Outage and Throughput Bounds for Stochastic Wireless Networks

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***Abstract*— We derive outage expressions and throughput bounds for wireless networks subject to different sources of non-determinism. The degree of uncertainty is characterized by the location of the network in the *uncertainty cube* whose three axes represent the three main sources of uncertainty in interference-limited networks: the node distribution, the channel gains, and the channel access. The range for the coordinates is [0,1], where 0 indicates complete determinism, and 1 a maximum degree of randomness (nodes distributed in a Poisson point process, fading with fading figure 1, and ALOHA channel access, respectively).**

I. Introduction

A. Background

In many large wireless networks, the achievable performance is limited by the inter-node interference. While the *scaling behavior* of the network throughput or transport capacity is a well-studied area [1]–[6], relatively few *quantitative* results on outage and throughput are available. We extend the results in [7]–[10] to general stochastic networks with up to three dimensions of uncertainty: node placement, channel characteristics, and channel access.

B. The uncertainty cube

The level of uncertainty of a network is determined by its position in the *uncertainty cube*. The three coordinates (l, c, t), $0 \leqslant l, c, t \leqslant 1$, denote the degree of uncertainty in the node placement, the channels, and the channel access scheme, respectively. Values of 1 indicate complete uncertainty (and independence), as specified in Table I.

Location of nodes	$l = 0$ $l = 1$	Deterministic node placement Poisson point process
Channel (fading figure)	$c = 0$ $c = 1$	Entirely deterministic Rayleigh (block) fading
Channel access	$t = 0$ $t = 1$	TDMA slotted ALOHA

TABLE I
Specification of the uncertainty cube.

Note that the value of the c-coordinate corresponds to the fading figure (amount of fading). A network with $(l, c, t) = (1, 1, 1)$, for example, has its nodes distributed according to a Poisson point process, all channels are Rayleigh (block) fading, and the channel access scheme is slotted ALOHA. Note that any point in the unit cube corresponds to a meaningful network—the three axes are independent. To completely characterize the network, the number of nodes or the intensity of the Poisson point process, the transmit probabilities (slotted ALOHA), rates and power levels, and the deterministic part of the channel (path loss exponent) need to be given. Our objective is to characterize outage and throughput for the interesting corners of this uncertainty cube. It can be expected that the corners bound the performance of networks whose uncertainty lies inside the cube, and that performance is monotonically decreasing along the l- and t-axes. This is generally not the case along the channel axis, since fading may improve the throughput (even without opportunistic transmission).

We focus on the interference-limited case, so we do not consider noise[1]. It is assumed that all nodes transmit at the same power level that can be set to 1 since only relative powers matter. The performance results are also independent of the absolute scale of the network, since only relative distances matter.

C. Models, notation, and definitions

Channel model. For the large-scale path loss (deterministic channel component), we assume the standard power law where the received power decays with d^α for a path loss exponent α. If all channels are Rayleigh, this is sometimes referred to as a "Rayleigh/Rayleigh" model. If either only the desired transmitter or the interferers are subject to fading, we speak of *partial fading*.

Network model. We consider a single link of distance 1, with a (desired) transmitter and receiver in a large network with n other nodes as potential interferers. For infinite networks, $n \to \infty$. A network with $l = 1$, where the interferers are distributed according to a Poisson point process, is denoted as Poisson random network. The distances to the interferers are denoted by r_i. The signal power (deterministic channel) or average signal power (fading channel) at the receiver is 1 (irrespective of α).

[1] In the Rayleigh fading case, the outage expressions factorize into a noise part and an interference part, see (4). So, the noise term is simply a multiplicative factor to p_s.

Transmit probability p. In slotted ALOHA, every node transmits independently with probability p in each timeslot. Hence for Poisson random networks the set of transmitting nodes in each timeslot form a Poisson point process of intensity p. Practical values of p are small, *i.e.*, $p \lessapprox 1/3$ due to interference and throughput considerations. This permits certain approximations that would not hold for $p \approx 1$. The mean number of interferers is np, and the interference from node i is $I_i = B_i S_i r_i^{-\alpha}$, where B_i is iid Bernoulli with parameter p and S_i is iid exponential with mean 1.

