Single-Hop Connectivity in Interference-Limited Hybrid Wireless Networks

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***Abstract*— We consider a hybrid wireless network, in which the low-power nodes collect data and pass it to the data aggregation nodes (base stations). The low-power nodes are assumed to form a Poisson point process and communicate to the base stations using spread spectrum. We derive bounds to the probability of sensor node isolation, when the base stations are arranged randomly or in a regular fashion.**

I. Introduction and System Model

A. Motivation

Hybrid sensor networks consist of two groups of nodes, one group of low powered sensors which sense and forward the data to the base stations or the data aggregation nodes [1]. The base stations are more powerful nodes which are connected to each other by a higher-bandwidth network. It has been shown in [2] that the throughput of hybrid networks is higher than the conventional network. The paper by Dousse et al. [3] analyzes the connectivity of hybrid networks using percolation theory. They consider a disk model without fading for connectivity and allow for multihop routing. The connectivity of an ad hoc network without interference is considered with fading in [4] and without fading in [5]. Baccelli et al. [6, Sec 5.3], give a non-explicit expression for the volume fraction of stationary coverage processes, which is closely related to the *node-isolation* probability when the base stations are *randomly* deployed.

In this paper we assume that the radio uses spread spectrum and that the sensor nodes simultaneously transmit data to the base stations which makes the communication interference-limited. A sensor node is said to be *connected* if it can directly communicate with at least one base station. We also assume that there is no power control, since the senor nodes do not know which base station to communicate with. The sensor nodes are assumed to be *randomly* placed on the plane. Can the base stations be also randomly placed or should they be placed in a more regular fashion to provide a certain probability of connectivity for the sensor nodes? In this paper, we shall be primarily dealing with sensor networks, but these results also apply to similar problems in cellular systems and wireless Internet access.

B. System Model

The location of sensor nodes is modeled as a homogeneous Poisson point process $\phi_1 = \{x_i\}$ on $\mathbb{R}^2$ of density λ_1. Each node is assumed to transmit at unit power. The power received by a receiver located at z due to a transmitter at x_i is modeled as $h_{xz}^2 g(x-z)$, where h_{xz} denotes Rayleigh fading and $g(x) = \|x\|^{-\alpha}$. We assume $\alpha > 2$. The base stations are assumed to form a point process ϕ_2 on the plane. The sum-power at point z on the plane is given by

$$I_{\phi_1}(z) \quad = \quad \sum_{x_i \in \phi_1} h_{x_i z}^2 g(x_i - z) \tag{1}$$

We say that the communication from a transmitter at location $x \in \phi_1$ to a base station situated at $z \in \phi_2$ is successful [7] if and only if

$$\frac{h_{xz}^2 g(x-z)}{I_{\phi_1 \setminus \{x\}}(z)} \quad \geq \quad \beta \tag{2}$$

where h_{xz} is the fading coefficient between node x and base station z. We assume the noise power to be negligible compared to interference. In the non-fading case, $h_{x_i z}, h_{xz}$ are identically taken to be one. We assume that the communication between the nodes and the base station utilizes spread spectrum.

Let $P(x)$ denote the probability that a sensor node located at x cannot connect to any base station. We define the probability of non-connectivity P of the sensor nodes as

$$P = \frac{1}{\lambda_1 R^2} E\Big[\sum_{x \in \phi_1 \cap A} P(x)\Big] = \frac{1}{R^2}\int_A P(x)dx \tag{3}$$

where $A = [-R/2, R/2]^2$, and R is chosen depending on the arrangement of base stations. The choice of R and, in turn, A stems from the symmetry of the arrangement of base stations. This definition is useful when the base station process ϕ_2 is not stationary. When ϕ_2 is stationary, $P = P(0)$. We also consider only single-hop connectivity, *i.e.*, the sensor nodes try to connect directly to the base stations, without using intermediate nodes for routing. This assumption may look restrictive, but some set of nodes have to directly connect with the base stations and one can consider ϕ_1 to consist of these nodes only.

In Sections II and III we calculate the isolation probability P when the base stations ϕ_2 are modeled as

- Poisson point process (PPP) : This means that the base stations are deployed randomly on the plane.
- Lattice point process: The base stations are meticulously placed in the lattice grid.

