Geometry, Connectivity, and Broadcast Transport Capacity of Random Networks with Fading

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***Abstract*— In ad hoc networks with random node distribution, the underlying point process model and the channel fading process are usually considered separately. We provide a unified framework for the geometric characterization of fading by incorporating the fading process into the point process model. Concretely, assuming nodes are distributed in a stationary Poisson point process in $\mathbb{R}^d$, we analyze the properties of the point processes that describe the path loss with fading. Applications include connectivity and broadcasting.**

I. Introduction and System Model

A. Motivation

When two wireless transceivers communicate, the path loss is well modeled by the product of a distance component (often called large-scale path loss) and a fading component (called small-scale fading or shadowing). It is usually assumed that the distance part is deterministic while the fading part is modeled as a random process. This distinction, however, does not apply to most ad hoc networks, where the distance itself is subject to uncertainty. In this case it may be beneficial to consider the distance and fading uncertainty jointly, *i.e.*, to define a point process that incorporates both. We introduce a framework that offers a geometrical interpretation of fading and some new insight into its effect on the network. In particular, we focus on two applications: connectivity and broadcasting. To obtain concrete results, we will often use the Nakagami-m fading model, which is quite general and has the advantage of including the special case of no fading ($m \to \infty$).

Connectivity. We characterize the geometric properties of the set of nodes that are directly connected to the origin for arbitrary fading models, generalizing the results in [1], [2]. We also show that if the path loss exponent equals the number of network dimension, any fading model (with unit mean) is distribution-preserving in a sense made precise later.

Broadcasting. We are interested in the single-hop *broadcast transport capacity*, *i.e.*, the cumulated distance-weighted rate summed over the set of nodes that can successfully decode a message sent from a transmitter at the origin. In particular, we prove that if the path loss exponent is smaller than the number of network dimensions plus one, this transport capacity can be made arbitrarily large by letting the rate of transmission approach 0.

B. Notation and symbols

For convenient reference, we provide a list of the symbols and variables used in the paper. Most of them are also explained in the text.

Symbol	Definition/explanation
$\mathbf{1}_A(x)$	indicator function
$u(x)$	$\triangleq \mathbf{1}_{\{x\geqslant 0\}}(x)$ (unit step function)
d	number of dimensions of the network
o	origin in $\mathbb{R}^d$
B	a Borel subset of $\mathbb{R}$ or $\mathbb{R}^d$
c_d	$\triangleq \pi^{d/2}/\Gamma(1+d/2)$ (volume of the d-dim. unit ball)
α	path loss exponent
δ	$\triangleq d/\alpha$
Δ	$\triangleq (d+1)/\alpha$
s	minimum path gain for connection
F, f	fading distribution (cdf), fading r.v.
F_X	distribution of random variable X (cdf)
$\Phi = \{x_i\}$	path loss process before fading (PLP)
$\Xi = \{\xi_i\}$	path loss process with fading (PLPF)
$\hat{\Phi} = \{\hat{x}_i\}$	points in Φ connected to origin
$\hat{\Xi} = \{\hat{\xi}_i\}$	points in Ξ connected to origin
Λ, λ	counting measure and density for Φ
M, μ	counting measure and density for Ξ
$\hat{N}$	number of nodes connected to o
$\#A$	number of elements in the set A

C. Poisson point process model

Node distribution. Let the set $\{y_i\}$, $i \in \mathbb{N}$ consist of the points of a stationary Poisson point process in $\mathbb{R}^d$ of intensity 1, ordered according to their Euclidean distance $\|y_i - o\|$ to the origin o. Define a new one-dimensional (generally inhomogeneous) PPP $\{r_i \triangleq \|y_i - o\|\}$ such that $0 < r_1 < r_2 < \ldots$ a.s. Let $\alpha > 0$ be the path loss exponent of the network and $\Phi = \{x_i \triangleq r_i^\alpha\}$ be the *path loss process* (before fading) (PLP). Let $\{f, f_1, f_2, \ldots\}$ be an iid stochastic process with f drawn from a distribution F with unit mean, *i.e.*, $\mathbb{E}f = 1$. Finally, let $\Xi = \{\xi_i \triangleq x_i/f_i\}$ be the *path loss process with fading* (PLPF). In order to treat the case of no fading in the same framework, we will allow the degenerate case $F(x) = u(x-1)$, resulting in $\Phi = \Xi$. Note that the fading is static (unless mentioned otherwise), and that $\{\xi_i\}$ is no longer ordered in general. We will also interpret these point processes as *random counting measures*, *e.g.*, $\Phi(B) = \#\{\Phi \cap B\}$ for any Borel subset B of $\mathbb{R}$.