Success probability p_s. A transmission is successful if the channel is not in an outage, *i.e.*, if the (instantaneous) SIR $\gamma = S_0/I$ exceeds a certain threshold Θ: $p_s = \mathbb{P}[\gamma > \Theta]$, where $I = \sum_{i=1}^n I_i$. This is the reception probability given that the desired transmit-receiver pair transmits and listens, respectively. It is usually assumed that $\Theta > 1$. Note that the transmission rate is assumed to be fixed, and no CSI is assumed anywhere in the network.

(Local) throughput g. The average local throughput, denoted simply as throughput, is defined to be the success probability multiplied by the probability that the transmit-receiver pair actually transmits and listens (the unconditioned reception probability). This is the throughput achievable with a simple ARQ scheme (with error-free feedback) [11]. For the ALOHA scheme $g := p(1-p)p_s$, whereas for a TDMA line network where nodes transmit in every m-th timeslot, $g := p_s/m$. Note that in networks with multiple dimensions of uncertainty, the throughput is averaged over all network realizations.

Spatial efficiency σ. As will be derived, the success probability for slotted ALOHA can be expressed or approximated as $p_s = e^{-p/\sigma}$. The parameter σ determines the degree of spatial reuse in a network, see Table II.

Effective distances ξ_i. The *effective distance* ξ_i of a node to the receiver is defined as $\xi_i := r_i^\alpha/\Theta$.

II. Related Work

A. $(1,0,1)$: Infinite non-fading random networks with $\alpha = 4$ and slotted ALOHA

This case is studied in [7]. The characteristic function of the interference is determined to be[2]

$$\mathbb{E}[e^{j\omega I}] = \exp\left(-\pi p \Gamma(1-2/\alpha) e^{-j\pi/\alpha} \omega^{2/\alpha}\right) \tag{1}$$

and, for $\alpha = 4$,

$$= \exp\left(-\pi\sqrt{\pi/2}(1-j)p\sqrt{\omega}\right). \tag{2}$$

It is easily seen that in this case, even though the path loss exponent is relatively large, $\mathbb{E}[I] = \infty$, which is due to the path loss model and the fact that interferers may be arbitrarily close to the transmitter.

[2]Note that their notation is adapted to ours. Also, a small mistake in [7, Eqn. (18)] is corrected here.

B. $(0,1,1)$: Regular fading networks with $\alpha = 2$ and slotted ALOHA

In [9], the authors derive the distribution of the interference power for one- and two-dimensional Rayleigh fading networks with slotted ALOHA and $\alpha = 2$. Closed-form expressions are derived for regular line networks with $r_i = i$. The Laplace transform of the interference is [9, Eqn. (8)]

$$\mathcal{L}_I(s) = \frac{\sinh\left(\pi\sqrt{s(1-p)}\right)}{\sqrt{1-p}\sinh\left(\pi\sqrt{s}\right)}. \tag{3}$$

For $p = 1$, the mean interference is simply $\sum_{i=1}^\infty i^{-2} = \zeta(2) = \pi^2/6$, and the variance is $\zeta(4) = \pi^4/90$, where ζ is the Riemann zeta function.

C. $(1,1,1)$: Random fading networks with slotted ALOHA

In [10], the success probability $p_s = \mathbb{P}[S \geqslant \Theta(N+I)]$ of a transmission over unit distance in a two-dimensional random network with Rayleigh fading and slotted ALOHA and a noise process N is expressed as

$$ps = \int_0^\infty e^{-s\Theta} \mathrm{d}\mathbb{P}[N + I \leqslant s] = \mathcal{L}_I(\Theta) \cdot \mathcal{L}_N(\Theta). \tag{4}$$

