the primary users is the Laplace transform evaluated at $\frac{\theta_p r_p^\alpha}{\mu_p}$ (see [8] for more details). Since the interference from the primary transmitters and the interference from the cognitive transmitters are independent, the outage probability ϵ_p is upper-bounded by $\hat{\epsilon}_p = 1 - \mathcal{L}_{I_{pp}}\left(\frac{\theta_p r_p^\alpha}{\mu_p}\right) \cdot \mathcal{L}_{\hat{I}_{cp}}\left(\frac{\theta_p r_p^\alpha}{\mu_p}\right)$. ■

Although the point process of active cognitive users Φ_a is not a PPP, independent thinning of the cognitive users outside the exclusion regions with probability $\exp(-\lambda_p \pi D^2)$ yields a good approximation on I_{cp}, since the higher-order statistics of the point process become less relevant if D is not too small. Thus the Laplace transform of the approximated interference $\tilde{I}_{cp}$ is

$$\mathcal{L}_{\tilde{I}_{cp}}(s) = \mathcal{L}_1\left(\lambda_c \exp(-\lambda_p \pi D^2), \mu_c, D, s\right). \quad (8)$$

Therefore, $\epsilon_p \approx 1 - \mathcal{L}_{I_{pp}}\left(\frac{\theta_p r_p^\alpha}{\mu_p}\right) \cdot \mathcal{L}_{\tilde{I}_{cp}}\left(\frac{\theta_p r_p^\alpha}{\mu_p}\right)$.

B. Interference to Cognitive Users

Lemma 5: Let $\bar{D} = D - r_p - r_c$ ($\bar{D} > 0$ since $D > r_p + r_c$ as described in Section II). The interference I_{pc} to a cognitive receiver from the primary transmitters is stochastically dominated by the random variable $\hat{I}_{pc}$ with Laplace transform

$$\mathcal{L}_{\hat{I}_{pc}}(s) = \mathcal{L}_1(\lambda_p, \mu_p, \bar{D}, s). \quad (9)$$

Proof: Since a cognitive transmitter is at least at distance D from a primary receiver, and the distance between a primary transmitter-receiver pair is r_p, the distance between a primary transmitter and a cognitive transmitter is at least $D - r_p$. Furthermore, the distance between a cognitive transmitter and its corresponding cognitive receiver is r_c, so the distance to the nearest primary transmitter for a cognitive receiver is at least $D - r_p - r_c$. Denote by $\hat{I}_{pc}$ the random variable whose Laplace transform is the same as in the CSMA-type MAC scheme by replacing the carrier sensing range with $D - r_p - r_c$. Since the location of the transmitter is not at the center of the exclusion region, $I_{pc} \overset{s}{<} \hat{I}_{pc}$. ■

Lemma 6: The interference to a cognitive receiver from other cognitive transmitters is stochastically dominated by the random variable $\hat{I}_{cc}$ with Laplace transform

$$\mathcal{L}_{\hat{I}_{cc}}(s) = \mathcal{L}_0(\lambda_c, \mu_c, s). \quad (10)$$