Connectivity. We are interested in connectivity to the origin. A node i is connected if the path loss is smaller than $1/s$, *i.e.*, if $\xi_i < 1/s$. The processes of connected nodes are denoted as $\hat{\Phi} = \{x_i : \ \xi_i < 1/s\}$ (PLP) and $\hat{\Xi} = \{\xi_i : \ \xi_i < 1/s\} = \Xi \cap [0, 1/s)$ (PLPF).

Counting measures. Let Λ be the counting measure associated with Φ, *i.e.*, $\Lambda(B) = \mathbb{E}\Phi(B) = \mathbb{E}\#\{\Phi \cap B\}$ for Borel B. For $\Lambda([0,a)) = \mathbb{E}\Phi([0,a))$, we will also use the shortcut $\Lambda(a)$. Similarly, let $\hat{\Lambda}$ be the counting measure for $\hat{\Phi}$. All the processes considered admit a *density*. Let $\lambda(x) = \mathrm{d}\Lambda(x)/\mathrm{d}x$ and and $\hat{\lambda}(x) = \mathrm{d}\hat{\Lambda}(x)/\mathrm{d}x$ be the densities of Φ and $\hat{\Phi}$, respectively. Further, let M be the counting measure for Ξ, $\hat{M}$ for $\hat{\Xi}$, and let $\mu(x)$ and $\hat{\mu}(x)$ denote the corresponding densities, *i.e.*, $\mu(x) = \mathrm{d}M(x)/\mathrm{d}x$ and $\hat{\mu}(x) = \mathrm{d}\hat{M}(x)/\mathrm{d}x$.

D. The standard network

For ease of exposition, we often consider a *standard network*[1] that has the following parameters: $d = \alpha = 2$, Rayleigh fading, *i.e.*, $F(x) = (1 - e^{-x})u(x)$. For comparison with the non-fading case, we use the term *standard network without fading* if there is no fading and $d = \alpha = 2$. For the standard network, $\delta = 1$ and $\Delta = 3/2$.

II. Basic Properties

Proposition 1 *The processes Φ, Ξ, and $\hat{\Xi}$ are Poisson.*

Proof: $\{y_i\}$ is Poisson by definition, so $\{r_i\}$ and $\Phi = \{x_i\}$ are Poisson by the mapping theorem [3]. Ξ is Poisson since f_i is iid, and $\hat{\Xi}(\mathbb{R}) = \Xi([0, 1/s))$. □

Cor. 2 states some basic facts about these point processes that result from their Poisson property. Note that (b) has been shown previously [4], and (c) is a generalization of a result in [2].

Corollary 2

(a) $\Lambda(x) = \mathbb{E}\Phi([0,x)) = c_d x^\delta$ *and* $\lambda(x) = c_d \delta x^{\delta-1}$. *In particular, for $\delta = 1$, Φ is stationary.*

(b) r_i *is governed by the generalized gamma pdf*

$$f_{r_i}(r) = e^{-c_d r^d} \frac{d\,(c_d r^d)^i}{r\Gamma(i)}\,, \tag{1}$$

and x_i is distributed according to the cdf

$$F_{x_i}(x) = 1 - \frac{\Gamma_{\mathrm{ic}}(i, c_d x^\delta)}{\Gamma(i)}\,, \tag{2}$$

where Γ_{ic} denotes the upper incomplete gamma function. The expected path loss without fading is

$$\mathbb{E}x_i = c_d^{-1/\delta} \frac{\Gamma(i + 1/\delta)}{\Gamma(i)}\,. \tag{3}$$