So, remarkably, the success probability for Rayleigh fading can be expressed as the product of the Laplace transforms of the noise N and interference I.[3] Ignoring the noise term, (4) evaluates to [10, Eqn. (6)]

$$p_s = e^{-p\Theta^{2/\alpha}C(\alpha)} \tag{5}$$

with $C(\alpha) = (2\pi\Gamma(2/\alpha)\Gamma(1-2/\alpha))/\alpha$. For example, $C(3) = 4\pi^2/3\sqrt{3} \approx 7.6$ and $C(4) = \pi^2/2 \approx 4.9$. $\lim_{\alpha\to 2} C(\alpha) = \infty$, so $p_s \to 0$ as $\alpha \to 2$ for any Θ. The spatial efficiency is $\sigma = 1/(\Theta^{2/\alpha}C(\alpha))$.

III. Networks with Random Node Distribution

A. $(1,0,1)$: Non-fading random networks with $\alpha = 4$ and slotted ALOHA

From [7, Eqn. (21)], $\gamma = 1/I$ has the cdf

$$F_\gamma(\Theta) = \mathbb{P}[1/I < \Theta] = \operatorname{erf}\left(\frac{\pi^{3/2}p\sqrt{\Theta}}{2}\right), \tag{6}$$

which is the outage probability for non-fading channels for a transmitter-receiver distance 1.

Note that since $p_s = 1 - F_\gamma(\Theta)$ is given by the error function rather than an exponential, the spatial efficiency is not defined. However, with the fairly sharp approximation $1 - \operatorname{erf} x \approx e^{-3x/\sqrt{\pi}}$, we obtain $\sigma \approx 2/(3\pi\sqrt{\Theta})$.

B. $(1,1,1)$: Partially fading random networks with slotted ALOHA

If only the desired link is subject to fading and $\alpha = 4$, we can exploit (2), replacing $j\omega$ by $-\Theta$, to get

$$p_s = \mathcal{L}_I(\Theta) = e^{-p\sqrt{\Theta}\pi^{3/2}}. \tag{7}$$

[3]This elegant equivalence of the Laplace transform evaluated at the SIR threshold and the success probability was also pointed out in [8].

Uncertainty	Spatial efficiency σ	Eqn.	Remark
$(1,1,1)$	$\frac{\alpha}{2\pi\Theta^{2/\alpha}\Gamma(2/\alpha)\Gamma(1-2/\alpha)}$	(5)	From [10], for general α
	$\frac{2}{\pi^2\sqrt{\Theta}}$	(5)	For $\alpha=4$
	$\frac{1}{\pi^{3/2}\sqrt{\Theta}}$	(7)	For $\alpha=4$ and non-fading interferers
$(1,0,1)$	$\approx \frac{2}{3\pi\sqrt{\Theta}}$	(6)	For $\alpha=4$ (approx. of error function)
$(0,1,1)$	$\left(\sum_{i=1}^n 1/(1+\xi_i)\right)^{-1}$	(9)	General deterministic node placement, n nodes.
	$\frac{2}{\pi\sqrt{\Theta}\coth(\pi\sqrt{\Theta})-1} \approx \frac{2}{\pi\sqrt{\Theta}-1}$	(12)	One-sided infinite regular line network with $\alpha=2$
	$\frac{2}{\pi\Theta^{1/4}/\sqrt{2}-1}$	(15)	One-sided infinite regular line network with $\alpha=4$
	$\left(\sum_{i=1}^n 1/\xi_i\right)^{-1}$	(16)	General detetерministic node placement, non-fading interferers
	$1/(\Theta\zeta(\alpha))$	(16)	Same, infinite number of nodes.
$(0,1,0)$	$p_s \gtrapprox e^{-\zeta(\alpha)\Theta/m^\alpha}$	(19)	m-phase TDMA in infinite one-sided regular line networks.

TABLE II

SPATIAL EFFICIENCY FOR DIFFERENT TYPES OF SLOTTED ALOHA NETWORKS. FOR $(1,1,1)$ NETWORKS, $p_s = e^{-p/\sigma}$ IS EXACT, FOR THE OTHER ONES, IT IS AN APPROXIMATION. FOR COMPARISON, THE TDMA CASE IS ADDED.