II. Random deployment of the base stations

The base stations form a PPP with density λ_2 on the plane. Since the base station process is stationary we have $P = P(0)$, *i.e.*, we need to only calculate the probability that a sensor node located at the origin is not able to connect to any base station in the presence of interference. We first calculate P under fading.

A. Rayleigh Fading

In Rayleigh fading, the probability $F_f(x)$ that the communication between a transmitter located at the origin and a receiver located at x is successful is given by [7], [8]

$$F_f(x) = \exp(-\lambda_1 \|x\|^2 \beta^{2/\alpha} C(\alpha)) \tag{4}$$

where $C(\alpha) = 2\pi^2/(\alpha \sin(2\pi/\alpha))$. Let $B(\phi_1, z) = \{y : |h_{yz}|^2 g(y-z) \geq \beta \sum_{x\in\phi_1} |h_{xz}|^2 g(x-z)\}$, where h_{ab} are independent Rayleigh fading random variables. $B(\phi_1, z)$ denotes the set of all node locations on the plane which can connect to the base station z when the interference is caused by the process ϕ_1. Let $1_A(x)$ denote the indicator function of set A. Then $1 - 1_{B(\phi\setminus\{0\},\eta)}(0)$ is equal to 1 if and only if a sensor node at 0 cannot connect to the base station at η. Hence we have

$$P = E \prod_{\eta\in\phi_2} 1 - 1_{B(\phi_1\setminus\{0\},\eta)}(0) \tag{5}$$

$$= E^{!0}_{\phi_1}\Big[E_{\phi_2}[\prod_{\eta\in\phi_2} 1 - 1_{B(\phi_1,\eta)}(0)|\phi_1]\Big] \tag{6}$$

where $E^{!0}_{\phi_1}$ denotes the conditional expectation (Palm probability) [9], [10], that the process ϕ_1 has a point at origin. For a Poisson point process, by Slivnyak's theorem [9], we have $E^{!0}(.) = E(.)$. For a point process ϕ, the moment generating functional $G_\phi(f)$ is given by $G_\phi(f) = E_\phi[\prod_{x\in\phi} f(x)]$ [9], [10]. So we have

$$P = E^0_{\phi_1}[G_{\phi_2}(1 - 1_{B(\phi_1,\eta)}(0))]$$

For a PPP, the generating functional $G_\phi(f) = \exp(-\int(1 - f(x))\lambda(x)dx)$ where $\lambda(x)$ is the density function of the process. Since the base stations ϕ_2 are Poisson with density λ_2, we have

$$P = E_{\phi_1}[\exp\Big[-\lambda_2 \int_{\mathbb{R}^2} 1_{B(\phi,\eta)}(0) d\eta\Big]]$$

$$\overset{(a)}{\geq} \exp\Big[-\lambda_2 \int_{\mathbb{R}^2} F_f(\eta) d\eta\Big]$$

where (a) follows from Jensen's inequality and (4). Hence we have

$$P \geq \exp(-\rho q) \tag{7}$$

where $\rho \triangleq \lambda_2/\lambda_1$ and $q \triangleq \pi\beta^{-2/\alpha}C(\alpha)^{-1}$. We can derive an upper bound as follows. The first contact distribution [9] $H(r)$ of ϕ_2 is Rayleigh. We have

$$P \leq \mathbb{P}(\text{Origin is unable to connect to nearest BS})$$
$$= \int_{\mathbb{R}_+} (1 - F_f(r))H(r)dr$$
$$= \frac{1}{1 + \pi\lambda_1^{-1}\lambda_2\beta^{-2/\alpha}C(\alpha)^{-1}} = \frac{1}{1+\rho q}$$

From the upper and lower bound we observe that ρ, *i.e.*, the number of base stations per sensor node is an important parameter. To maintain a minimum degree of connectivity, the number of base stations should scale proportionally to the number of sensor nodes. We observe from Figure 5 that the lower bound is very close to the actual isolation probability. This is intuitive since many of the terms in (6) will be independent, the expectation with respect to ϕ_1 can be moved inside, and the inequality (7) becomes an equality. So we will use $\exp(-\rho q)$ as the probability of sensor node isolation, when the base stations are randomly deployed. Using the above procedure, one can show that $\delta_r = \mathbb{P}(\text{A node cannot connect to any base station within a distance } r) \approx \exp(-\rho q(1 - e^{-r^2\lambda_1/q}))$.