(c) ξ_i *is distributed according to the cdf*

$$F_{\xi_i}(x) = 1 - \int_0^\infty F(r/x) \left(\frac{c_d^i \delta r^{\delta i-1} \exp(-c_d x r^\delta)}{\Gamma(i)} \right) \mathrm{d}x\,. \tag{4}$$

For $\delta = 1$ and Rayleigh fading,

$$F_{\xi_i}(x) = \frac{(c_d x)^i}{(c_d x + 1)^i}\,. \tag{5}$$

Proof:

(a) Since the original process $\{y_i\}$ is stationary, the expected number of points in a ball of radius x around the origin is $c_d x^d$. The one-dimensional process $\{r_i\}$ has the same number of points in $[0,x)$, and $x_i = r_i^\alpha$, so $\mathbb{E}\Phi([0,x)) = c_d x^{d/\alpha}$. For $\delta = 1$, $\lambda(x) = c_d$ is constant.

(b) Follows directly from the fact that $\{y_i\}$ is stationary Poisson.

(c) The cdf is $1 - \mathbb{E}(F(x_i/x))$ with x_i distributed according to (2). For Rayleigh fading and general d, α, there is a pseudo-analytic form of F_{ξ_i} available using hypergeometric functions ${}_d\mathrm{F}_\alpha$. Note that for the standard network $\mathbb{E}\xi_i$ does not exist for any i. □

Proposition 3 *For $\delta = 1$ and any fading distribution F with mean 1,*

$$\Xi(B) \overset{d}{=} \Phi(B) \qquad \forall \text{Borel } B \subset \mathbb{R}\,,$$

i.e., fading is distribution-preserving.

Proof: Since Ξ is Poisson, independence of $\Xi(B_1)$ and $\Xi(B_2)$ for $B_1 \cap B_2 = \emptyset$ is guaranteed. So it remains to be shown that the intensities (or, equivalently, the counting measures on Borel sets) are the same. This is the case if for all $a > 0$,

$$\mathbb{E}\left(\#\{x_i : x_i > a,\ \xi_i < a\}\right) = \mathbb{E}\left(\#\{x_i : x_i < a,\ \xi_i > a\}\right)\,,$$

i.e., the expected numbers of nodes crossing a from the left (leaving the interval $[0,a)$) and the right (entering the same interval) are equal. This condition can be expressed as

$$\int_0^a \lambda(x) F(x/a) \mathrm{d}x = \int_a^\infty \lambda(x)(1 - F(x/a)\mathrm{d}x \quad \forall a > 0\,.$$

If $\delta = 1$, $\lambda(x) = c_d$, and the condition reduces to

$$\int_0^1 F(x)\mathrm{d}x = \int_1^\infty (1 - F(x))\mathrm{d}x\,,$$

which holds since

$$\underbrace{\int_0^1 (1 - F(x))\mathrm{d}x}_{1 - \int_0^1 F(x)\mathrm{d}x} + \int_1^\infty (1 - F(x))\mathrm{d}x = \mathbb{E}f = 1\,.$$

□

[1]The term "standard" here refers to the fact that in this case the analytical expressions are particularly simple. We do not claim that a path loss exponent of 2 is the one most frequently observed.

Corollary 4 *For Nakagami-m fading, $\delta = 1$, and any $a > 0$, the expected number of nodes that leave the interval $[0, a)$ is*

$$c_d a \frac{m^{m-1}}{\Gamma(m)} e^{-m} \,. \tag{6}$$

The same number of nodes is expected to enter this interval. For Rayleigh fading ($m = 1$), the fraction of nodes leaving the interval is $1/e$.

Proof: For Nakagami-m,

$$\int_0^1 F(x) = 1 - \int_0^1 \frac{\Gamma_{\text{ic}}(m, mx)}{\Gamma(m)} \mathrm{d}x = \frac{m^{m-1}}{\Gamma(m)} e^{-m} \,.$$

□

Note that the degenerate case of no fading is retrieved as $m \to \infty$.

Due to the ordering of $\{x_i\}$, the random variables ξ_i are not independent. It is often useful to consider a set of *independent* random variables, obtained by conditioning the process on having a certain number of nodes n in an interval $[0, a)$ (or, equivalently, conditioning on $x_{n+1} = a$). With this conditioning, the n nodes $\{x_i\}$, $i = 1, 2, \ldots, n$ are iid distributed as follows.