IV. NETWORKS WITH DETERMINISTIC NODE PLACEMENT

In this section, we assume that n interferers are placed at fixed relative distances r_i from the intended receiver.

A. $(0,1,1)$ *Fading networks with slotted ALOHA*

In this case, $p_s = \mathbb{P}[S \geqslant \Theta I]$ for $I = \sum_{i=1}^n S_i r_i^{-\alpha}$ and S_i iid exponential with mean 1. For general r_i and α, we obtain from $p_s = \mathbb{E}[e^{-\Theta I}] = \mathcal{L}_I(\Theta)$ (see also [12]):

$$p_s = \prod_{i=1}^n \Big(1 - \frac{p}{1 + r_i^\alpha/\Theta}\Big) = \prod_{i=1}^n \Big(1 - \frac{p}{1+\xi_i}\Big) \qquad (8)$$

where $\xi_i = r_i^\alpha/\Theta$ is the effective distance.

Since we are mostly interested in the behavior for small p (and $\xi_i \gg 1$ for most i, *i.e.*, most interferers are far for non-negligible success probabilities), we approximate $\log p_s$ as a sum of terms $\log(1 - p/(1+\xi)) \lessapprox -p/(1+\xi)$, such that p_s can be expressed as $p_s \lessapprox e^{-p/\sigma}$ for

$$\sigma = \frac{1}{\sum_{i=1}^n \frac{1}{1+\xi_i}} \,. \qquad (9)$$

So, the approximation shows that p_s has the same exponential form as for random networks, and the spatial efficiency is given by the "parallel connection" (or $1/n$ times the harmonic mean) of $1+\xi_i$.

B. $(0,1,1)$*: Infinite regular line networks with fading and slotted ALOHA*

In one-sided regular networks, $r_i = i$.

Special case 1: $\alpha = 2$. This Laplace transform is given in (3), so we simply have to substitute s with Θ. To find an exponential approximation, we note that $\exp(-\pi\sqrt{\Theta}) \ll 1$, hence

$$p_s \lessapprox \frac{\exp(\pi\sqrt{\Theta}\sqrt{1-p})}{\sqrt{1-p}\,\exp(\pi\sqrt{\Theta})} \,, \qquad (10)$$

and employing the linear approximations $\sqrt{1-p} \approx 1 - p/2$ and $\log\sqrt{1-p} \approx -p/2$ yields

$$\log p_s \approx -p\pi\sqrt{\Theta}/2 + p/2 = -p\Big(\frac{\pi\sqrt{\Theta}}{2} - \frac{1}{2}\Big) \,. \qquad (11)$$

Note that we can derive the same expression by starting with the sum (9). For $\alpha = 2$, it has the closed-form

$$\sigma = \frac{2}{\pi\sqrt{\Theta}\coth(\pi\sqrt{\Theta}) - 1} \lessapprox \frac{2}{\pi\sqrt{\Theta} - 1} \,. \qquad (12)$$

Special case 2: $\alpha = 4$.

Proposition 1 *For one-sided infinite regular line networks ($r_i = i$, $i \in \mathbb{N}$) with slotted ALOHA and $\alpha = 4$,*

$$p_s = \frac{\cosh^2\left(y(1-p)^{1/4}\right) - \cos^2\left(y(1-p)^{1/4}\right)}{\sqrt{1-p}\,(\cosh^2 y - \cos^2 y)} \qquad (13)$$

with $y := \pi\Theta^{1/4}/\sqrt{2}$.