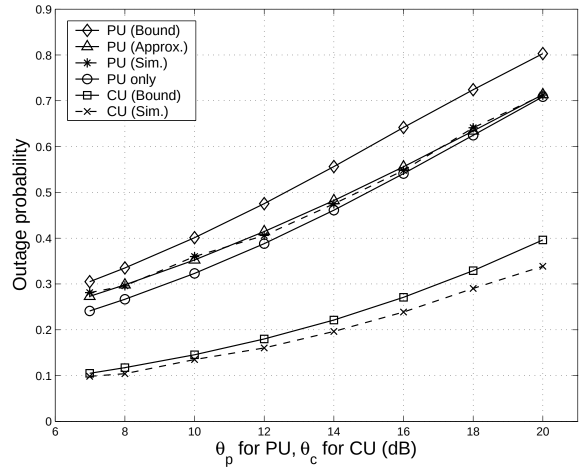


Fig. 2. Bounds and simulation results of the outage probabilities of the primary and the cognitive users. For comparison, the outage probability in the primary network without the presence of cognitive users ("PU only" in the figure) is also shown. The simulation parameters are: $\lambda_p = 0.1$, $\lambda_c = 1$, $\mu_p = 1$, $\mu_c = 0.2$, $r_p = 0.5$, $r_c = 0.1$, $\beta = 81$, and $\alpha = 4$. When calculating the outage probability of the cognitive users, θ_p is set to 10.

Proof: Let $\hat{I}_{cc}$ be the interference generated by the process Φ_c. Since $\Phi_a \subset \Phi_c$, I_{cc} is stochastically dominated by $\hat{I}_{cc}$. Since Φ_c is a PPP, the Laplace transform of $\hat{I}_{cc}$ follows from Lemma 1. ■

Theorem 2: Let $\xi = \frac{\theta_c \mu_p}{\mu_c}\left[\left(\frac{\theta_p \beta \mu_c}{\mu_p}\right)^{\frac{1}{\alpha}}\left(\frac{r_p}{r_c}\right) - \frac{r_p}{r_c} - 1\right]^{-\alpha}$. The outage probability of the cognitive users ϵ_c is upper-bounded as

$$\epsilon_c < 1 - \exp\left\{ -\lambda_p \pi \left[\theta_c^\delta \left(\frac{\mu_p}{\mu_c}\right)^\delta r_c^2 \mathbb{E}_h\left[h^\delta \gamma(1-\delta, \xi h)\right] - r_p^2 \left[\left(\frac{\theta_p \beta \mu_c}{\mu_p}\right)^{\frac{1}{\alpha}} - \frac{r_c}{r_p} - 1\right]^2 \left(\frac{\xi}{1+\xi}\right) \right] - \lambda_c \frac{\pi^2 \delta}{\sin(\pi\delta)} \theta_c^\delta r_c^2 \right\}. \quad (11)$$

Proof: The success transmission probability of the cognitive users is the Laplace transform evaluated at $\frac{\theta_c r_c^\alpha}{\mu_c}$. Since the interference from the primary transmitters and the interference from the cognitive transmitters are independent, the outage probability ϵ_c is upper-bounded by $\hat{\epsilon}_c = 1 - \mathcal{L}_{\hat{I}_{pc}}\left(\frac{\theta_c r_c^\alpha}{\mu_c}\right) \cdot \mathcal{L}_{\hat{I}_{cc}}\left(\frac{\theta_c r_c^\alpha}{\mu_c}\right)$. ■

Fig. 2 shows the simulation results and the upper bounds of the outage probabilities of the primary and cognitive users for different θ_p and θ_c. It also shows the approximation of the primary user outage probability and the simulation results for the primary user-only network. The simulation parameters are: $\lambda_p = 0.1$, $\lambda_c = 1$, $\mu_p = 1$, $\mu_c = 0.2$, $r_p = 0.5$, $r_c = 0.1$, $\beta = 81$, and $\alpha = 4$. D is determined using (1). We observe that for large θ_p the primary user outage is dominated by the interference from the primary users, since a large θ_p implies a large guard zone radius D, which means that few secondary users are active.

IV. Analysis of the Nearest-Neighbor Model

Now we turn our attention to the NN model in which nodes only transmit to their nearest neighbors and the transmitter-receiver distance is a random variable. The following three lemmas are required in order to analyze the interference and outage probability. The first gives the Laplace transform of the interference observed from a typical node at the origin for the UNN case with power control, the second derives the fraction of cognitive users that are active, and the last determines the density of the MNN pairs.

