Fast Transmission in Ad Hoc Networks

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I. Introduction

We are interested in various transmission strategies for sending information from a *source* s over a large distance to a *target* t in ad hoc wireless networks where the nodes are distributed as a Poisson process of intensity 1. We wish to transmit reliably (s and t lie in the same component with probability $1-\epsilon$), quickly (through few hops), and economically (using little power).

In Gilbert's disk model [4] for omni-directional transmission, for each point x of the Poisson process, we take a disk D_x of area a centred at x, and postulate that two points x and y of the process can communicate if $y \in D_x$ or, equivalently, $x \in D_y$. Joining two points by an edge if they can communicate, we obtain an infinite geometric random graph G_a. Note that the area a is exactly the expected degree of a vertex.

Gilbert proved that there exists a critical area a_c such that if $a > a_c$ then G_a almost surely has an infinite component, and if $a < a_c$ then G_a almost surely has only finite components.

The existence of an infinite component, i.e., *percolation*, is not sufficient for our problem since the proportion of vertices in the infinite component may be very low. To achieve *connectivity* the power must increase with the number of vertices, since there is some positive chance that a vertex is isolated. Independently, Penrose [6,7], and Xue and Kumar [5] proved that the threshold on the degree is $\log n$ where n is the number of vertices in the region. So to guarantee that s and t are connected we need enormous power, and we do not know anything about transmission speed[5].

II. Directional Transmitters

If has been shown that percolation can occur if a vertex can, on average, broadcast to $1+o(1)$ neighbors if we have transmitters that transmit (1) to a randomly oriented sector of angle δ and radius r or (2) to an annulus of inner and outer radii 1 and $1+\delta$. These results[6] show that as soon as the average degree exceeds *one*, there is an infinite component.

Result (1) shows that with directional transmissions, even with very low power there exist points at arbitrarily large distance that can communicate. It does not give us any bound on how many hops this will take or how likely this transmission is. Indeed if the power is such that we expect only $1+\eta$ neighbours then there is a significant probability ($e^{-(1+\eta)}$) that s has no neighbours at all, and hence definitely can not talk to t. Hence even to hope for reliable transmission with reliability $1-\varepsilon$ the vertex s must expect $\log(1/\varepsilon)$ neighbours, and similarly for t. Rather than pushing the power this high, we make s and t better nodes than the rest. With a more powerful transmitter s and a more sensitive receiver t there is a high chance of s and t communicating even with all the rest of the nodes being low power:

Theorem 1 *Fix an angle ϕ (small but not tiny, i.e., not tending to zero). Suppose that all the transmitters nearer s than t broadcast directionally into a sector of radius r and angle δ oriented randomly inside the sector of angle ϕ pointed in the st direction, and that all the transmitters nearer t than s receive directionally, again with radius r and angle δ this time randomly oriented in a sector of angle ϕ pointed in the ts direction. Finally suppose that s can transmit to any point within distance R and that t can receive from any point within distance R. Then, provided that R is large enough* independently *of the distance from s to t and that the area of a sector, $\delta r^2/2$, is greater than 1, node s can communicate with the node t with probability arbitrarily close to one. Moreover the number of hops is at most $(1+o(1))\left(\frac{3\phi}{4r\sin(\phi/2)}\right)d(s,t)$.*

This is the main result of this paper. Since the expected distance of a neighbour of a vertex is $4r\sin(\phi/2)/3\phi$, this is the best that we could hope for.

Note that the transmitters and receivers are directed randomly: they do *not* need to know the locations of their nearby neighbours. Indeed, if they all point directly towards t we need a higher density of transmitters; i.e., the critical area is bounded away from one.

We conjecture that Theorem 1 holds using only directional transmitters (i.e., all receivers are omni-directional).

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[5] Although a reasonable bound could be proved.

[6] They are examples of a more general result [2], and part (2) was independently and simultaneously proved in [3] and [1].

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