Proof: First, rewrite (8) as

$$p_s = \frac{\prod_{i=1}^n (1 + (1-p)\Theta/i^4)}{\prod_{i=1}^n (1 + \Theta/i^4)} \,. \qquad (14)$$

The factorization of both numerator and denominator according to $(1 - z^4/i^4) = (1 - z^2/i^2)(1 + z^2/i^2)$ permits the use of Euler's product formula $\sin(\pi z) \equiv \pi z \prod_{i=1}^\infty (1 - z^2/i^2)$ with $z = \sqrt{\pm j}((1-p)\Theta)^{1/4}$ (numerator) and $z = \sqrt{\pm j}\Theta^{1/4}$ (denominator). The two resulting expressions are complex conjugates, and $|\sin(\sqrt{j}x)|^2 = \cosh^2(x/\sqrt{2}) - \cos^2(x/\sqrt{2})$. $\square$

For small p, the $\cosh$ terms dominate the $\cos$ terms, and with $\cosh^2(x) \approx e^{2x}/4$, $1 - (1-p)^{1/4} \approx p/4$, and $(1-p)^{-1/2} \approx e^{p/2}$ we obtain

$$p_s \approx e^{-p\left(\frac{\sqrt{2}}{4}\pi\Theta^{1/4} - 1/2\right)} \,. \qquad (15)$$

So, for $\alpha = 4$, $\sigma = 2/(\pi\Theta^{1/4}/\sqrt{2} - 1)$.

C. $(0,1,1)$: *Partially fading regular networks*

If only the desired link is subject to fading, the success probability is given by

$$p_s = e^{-p\Theta \sum_{i=1}^{n} r_i^{-\alpha}}\,, \tag{16}$$

thus $\sigma = (\sum_{i=1}^{n} 1/\xi_i)^{-1}$. Compared with (9), $1+\xi$ is replaced by ξ. So, σ is the "parallel connection" of all the ξ_i, and it is trivially upperbounded by $\min_i\{\xi_i\}$. For $n \to \infty$, $p_s = e^{-p\Theta\zeta(\alpha)}$, and $\sigma = 1/(\Theta\zeta(\alpha))$.

D. $(0,1,0)$: *Regular line networks with fading and TDMA*

If in a TDMA scheme, only every m-th node transmits, the relative distances of the interferers are increased by a factor of m. Since $(mr)^\alpha/\Theta = r^\alpha/(\Theta m^{-\alpha})$, having every m-th node transmit is equivalent to reducing the threshold Θ by a factor m^α and setting $p=1$.

Proposition 2 *The success probability for one-sided infinite regular line networks with Rayleigh fading and m-phase TDMA is: For $\alpha = 2$:*

$$p_s = \frac{y}{\sinh y}\,, \quad \text{where } y := \frac{\pi\sqrt{\Theta}}{m}\,, \tag{17}$$

and for $\alpha = 4$:

$$p_s = \frac{2y^2}{\cosh^2 y - \cos^2 y}\,, \quad \text{where } y := \frac{\pi\Theta^{1/4}}{\sqrt{2}m}\,. \tag{18}$$

Proof: Apply L'Hôpital's rule for $p=1$ in (3) and (13) (for $\alpha = 2,4$, respectively) and replace Θ by $\Theta m^{-\alpha}$. $\square$

The following proposition establishes sharp bounds for arbitrary α.

Proposition 3 *The success probability for one-sided infinite regular line networks, Rayleigh fading, and m-phase TDMA is bounded by*

$$e^{-\zeta(\alpha)\Theta/m^\alpha} \lessapprox p_s \lessapprox \frac{1}{1+\zeta(\alpha)\frac{\Theta}{m^\alpha}}\,. \tag{19}$$

A tighter upper bound is

$$p_s \lessapprox \frac{1}{1+\zeta(\alpha)\frac{\Theta}{m^\alpha} + (\zeta(\alpha)-1)\frac{\Theta^2}{m^{2\alpha}}}\,. \tag{20}$$