B. No Fading

In the case when there is no fading, we do not have an equivalent of the success probability $F_f(x)$. The interference is an alpha-stable process [11], [12], [13]. But we have the characteristic function and the Laplace transform of the interference distribution. Let $F(y) = \mathbb{P}(\sum_{x\in\phi_1} g(x-z) \leq y)$, *i.e.*, the CDF of the interference and $f(y) = dF(y)/dy$, the probability density function. Since ϕ_1 is a PPP, $F(y)$ is independent of z. Then the Laplace transform of $f(y)$ [11], [14] is

$$\mathcal{L}(s) = \exp(-\pi\lambda_1 s^{2/\alpha}\Gamma(1 - 2/\alpha)),$$

and the characteristic function is given by

$$\hat{f}(\eta) = \exp\Big[-2\pi\lambda_1 \int_0^\infty (1 - e^{-2\pi i\eta g(r)})rdr\Big] \tag{8}$$

Let $B_1(\phi_1, z) = \{y : g(y-z) \geq \beta \sum_{x\in\phi_1} g(x-z)\}$, *i.e.*, the set of all sensor nodes that can connect to the base station at z under the interference by transmitters in ϕ_1. Then as in the fading case we have

$$P = E \prod_{\eta\in\phi_2} 1 - 1_{B_1(\phi_1\setminus\{0\},\eta)}(0)$$

By similar arguments as in the fading case, we have

$$P \geq \exp\Big[-\lambda_2 \int_{\mathbb{R}^2} F(g(x)/\beta)dx\Big]$$

The technique to evaluate $\int_{\mathbb{R}^2} F(g(x)/\beta)dx$ using the characteristic function $\hat{f}$ is given in Appendix A. We obtain

$$P \geq \exp(-\rho q). \tag{9}$$

As in the fading case, we have

$$P \leq 1 - E_r[F(g(r)/\beta)]$$

where the expectation is with respect to the Rayleigh distribution function $H(r)$. We also have, for any random variable $Y \geq 0$,

$$\mathbb{P}(Y < a) \geq (e/(e-1))\mathcal{L}_Y(1/a) - 1/(e-1)$$

[15]. So we have

$$P \leq \frac{e}{e-1} - \frac{e}{e-1} E_r \mathcal{L}_f(\beta/g(r))$$
$$= \frac{e}{e-1}\Big[\frac{\alpha}{\alpha + 2\pi\rho q\Gamma(2/\alpha)}\Big]$$

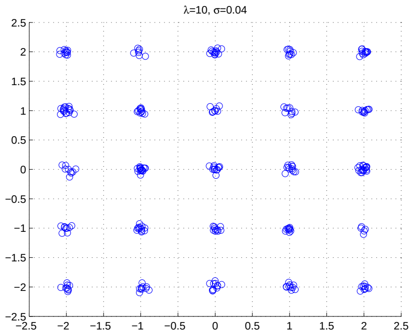

Fig. 1. Poisson lattice process with $\sigma = 0.04, \lambda = 10, \lambda_2 = 1$

Also in the non-fading case we observe that λ_2/λ_1 is a important system parameter. We also observe that the lower bounds for the fading and non-fading cases match.

III. Regular deployment of base stations

We would like to analyze the connectivity when the base stations ϕ_2 are arranged in a lattice process. To overcome some technical difficulties, we introduce the thinned lattice model, for which the lattice is a limiting process. Let $\phi_2 = \mathbb{Z}^2/\sqrt{\lambda_2}$ and remove a lattice point with probability p independent of other points (thinning the lattice). The final density of ϕ_2 is $\lambda_2(1-p)$. We obtain the normal lattice when $p \to 0$.

Let $\nu(x) = 1/2\pi\sigma^2 \exp(-\|x\|^2/2\sigma^2), x \in \mathbb{R}^2$ denote a two dimensional Gaussian function. A *Poisson lattice process* (see Fig. 1) $\hat{\phi}$ is a Poisson process with density function $\lambda(x) = \lambda \sum_{k\in\mathbb{Z}^2} \nu(x + k/\sqrt{\lambda_2})$. We also have

$$\hat{\phi} \approx \cup_{i=1}^N \phi_2^i$$

where $\phi_2^i, i = 1\ldots N$ denotes i.i.d copies of ϕ_2 and $N \sim \text{Poi}(\lambda/(1-p))$. This approximation in distribution becomes tight as $\sigma \to 0$ [1].