Corollary 5 *Conditioned on $x_{n+1} = a$:*

(a) The nodes $\{x_i\}_{i=1}^n$ are iid distributed with

$$f_{x_i}^a(x) = \frac{\lambda(x)}{\Lambda(x)} = \delta a^{-\delta} x^{\delta-1} \,, \quad 0 \leqslant x < a \tag{7}$$

and cdf $F_{x_i}^a(x) = (x/a)^\delta$.

(b) The path loss with fading $\{\xi_i\}_{i=1}^n$ is distributed as

$$F_{\xi_i}^a(x) = 1 - \int_0^a F(y/x) \delta a^{-\delta} y^{\delta-1} \mathrm{d}y \,. \tag{8}$$

(c) For the standard network,

$$F_{\xi_i}^a(x) = \frac{x}{a}\left(1 - e^{-a/x}\right) \tag{9}$$

(d) For Rayleigh fading, $d = 2$, and $\alpha = 4$ ($\delta = 1/2$),

$$F_\xi^a(x) = \frac{\sqrt{\pi}}{2}\sqrt{\frac{x}{a}}\, \text{erf}\left(\sqrt{\frac{a}{x}}\right) \,. \tag{10}$$

Proof: As in (4), the cdf is given by $1 - \mathbb{E}(F(y/x))$ with y distributed as (7). □

III. Connectivity

Here we investigate the processes $\hat{\Phi}$ and $\hat{\Xi} = \Xi \cap [0, 1/s)$ of connected nodes. Note that $\hat{\mu}(x) = \mu(x)(1 - u(x - 1/s))$.

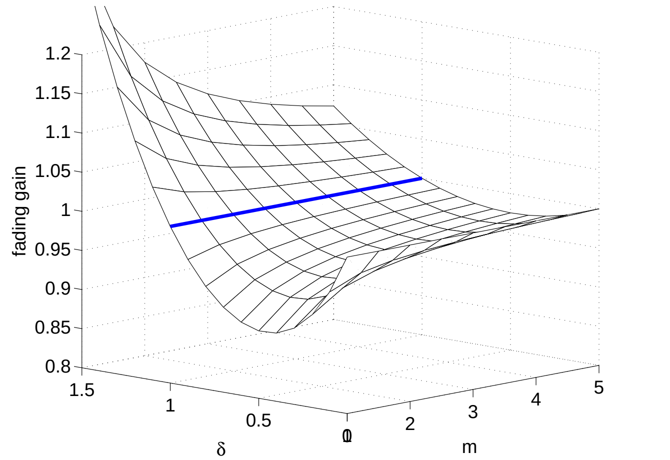

Fig. 1. Connectivity fading gain for Nakagami-m fading as a function of $\delta \in [0, 3/2]$ and $m \in [1, 5]$. For $\delta = 1$, the gain is 1 independent of m (thick line).

A. Single transmission and fading gain

Proposition 6

(a) $\hat{\lambda}(x) = \lambda(x)(1 - F(sx))$.

(b) With Nakagami-m fading, the number $\hat{N} = \hat{\Phi}(\mathbb{R})$ of connected nodes is Poisson with mean

$$\mathbb{E}\hat{N}_m = \frac{c_d}{(ms)^\delta} \frac{\Gamma(\delta + m)}{\Gamma(m)} \tag{11}$$

and the connectivity fading gain, *defined as the ratio of the expected numbers of connected nodes with and without fading, is*

$$\frac{\mathbb{E}\hat{N}_m}{\mathbb{E}\hat{N}_\infty} = \frac{1}{m^\delta} \frac{\Gamma(\delta + m)}{\Gamma(m)} = \mathbb{E}(f^\delta) \,. \tag{12}$$

Proof:

(a) The effect of fading on the connectivity is independent thinning by $1 - F(sx) = \mathbb{P}[x/f < 1/s]$.