Lemma 7: Let Φ be a PPP. Suppose nodes in Φ talk to their nearest neighbors (the UNN case) with power control applied in order to compensate for the large-scale path loss. Let

$$\mathcal{L}_2(s) \triangleq \exp\left(-\frac{\pi\delta}{\sin(\pi\delta)}s^{\delta}\right). \tag{12}$$

Then the Laplace transform of the interference observed from a typical node at the origin is $\mathcal{L}_2(s)$.

Proof: Follows from Eq. (3.37) in [8]. ∎

Lemma 8: Assume the primary users intend to transmit to their nearest neighbors and the primary users apply some protocol (such as ALOHA) or some restriction (such as mutual nearest neighbors) that results in the thinning of the transmitters with probability ϑ. Then the fraction q of cognitive users that are active is $\exp(-\vartheta)$.

Proof: A cognitive user can be active only when it is outside of all the exclusion regions. Equivalently, the distance to a primary receiver located at $x \in \mathbb{R}^2$ from the CU must be larger than the radius of the exclusion region $r_p(x)$. Since Φ_c is stationary, we can consider a typical cognitive transmitter at the origin. The probability of $r_p(x)$ being smaller than $\|x\|$ is $1-\exp(-\lambda_p\pi\|x\|^2)$. Let Φ_p' be the point process after thinning. From the probability generating functional for PPPs,

$$\begin{aligned} q &= \mathbb{E}\left[\prod_{x\in\Phi_p'}\left\{1-\exp\left(-\lambda_p\pi\|x\|^2\right)\right\}\right] \\ &= \exp\left(-\lambda_p\vartheta\int_{\mathbb{R}^2}\exp(-\lambda_p\pi\|x\|^2)\mathrm{d}x\right) \\ &= \exp(-\vartheta). \end{aligned} \tag{13}$$

∎

Lemma 9: In a PPP with density λ, the density of MNN pairs is $\zeta\lambda$, where $\zeta = 3\pi/\left(8\pi+3\sqrt{3}\right) \approx 0.3107$.

Proof: Suppose the nearest neighbor of a node $\mathcal{A}$ is node $\mathcal{B}$. Let $P_M = 2\zeta$ be the probability that $\mathcal{A}$ is the nearest neighbor of $\mathcal{B}$. Straightforward calculation yields

$$P_M = \int_0^\infty e^{-r^2(\pi/3+\sqrt{3}/2)}2\pi r e^{-r^2\pi}\mathrm{d}r = \frac{6\pi}{8\pi+3\sqrt{3}}. \tag{14}$$

∎

A. Interference

Now we are ready to characterize the interference. We first derive a bound for the interference from the primary transmitters to both the primary and cognitive receivers, and then we give a bound for the interference from the cognitive transmitters. Due to the protection by the exclusion regions, the second bound is looser for the primary users compared to the cognitive users. A tighter bound for the interference from the cognitive transmitters to the primary users is then proposed. Since either ALOHA or MNN is a thinning of the UNN case, this allows us to obtain a general expression as described by the next three lemmas.

Lemma 10: Suppose the primary users transmit to their nearest neighbors with power control applied in order to compensate for the large-scale path loss. If every primary user is active with probability ϑ, the interference from the primary transmitters to either a primary or a cognitive receiver is stochastically dominated by the random variable with the Laplace transform

$$\mathcal{L}_{\hat{I}_p}(s) = \exp\left(-\vartheta\frac{\pi\delta}{\sin(\pi\delta)}s^{\delta}\right). \tag{15}$$

Proof: By modifying (12) to incorporate the density $\vartheta\lambda$ of primary transmitters, where λ is the original density, we get (15). However, (15) characterizes the interference observed at a typical node. When conditioned on a primary receiver (not a typical node), the distance profile would change since the nearest node is likely to be its corresponding transmitter and the nearest interferer is now farther away. As a result, the interference one may observe at the primary receiver is lower than what is calculated using (15). On the other hand, the exclusion region decreases the chance of cognitive receivers being close to the primary transmitters. This is because the primary transmitters are at the edges of the exclusion regions, and by inspecting the shape of an exclusion region, it is easily seen that more cognitive users close to the primary transmitters are silenced. Therefore, the interference to the cognitive users is again over-estimated. ∎

Remark 1: Suppose ALOHA with parameter p_p is applied. If every primary user transmits to its nearest neighbor (the UNN case), $\vartheta = p_p$; if the primary users form mutual nearest-neighbor transmitter-receiver pairs, $\vartheta = p_p\zeta$.

