Efficient Routing in Wireless Networks with Random Node Distribution

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Abstract — **After deriving the distribution of the distance to the n-th nearest neighbor in uniformly random networks of any dimension we establish that nearest-neighbor routing schemes perform poorly in random networks. We suggest and analyze an improved scheme that approaches the performance of regular networks.**

I. Distances in Random Networks

For large m-dimensional networks with uniformly random node distribution of density λ, the probability of finding k nodes in a subset of measure A is given by the Poisson distribution $e^{-\lambda A}\frac{(\lambda A)^k}{k!}$.

Theorem 1 *The distance R_n between a node and its n-th neighbor has the pdf* [2]

$$f_{R_n}(r) = e^{-\lambda c_m r^m} \frac{m\,(\lambda c_m r^m)^n}{r(n-1)!}\,, \quad r \geqslant 0\,, \tag{1}$$

where $c_m r^m$ is the volume of the m-sphere of radius r.

Proof: Let $c_m r^m =: A_m(r)$. The complementary cdf of R_n is the probability P_n that there are less than n nodes closer than r. We have $P_n := e^{-\lambda A_m(r)} \sum_{k=0}^{n-1} (\lambda A_m(r))^k/k!$, from which (1) follows. □

To ensure routing progress, we have to restrict the angle ϕ between the direction of a link and the source-destination axis, which entails a change of the volume from an m-sphere to an m-sector with volume $c_{\phi,m} r^m$ in (1). The energy consumption is (proportional to) $\mathbb{E}[R_n^\alpha] = (\lambda c_{\phi,m})^{-\alpha/m}\frac{\Gamma(n+\alpha/m)}{\Gamma(n)}$, where α is the path loss exponent. The main problem of nearest-neighbor (or n-th neighbor) routing is the large variance in the expected energy consumption. Furthest-neighbor routing (within a maximum distance) is preferred.

Theorem 2 *Let R_d denote the distance to the furthest node within the sector ϕ such that $R_d \leqslant d$, conditioned on having at least one node within distance d. The pdf of R_d is*

$$f_{R_d}(r) = \frac{\lambda c_{\phi,m} m r^{m-1} e^{\lambda c_{\phi,m} r^m}}{e^{\lambda c_{\phi,m} d^m} - 1}\,, \quad r \in [0,d]\,. \tag{2}$$

Proof: The complementary cdf $\mathbb{P}[R_d > r]$ is given by the probability that there is at least one node in the volume $c_{\phi,m}(d^m - r^m)$ divided by the probability that there is at least a node in $c_{\phi,m} d^m$. □

The fact that the pdf increases with e^{r^m} suggests that the expected value is close to d and that the variance is small. So, in this furthest-neighbor routing scheme, all nodes in a route transmit over approximately a distance d. So, furthest-neighbor routing results in balanced energy consumption and reduced delay. For $m = 2$ and $\phi = \pi/4$, the expected maximum of k RVs R_1 (k nearest-neighbor hops) is lowerbounded by $\sqrt{\ln(k)+1}$. For $k = 20$, this is 2, while $\mathbb{E}[R_1] = 1$. Thus choosing $d = 2$ cuts the delay in half at no cost in lifetime.

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[2]This distribution generalizes the Erlang ($m = 1$), Weibull ($n = 1$), exponential ($m = n = 1$), the Rayleigh ($n = m/2 = 1$), and the Γ (non-integer n) distribution.

II. Routing over Rayleigh Fading Channels

We assume a narrowband Rayleigh block fading channel. Assuming a transmission is successful if the SINR exceeds some threshold Θ, the mean packet reception probability p_r can be factorized into a zero-noise part p_r^I and a zero-interference part p_r^N, *i.e.*, $p_r = p_r^I p_r^N$ [1, Theorem 1]. Since we are concerned with energy consumption, we focus on the noise part $p_r^N = \exp(-\Theta N d^\alpha / P_0)$, where P_0 is the transmit power and d is the distance.

Assume an n-hop route from node 0 to node n, and let p_D denote the desired (end-to-end) reliability. The reception probability of a chain of n nodes is $p_n = \prod_{i=1}^n e^{-\Theta/\bar{\gamma}_i} = e^{-\Theta \sum_{i=1}^n \frac{1}{\bar{\gamma}_i}}$, where $\bar{\gamma}_i$ denotes the mean SNR at link i.

Denote the ratio of the expected per-hop distances for nearest- and furthest-neighbor routing by ρ, *i.e.*, $\rho = d/\mathbb{E}[R_n]$. Given an end-to-end reliability p_D, the per-hop reception probability is $p_r^{\mathsf{NEAR}} = p_D^{1/k}$, whereas $p_r^{\mathsf{FAR}} = p_D^{\rho/k}$. Since the energy consumption is proportional to $-1/\ln p_r$, this reduces the energy consumption for furthest-neighbor routing by a factor ρ.

III. Delay Considerations

Routing schemes with less hops can exploit time diversity in the form of retransmissions. Consider an n-hop strategy and a single-hop strategy, both covering a distance d. The single-hop scheme can transmit n times. The required single-use reception probability $p_{D,1}$ is $p_{D,1} = 1-(1-p_D)^{\frac{1}{n}}$. Compared with the single-transmission case, this leads to an energy gain of $G = \frac{\log p_{D,1}}{n \log p_D}$, which increases with increasing p_D (diversity benefit) and, as a function of n, has a maximum for small n. If CSI is available, a single transmission can be scheduled optimistically, and the gain increases by a factor of n.

IV. Concluding Remarks

Due to the variance in the node distances and the large number of hops, nearest-neighbor routing may be very inefficient in both energy (lifetime) and delay. A furthest-neighbor routing approach performs much better, in particular in fading environments, where the increased transmission speed (fewer hops) can be used for time diversity, *e.g.*, for retransmission in block fading channels.

References

[1] M. Haenggi, "On Routing in Random Rayleigh Fading Networks," *IEEE Transactions on Wireless Communications*, 2003. Submitted for publication. Available at http://www.nd.edu/~mhaenggi/routing.pdf.