Let $P(x)$ denote the probability that a sensor node located at x cannot connect to any base station in the process ϕ_2, and $\hat{P}(x)$ denote the same, when the base stations are arranged as $\hat{\phi}$. By using the moment generating function of a Poisson random variable, it can be shown that

$$\hat{P}(x) \approx \exp\left[\frac{-\lambda}{1-p}(1-P(x))\right] \tag{10}$$

Using the same notation and as for the PPP with fading, we have

$$\hat{P}(x) = E\prod_{\eta\in\hat{\phi}} 1 - 1_{B(\phi_1\setminus\{x\},\eta)}(x)$$

Using similar techniques as in Section II-A and from Appendix I-B, we have

$$\hat{P}(x) \geq \exp(-\lambda\lambda_2\psi(x)) \tag{11}$$

[1] $\hat{\phi}$ tends to a perfect lattice as $\sigma \to 0$ and $\lambda \to \infty$. Even then it would be an overlap of multiple ϕ_2. For example consider a ball $\delta(0,\epsilon)$, where ϵ is small but larger than 5σ. Then the probability that no point lies in this ball is approximately $\exp(-\lambda\int_{\delta(0,\epsilon)}\sum_{k\in\mathbb{Z}^2}\nu(x+k)dx) \approx \exp(-\lambda)$. Also the number of points inside this ball is Poisson distributed with mean λ.

where $\psi(x) = \sum_{\xi\in\mathbb{Z}^2}\hat{\nu}(\sqrt{\lambda_2}\xi)\widehat{F_f}(-\sqrt{\lambda_2}\xi)\cos(2\pi\sqrt{\lambda_2}\langle\xi,x\rangle)$. $\widehat{F_f}(\xi)$ is the Fourier transform of $F(x)$, and $\hat{\nu}(\xi)$ is the Fourier transform of the Gaussian ν. From (10) and (11), we have $P(x) \geq 1-(1-p)\lambda_2\psi(x)$. Using the definition of P from (3) and choosing $R = 1/\sqrt{\lambda_2}$ (by symmetry of the lattice), we have

$$\begin{aligned} P &\geq 1 - \frac{(1-p)\lambda_2}{R^2}\int_{[-R/2,R/2]^2}\psi(x)dx \\ &\overset{(a)}{=} 1-(1-p)\pi\lambda_2\lambda_1^{-1}\beta^{-2/\alpha}C(\alpha)^{-1} \end{aligned}$$

(a) follows from $\int_{[-R/2,R/2]^2}\cos(2\pi\sqrt{\lambda_2}\langle\xi,x\rangle)dx = 0, \forall\xi \neq (0,0)$. Taking the limit $p \to 0$ (corresponding to the regular lattice without holes), we have

$$P \geq [1-\rho q]^+ \tag{12}$$

where $[x]^+ = \max\{0,x\}$. This lower bound corresponds to the regular lattice process (since $p = 0$). From Fig 6, we see that the lower bound is tight for large $1/\rho$ and large β. For the regular lattice, as in the previous cases, we have the following upper bound

$$P \leq 1 - \frac{1}{R^2}\int_{[-R/2,R/2]^2}\exp(-\lambda_1\|x\|^2\beta^{2/\alpha}C(\alpha))dx$$

Choosing $R = 1/\sqrt{\lambda_2}$, we have $P \leq 1 - \text{erf}\left[\sqrt{\frac{\pi}{4\rho q}}\right]^2\rho q$, where erf(.) is the standard error function.