Proof: Upper bound: We only need to proof the tighter bound. Let $q := \Theta/m^\alpha$. The expansion of the product (8), $p_s^{-1} = \prod_{i=1}^{\infty} 1 + q/i^\alpha$, ordered according to powers of q, has only positive terms and starts with $1 + q\zeta(\alpha) + q^2(\zeta(\alpha) - 1)$. There are more terms with q^2, but their coefficients are relatively small, so the bound is tight. The lower bound follows immediately from

$$\log p_s = \log \prod_{i=1}^{\infty} \frac{1}{1+\Theta/(mi)^\alpha} \gtrapprox -\sum_{i=1}^{\infty} \Theta/(mi)^\alpha\,. \tag{21}$$

$\square$

Note that all bounds approach p_s as Θ/m^α decreases. They are loosest for $m = \alpha = 2$ and relatively large Θ. Even in this impractical case (for $\alpha = 2$, m should be chosen much larger for acceptable transmit efficiencies), the tighter upper bound and the lower bound are not off by more than 0.03. Interestingly, for $\alpha = 2,4$, the upper bound (19) corresponds exactly to the expressions obtained when the denominators in (17) and (18) are replaced by their Taylor expansions of order 2α. Higher-order Taylor expansions, however, deviate from the tighter bound (20).

V. THROUGHPUT

A. $(l,c,1)$: *Networks with slotted ALOHA*

For networks with slotted ALOHA, the throughput is given by $g(p) = p(1-p)p_s$. With $g \approx p(1-p)e^{-p/\sigma}$, maximizing $\log(g)$ yields the quadratic equation $p_{\text{opt}}^2 - p_{\text{opt}}(1+2\sigma) + \sigma = 0$. So, p_{opt} is given by

$$p_{\text{opt}} \approx \sigma + \frac{1}{2}\left(1 - \sqrt{1+4\sigma^2}\right). \tag{22}$$

The *transmit efficiency* η, defined as the success probability given that a transmission attempt has been made, *i.e.*, $\eta := g/p = (1-p)e^{-p/\sigma}$ is monotonically increasing from $\lim_{\sigma\to 0}\eta = e^{-1} \approx 37\%$ to $\lim_{\sigma\to\infty}\eta = 1/2$. The upper bound is achieved if the interference goes to zero, in which case $p = 1/2$ and $g = 1/4$.

B. $(0,1,0)$: *Two-sided regular line networks with TDMA*

Here we consider a two-sided infinite regular line network with m-phase TDMA. To maximize the throughput $g := p_s/m$, we use the bounds (19) for p_s. Since the network is now two-sided, the expressions need to be squared. Let $\tilde{m}_{\text{opt}} \in \mathbb{R}$ and $\hat{m}_{\text{opt}} \in \mathbb{N}$ be estimates for the true $m_{\text{opt}} \in \mathbb{N}$. We find

$$\left(\Theta\zeta(\alpha)(2\alpha-1)\right)^{1/\alpha} < \tilde{m}_{\text{opt}} < \left(\Theta\zeta(\alpha)2\alpha\right)^{1/\alpha}\,, \tag{23}$$

where the lower and upper bounds stem from maximizing the upper and lower bounds in (19), respectively. The factor 2 in 2α indicates that the network is two-sided. Rounding an average value to the nearest integer yields a good estimate for m_{opt}:

$$\hat{m}_{\text{opt}} = \left\lceil \left(\Theta\zeta(\alpha)(2\alpha - 1/2)\right)^{1/\alpha} \right\rfloor \tag{24}$$

Fig. 1 shows the bounds (23), $\hat{m}_{\text{opt}}$, and the true m_{opt} (found numerically) for $\alpha = 2$ as a function of Θ. For most values of Θ, $\hat{m}_{\text{opt}} = m_{\text{opt}}$. The resulting difference in the maximum achievable throughput $g_{\max}$ is negligibly small. Using the real estimate $\tilde{m}_{\text{opt}}$, we can obtain bounds on the average success probability $\bar{p}_s$ (averaged over Θ) by inserting (23) into (19):

$$\left(1 - \frac{1}{2\alpha}\right)^2 \lessapprox \bar{p}_s \lessapprox e^{-1/\alpha}\,. \tag{25}$$

In Fig. 2, the actual $p_s(\Theta)$ is shown with the bounds on $\bar{p}_s$ for $\alpha = 2$. Since m_{opt} is increasing with Θ, the relative error $\tilde{m}_{\text{opt}}/m_{\text{opt}} \to 0$, so $\bar{p}_s = \lim_{\Theta\to\infty} p_s(\Theta)$ will lie between the bounds (25).