(b) Using (a), the expected number of connected nodes is

$$\int_0^\infty \hat{\lambda}(x) \mathrm{d}x = \int_0^\infty c_d \delta x^{\delta-1} \frac{\Gamma_{\text{ic}}(m, msx)}{\Gamma(m)} \mathrm{d}x$$

which equals $\mathbb{E}\hat{N}_m$ in the assertion. Without fading, $\mathbb{E}\hat{N}_\infty = \lim_{m\to\infty} = \Lambda(1/s) = c_d s^{-\delta}$, which results in the ratio (12). □

Remarks.

1) Since $\hat{N}$ is Poisson, the probability of isolation is

$$\mathbb{P}(\hat{N} = 0) = \exp(-\mathbb{E}\hat{N}) \tag{13}$$

2) $\mathbb{E}\hat{N}$ can also be expressed as

$$\mathbb{E}\hat{N} = \sum_{i=1}^\infty \mathbb{P}[\xi_i < 1/s] \,. \tag{14}$$

3) The *connectivity fading gain* equals the δ-th moment of the fading distribution, which, by definition, approaches one as the fading vanishes, *i.e.*, as $m \to \infty$. For a fixed

δ, it is decreasing in m if $\delta > 1$, increasing if $\delta < 1$, and equal to 1 for all m if $\delta = 1$. It also equals 1 if $\delta = 0$. For a fixed m, it is not monotonic with δ, but exhibits a minimum at $\delta \in (0, 1)$. The fading gain as a function of δ and m is plotted in Fig. 1. For Rayleigh fading and $\delta = 1/2$, the factor is $\pi/2$, and the minimum is assumed at $\delta \approx 0.462$. So, depending on the type of fading and the ratio of the number of network dimensions to the path loss exponent α, fading can increase or decrease the number of connected nodes.

4) For the standard network, $\mathbb{E}\hat{N} = \pi/s$ and the probability of isolation is $e^{-\pi/s}$.
5) The expected number of connected nodes $\hat{N}^a$ with $x_i < a$ is

$$\mathbb{E}\hat{N}^a = c_d a^\delta F^a_{\xi_i}(1/s)\,. \tag{15}$$

where $F^a_{\xi_i}$ is given in (8).

Corollary 7 *Under Nagakami-m fading, a randomly chosen connected node $\hat{x} \in \hat{\Phi}$ has mean*

$$\mathbb{E}\hat{x} = \frac{\delta(\delta+m)}{ms(\delta+1)}\,, \tag{16}$$

which is $1 + \delta/m$ times the value without fading.

Proof: A random connected node $\hat{x}$ is distributed according to

$$f_{\hat{x}}(x) = \frac{\hat{\lambda}(x)}{\mathbb{E}\hat{N}}\,. \tag{17}$$

Without fading, the distribution is $s^\delta \delta x^{\delta-1}$, $0 \leqslant x \leqslant 1/s$, resulting in an expectation of $\delta/(s(\delta+1))$. □

For Rayleigh fading, for example, the distribution $f_{\hat{x}}$ is a gamma distribution with mean δ/s, so the average connected node is $1 + \delta$ times further away than without fading.

B. Connectivity with retransmissions

Assuming a block fading network and n transmissions of the same packet, what is the process of nodes that receive the packet at least once?

Corollary 8 *In a network with iid block fading, the density of the process of nodes $\hat{\lambda}^n$ that receive at least one of n transmissions is*

$$\hat{\lambda}^n = (1 - F(sx)^n) c_d \delta x^{\delta-1}\,. \tag{18}$$

Proof: This is a straightforward generalization of Prop. 6(a). □

So, in a standard network, the number of connected nodes with n transmissions $\mathbb{E}\hat{N}^n = \int_0^\infty \hat{\lambda}^n(x)\mathrm{d}x$ grows with $\log n$.