Lemma 11: Suppose both the primary and cognitive users transmit to their nearest neighbors with power control applied in order to compensate for the large-scale path loss. If a primary and a cognitive user are active with probability ϑ and ν, respectively, the interference from the cognitive transmitters to either a primary or a cognitive receiver is stochastically dominated by the random variable with the Laplace transform

$$\mathcal{L}_{\hat{I}_c}(s) = \exp\left(-\nu\exp(-\vartheta)\frac{\pi\delta}{\sin(\pi\delta)}s^{\delta}\right). \tag{16}$$

Proof: From Lemma 8, we know that the fraction of cognitive users that are active is $\exp(-\vartheta)$. Since the cognitive users themselves perform thinning with probability ν, (16) is obtained. Similar to what we explained in Lemma 10, the interference to cognitive users is over-estimated using this formula when conditioning on a cognitive receiver. The interference to the primary users is also over-estimated since the exclusion regions around the primary receivers are neglected. ∎

Remark 2: Suppose ALOHA with parameter p_p and p_c is applied to primary users and cognitive users, respectively. If every primary and cognitive user transmits to its nearest neighbor (the UNN case), $\vartheta = p_p$ and $\nu = p_c$; if every primary user transmits to its nearest neighbor (the UNN case) and the cognitive users form MNN transmitter-receiver pairs, $\vartheta = p_p$ and $\nu = p_c\zeta$. If the primary users form MNN transmitter-receiver pairs and every cognitive user transmits to its nearest neighbor (the UNN case), $\vartheta = p_p\zeta$ and $\nu = p_c$; if all the primary and cognitive users form MNN transmitter-receiver pairs, $\vartheta = p_p\zeta$ and $\nu = p_c\zeta$.

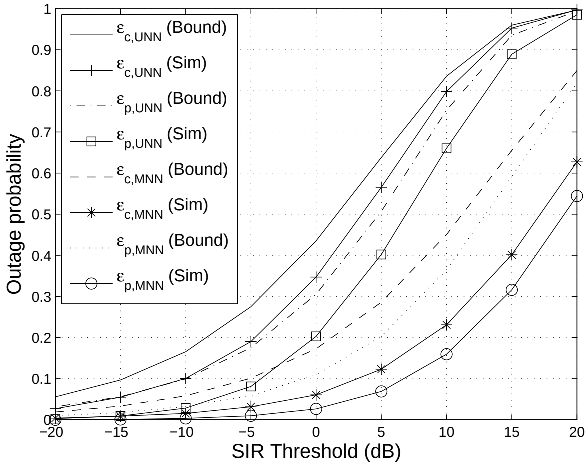


Fig. 3. Comparison of outage probabilities obtained by analysis and simulation for the cases that all the primary and cognitive users transmit to their nearest neighbors (the UNN case) and both the primary and cognitive users form mutual nearest-neighbor pairs (the MNN case). ϵ_c is the bound from (18) and ϵ_p is the bound from (19). The simulation parameters are $\lambda_p = 0.1$, $\lambda_c = 1$, $p_p = 0.2$, $p_c = 0.2$, and $\alpha = 4$.