Note that in the case of TDMA, the transmit efficiency η is identical to p_s. So, compared with throughput-optimized slotted ALOHA, where $1/2$ is an upper bound on η, irrespective of α, the efficiency is *at least* $1/2$ for TDMA, and it is increasing with α. These are the two extremes; the efficiency of other MAC schemes falls in between.

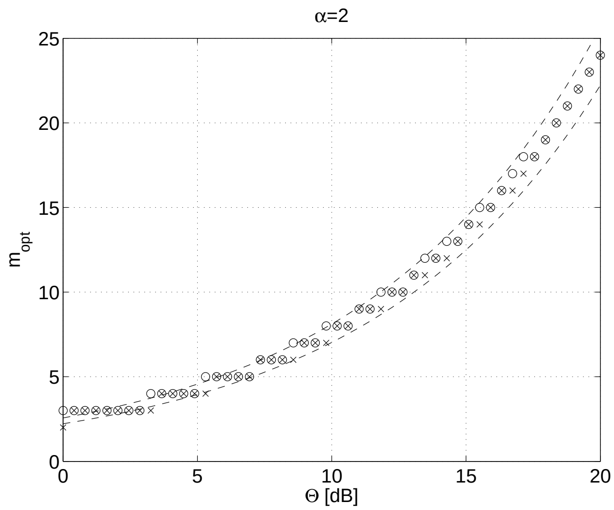

Fig. 1. Optimum TDMA parameter m as a function of Θ [dB] for $\alpha = 2$. The dashed lines show the bounds (23), the circles indicate the true optimum m_{opt}, the crosses the estimate $\hat{m}_{\text{opt}}$ in (24).

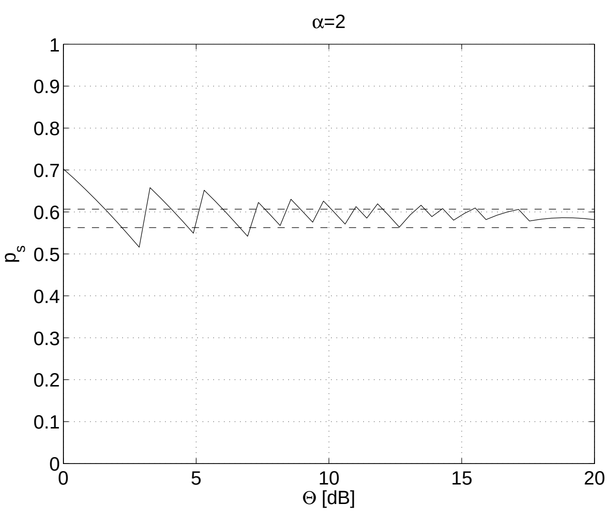

Fig. 2. p_s for the optimum m as a function of Θ [dB] for $\alpha = 2$. The dashed lines show the approximations (25), the solid line the actual value obtained numerically.

VI. Concluding Remarks

We have analyzed four corners of the uncertainty cube (see Table I). The case $(0, 0, 1)$ seems less interesting, and $(0, 0, 0)$ is totally deterministic so $p_s = 1$ can always be achieved if the TDMA scheme is properly chosen. On the other hand, TDMA for Poisson random networks is difficult to specify (and implement), so the $(1, c, 0)$ case is not discussed either.