IV. Broadcasting

A. Broadcast transport sum-distance and capacity

Assuming the origin o transmits, the set of nodes that receive the message is $\{\hat{x}_i\}$. We would like to determine the *broadcast transport sum-distance* D, *i.e.*, the expected sum over the all the distances $\hat{x}_i^{1/\alpha}$ from the origin:

$$D \triangleq \mathbb{E}\left(\sum_{\hat{x}_i \in \hat{\Phi}} \hat{x}_i^{1/\alpha}\right) \tag{19}$$

Proposition 9 *The broadcast transport sum-distance for Nakagami-m fading is*

$$D_m = c_d \frac{\delta}{\Delta} \frac{1}{(ms)^\Delta} \frac{\Gamma(m+\Delta)}{\Gamma(m)}\,, \tag{20}$$

and the (broadcast) fading gain D_m/D_∞ *is*

$$\frac{D_m}{D_\infty} = \frac{1}{m^\Delta}\frac{\Gamma(m+\Delta)}{\Gamma(m)} = \mathbb{E}(f^\Delta)\,. \tag{21}$$

Proof: From Campbell's theorem [5],

$$\begin{aligned}\mathbb{E}\left(\sum_{\hat{x}_i \in \hat{\Phi}} \hat{x}_i^{1/\alpha}\right) &= \int_0^\infty x^{1/\alpha}\hat{\lambda}(x)\mathrm{d}x \\ &= c_d\delta \int_0^\infty x^{1/\alpha+\delta-1}(1 - F(sx))\mathrm{d}x\,,\end{aligned}$$

which equals (20) for Nakagami-m fading.

Without fading, a node x_i is connected if $x_i < 1/s$, therefore

$$D_\infty = \int_0^{1/s} x^{1/\alpha}\lambda(x)\mathrm{d}x \tag{22}$$

$$= c_d \frac{\delta}{\Delta} s^{-\Delta} = c_d \frac{d}{d+1} s^{-\Delta}\,. \tag{23}$$

So the fading gain D_m/D_∞ is the Δ-th moment of f as given in (21). □

Remarks.

1) The fading gain is independent of the threshold s. $D_m \propto s^{-\Delta}$ for all m. It strongly resembles the connectivity gain (Prop. 6), the only difference being the parameter Δ instead of δ. See Remark 3 to Prop. 6 and Fig. 1 for a discussion and visualization of the behavior of the gain as a function of m and Δ.
2) For Rayleigh fading ($m = 1$), $D_1 = c_d \delta s^{-\Delta}$, and the fading gain is $\Gamma(1+\Delta)$. For the standard network, this is $\Gamma(5/2) = 3\sqrt{\pi}/4 \approx 1.33$. For $d = \alpha = 2$, $D_\infty = \frac{2\pi}{3s^{3/2}}$.
3) The formula for the broadcast transport sum-distance reminds of an interference expression. Indeed, by simply replacing $x^{1/\alpha}$ by x^{-1}, a well-known result on the mean interference is reproduced: Assuming each node transmits at unit power, the total interference at the origin is

$$\mathbb{E}\left(\sum_{x_i \in \Phi} x_i^{-1}\right) = \int_0^\infty x^{-1}\lambda(x)\mathrm{d}x = c_d \frac{\delta}{\delta-1} x^{\delta-1}\Big|_0^\infty$$

which for $\delta < 1$ diverges due to the lower bound integration bound (*i.e.*, the one or two closest nodes) and for $\delta \geqslant 1$ diverges due to the upper bound (*i.e.*, the large number of nodes that are far away).

So far, we have ignored the actual rate of transmission R and just used the threshold s for the sum-distance. To get to the single-hop broadcast transport capacity C (in bit-meters/s/Hz), we relate the (bandwidth-normalized) rate of transmission R and the threshold s by $R = \log_2(1+s)$ and define

$$C \triangleq \max_{R>0} RD = \max_{s>0} \log_2(1+s)D(s). \qquad (24)$$

Let D_m^1 be the broadcast transport sum-distance for $s = 1$ (see Prop. 9), *i.e.*, $D_m = D_m^1 s^{-\Delta}$.

Proposition 10 *For Nakagami-m fading:*

(a) For $\Delta \in (0,1]$, the broadcast transport capacity is achieved for

$$R_{\text{opt}} = \frac{\mathcal{W}\left(-\frac{e^{-1/\Delta}}{\Delta}\right) + \Delta^{-1}}{\log 2}, \quad \Delta \in (0,1], \qquad (25)$$

where $\mathcal{W}$ denotes the principal branch of the Lambert W function. The resulting broadcast transport capacity is tightly (within at most 0.13%) lower bounded by

$$C_m \geqslant \frac{D_m^1}{\log 2}(\Delta^{-1} - \Delta)\left(e^{\Delta^{-1}-\Delta} - 1\right)^{-\Delta}. \qquad (26)$$

(b) For $\Delta = 1/(2\log 2)$, $C_m = D_m^1$. For all other values of Δ, C_m is larger.