Lemma 12: An improved bound for the interference to a primary receiver from the cognitive transmitters is

$$\mathcal{L}_{\tilde{I}_c}(s) = \exp\Bigg\{-\nu\exp(-\vartheta) \times \left(s^\delta \mathbb{E}_h\left[h^\delta\gamma(1-\delta, sh\bar{r}^{-\alpha})\right] - \frac{s\bar{r}^2}{s+\bar{r}^\alpha}\right)\Bigg\}, \quad (17)$$

where $\bar{r} = \frac{1}{\sqrt{\pi\lambda_p}}$ is the mean distance between the primary transmitters and the receivers.

Proof: With the protection by the exclusion regions, a cognitive transmitter cannot get closer than r_p to a primary receiver. Since r_p is a random variable, one starts with (4) by plugging $\bar{r}$ into ρ and then follows the derivation similar to what is described in Section 3.5.2 of [8]. ■

B. Outage Probability

Theorem 3: The outage probability ϵ of the primary or cognitive users is upper-bounded as

$$\epsilon < 1 - \exp\left(-\left[\vartheta + \nu\exp(-\vartheta)\right]\frac{\pi\delta}{\sin(\pi\delta)}\theta^\delta\right). \quad (18)$$

Proof: The received power is 1 for both the primary and cognitive users ($\mu_p r_p^{-\alpha} = \mu_c r_c^{-\alpha} = 1$), so $\frac{\theta}{\mu_p r_p^{-\alpha}} = \frac{\theta}{\mu_c r_c^{-\alpha}} = \theta$. Therefore, the outage probability is upper-bounded as $\epsilon < 1 - \mathcal{L}_{I_p}(\theta)\cdot\mathcal{L}_{I_c}(\theta)$. ■

Theorem 4: The outage probability ϵ_p of primary users is upper-bounded as

$$\epsilon_p < 1 - \exp\Bigg\{-\vartheta\frac{\pi\delta}{\sin(\pi\delta)}\theta^\delta - \nu\exp(-\vartheta) \times \left(\theta^\delta\mathbb{E}_h\left[h^\delta\gamma(1-\delta, \theta h\bar{r}^{-\alpha})\right] - \frac{\theta\bar{r}^2}{\theta+\bar{r}^\alpha}\right)\Bigg\}. \quad (19)$$

Proof: The outage probability is upper-bounded as $\epsilon_p < 1 - \mathcal{L}_{I_p}(\theta)\cdot\mathcal{L}_{\tilde{I}_c}(\theta)$. ■

Fig 3 shows the outage probabilities obtained by analysis and simulation for the cases that all the primary and cognitive users transmit to their nearest neighbors (the UNN case) and both the primary and cognitive users form mutual nearest-neighbor pairs (the MNN case), where ϵ_c is the bound from (18) and ϵ_p is the bound from (19). The simulation parameters are $\lambda_p = 0.1$, $\lambda_c = 1$, $p_p = 0.2$, $p_c = 0.2$, and $\alpha = 4$. We observe that indeed the outage probability of the primary users is less than the outage probability of the cognitive users due to the protection by the exclusion regions. We also observe that the bounds are loose for the case when both the primary and cognitive users form mutual nearest-neighbor pairs but tight for the UNN case, especially for large SIR thresholds.

V. Conclusions

The interference in the cognitive radio network is hard to analyze due to the interaction between the primary and the cognitive users: the point processes of the primary users and the cognitive users are not independent. In this paper, we have bounded the four types of interference: the interference from the primary transmitters to the primary receivers, from the cognitive transmitters to the primary receivers, from the primary transmitters to the cognitive receivers, and from the cognitive transmitters to the cognitive receivers for two different network models: the bipolar model and the nearest-neighbor model. The outage probabilities for the primary and the cognitive users are also bounded. To our best knowledge, these are the first analytical results that consider all four types of auto- and cross-interference between primary and secondary users.

The results presented in this paper show the fundamental relationship between the wireless network parameters (such as densities and distances) and the outage probabilities. Network engineers may apply our results to designing the network parameters to ensure that the cognitive radio network works properly below the specified outage probability.

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