For the other three slotted ALOHA corners, the success probability can be expressed as $p_s = e^{-p/\sigma}$, where the *spatial efficiency* σ is approximately proportional to $1/\Theta^{2/\alpha}$ for two-dimensional networks. σ has different interpretations: (1) For $\sigma = 0$, no simultaneous transmissions are possible (no spatial reuse), whereas for $\sigma \to \infty$, there are no collisions at all. (2) In a Poisson random network, the success probability equals the probability that a disk of radius $r = 1/\sqrt{\pi\sigma}$ around the receiver is free from interferers. (3) σ determines how fast p_s decays as p increases from 0: $\partial p_s/\partial p|_{p=0} = -1/\sigma$.

Closed-form expressions and approximations for the achievable throughput for slotted ALOHA and deterministic m-phase TDMA are given. Not surprisingly, the TDMA scheme has a substantially better throughput performance. While the energy consumption is comparable if both schemes are optimized for throughput ($p_{\text{opt}} \approx 1/m_{\text{opt}}$), the efficiency of the random access scheme is substantially lower.

The success probability $p_s = e^{-p/\sigma}$ as a function of Θ can be interpreted as the complementary cumulative distribution of the SIR, which permits a complete characterization of the SIR and/or interference. This shows that the interference is far from Gaussian.

Many extensions are possible, such as the inclusion of power control and access schemes and node distributions whose uncertainty lies *inside* the uncertainty cube. Relating Θ to the rate of transmission permits the analysis of schemes with rate control.

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References

[1] P. Gupta and P. R. Kumar, "The Capacity of Wireless Networks," *IEEE Transactions on Information Theory*, vol. 46, pp. 388–404, Mar. 2000.

[2] P. Gupta and P. R. Kumar, "Towards an Information Theory of Large Networks: An Achievable Rate Region," *IEEE Transactions on Information Theory*, vol. 49, pp. 1877–1894, Aug. 2003.

[3] F. Xue, L.-L. Xie, and P. R. Kumar, "The Transport Capacity of Wireless Networks over Fading Channels," *IEEE Transactions on Information Theory*, 2003. Submitted.

[4] O. Dousse and P. Thiran, "Connectivity vs Capacity in Dense Ad Hoc Networks," in *IEEE INFOCOM*, (Hongkong), Mar. 2004.

[5] M. Franceschetti, O. Dousse, D. Tse, and P. Thiran, "Closing the gap in the capacity of random wireless networks," in *IEEE Symposium on Information Theory (ISIT'04)*, (Chicago, IL), June 2004.

[6] L. Xie and P. R. Kumar, "A Network Information Theory for Wireless Communication: Scaling Laws and Optimal Operation," *IEEE Transactions on Information Theory*, vol. 50, pp. 748–767, May 2004.

[7] E. S. Sousa and J. A. Silvester, "Optimum Transmission Ranges in a Direct-Sequence Spread-Spectrum Multihop Packet Radio Network," *IEEE Journal on Selected Areas in Communications*, vol. 8, pp. 762–771, June 1990.

[8] J.-P. M. G. Linnartz, "Exact Analysis of the Outage Probability in Multiple-User Radio," *IEEE Transactions on Communications*, vol. 40, pp. 20–23, Jan. 1992.

[9] R. Mathar and J. Mattfeldt, "On the distribution of cumulated interference power in Rayleigh fading channels," *Wireless Networks*, vol. 1, pp. 31–36, Feb. 1995.

[10] F. Baccelli, B. Blaszczyszyn, and P. Mühlethaler, "A Spatial Reuse Aloha MAC Protocol for Multihop Wireless Mobile Networks," Tech. Rep. 4955, Institut National de Recherche en Informatique et en Automatique (INRIA), Rocquencourt, Le Chesnay Cedex, France, Oct. 2003. Available at `http://www.terminodes.org/MV2003-Present/Me15/Spacial-Baccelli.pdf`.

[11] N. Ahmed and R. G. Baranjuk, "Throughput Measures for Delay-Constrained Communications in Fading Channels," in *Allerton Conference on Communication, Control and Computing*, (Monticello, IL), Oct. 2003.

[12] M. Haenggi, "On Routing in Random Rayleigh Fading Networks," *IEEE Transactions on Wireless Communications*, 2005. Accepted for publication.