(c) For $\Delta > 1$, the broadcast transport capacity increases without bounds as $R \to 0$, independent of the transmit power.

Proof:

(a) $D_m \propto s^{-\Delta}$, so $C_m \propto R(2^R - 1)^{-\Delta}$ which, for $\Delta \leqslant 1$, has a maximum at R_{opt} given in (25). The lower bound stems from an approximation of R_{opt} using $\mathcal{W}(-\exp(-1/\Delta)/\Delta) \approx -\Delta$.

(b) $\Delta = 1/(2\log 2)$ minimizes $R_{\text{opt}}(2^{R_{\text{opt}}} - 1)^{-\Delta}$ (not the bound (26)).

(c) For $\Delta > 1$, $R(2^R - 1)^{-\Delta}$ is decreasing with R, and $\lim_{R\to 0} R(2^R-1)^{-\Delta} = \lim_{R\to 0}(\log 2)^{-\Delta}R^{1-\Delta} = \infty$. □

Remark. (c) is also apparent from the expression $D(s)\log_2(1+s)$, which, for $s \to 0$, is approximately $D_m^1 s^{1-\Delta}/\log 2$. So, in this regime, the gain from reaching additional nodes more than offsets the loss in rate.

B. The benefit of retransmissions

Let $p(x) \triangleq 1 - F(sx)$. The density of nodes that receive k packets out of n transmissions is given by

$$\lambda_k^n(x) = \lambda(x)\left(\binom{n}{k}p(x)^k(1-p(x))^{n-k}\right). \qquad (27)$$

Note that summing λ_k^n from 1 to n reproduces Cor. 8. The densities of the nodes receiving exactly m of n messages is plotted in Fig. 2 for the standard network with $n = 10$.

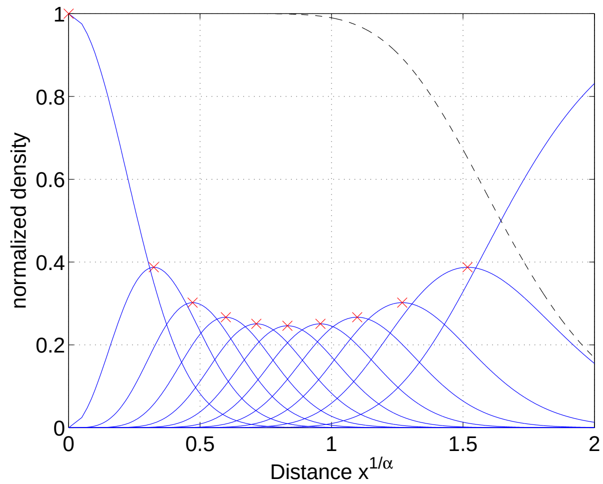

Fig. 2. Normalized node densities $\lambda_k^{10}(x^{1/\alpha})$ in a standard network that receive $k = 0, 1, \ldots 10$ out of 10 transmissions. The dashed line indicates the density of the nodes that receive at least one transmission.

This expression permits the evaluation of the contribution that each additional transmission makes to the broadcast transport sum-distance and capacity.

V. Concluding Remarks

We have studied some geometric properties of d-dimensional Poisson networks with fading. For connectivity and broadcast transport capacity, it is sufficient to consider one-dimensional distance processes with and without fading.

For Nakagami-m fading, it turns out that the *connectivity fading gain* is the δ-th moment of the fading distribution, while the fading gain in the *broadcast transport sum-distance* is its Δ-th moment. A path loss exponent larger than the number of dimensions d ($d+1$ for broadcasting) leads to a negative impact of fading.

Interestingly, the *broadcast transport capacity* turns out to be unbounded if $\Delta > 1$, *i.e.*, if the path loss exponent is smaller than $d+1$. While this result may be of interest for the design of efficient broadcasting protocols, it also raises doubts on the validity of transport capacity as a performance metric.

Acknowledgments

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