IV. Simulation and Observations

For the simulations, we consider a square of area 10×10 and place the bases stations and the sensor nodes appropriately. The fading h_{xy}^2 is taken to be exponential with mean 1. Figures 2 and 3 illustrate the single-hop connectivity when the base stations are arranged as PPP and lattice, respectively. Stars indicate base stations and circles indicate the sensor nodes. We see from the figures that the single-hop connectivity is better when the base stations are arranged as a lattice. Also we observe that most of the sensor nodes are connected to the nearest base stations even under fading. This is expected since $\delta_r = \mathbb{P}$(A node cannot connect to any base station within a distance r from itself) $\approx \exp(-\rho q(1-e^{-r^2\lambda_1/q}))$, is very close to P even at small r. For example in Fig 4, we observe that δ_r saturates at $r = 1$ for a wide range of β and λ_1, *i.e.*, whatever connectivity is available for a sensor node, it is available within its immediate vicinity, even with fading. This is interesting since, even under fading a base station can localize the positions of the sensor nodes that connect to it. The saturation point for δ_r scales approximately as $c+\sqrt{\log(\lambda_2)}$ for some positive c independent of λ_2.

That said, just communicating to the geographically nearest base station is not an optimal strategy for connectivity under fading (when there is no power control), because the upper bound to P which corresponds to this strategy is large compared to actual P (especially for smaller β). For example when the base stations are randomly deployed, we see from Fig 5, that for $\beta = 0.01, 1/\rho = 4$, the gap is about 0.2.

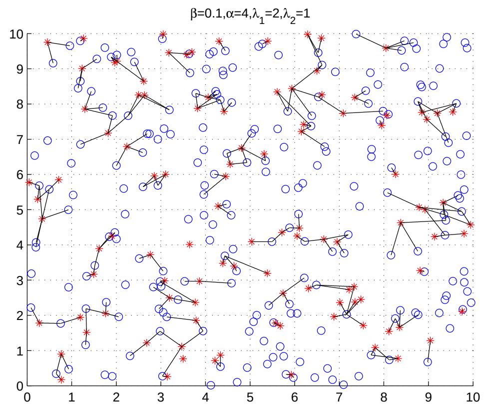

Fig. 2. Connectivity, Stars: base stations, circles: sensor nodes, base stations form a PPP with fading

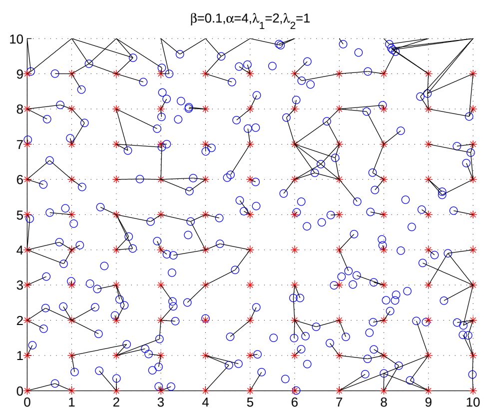

Fig. 3. Connectivity, Stars: base stations, circles: sensor nodes, base stations form a lattice process with fading

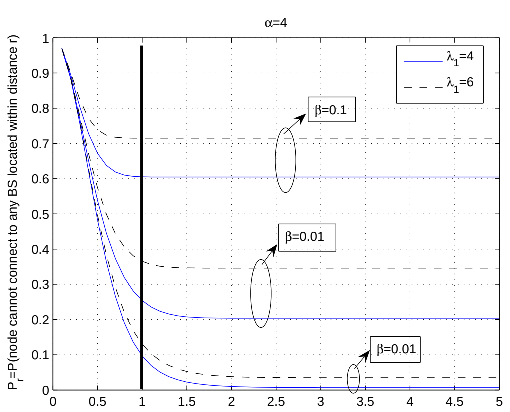

Fig. 4. δ_r versus r, random arrangement of BS with fading with $\lambda_2 = 1$

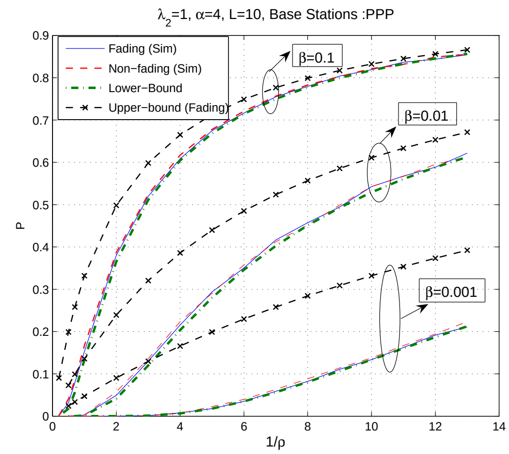

Fig. 5. P versus $1/\rho$. Comparison of simulation and theoretical bounds, when base stations are random.

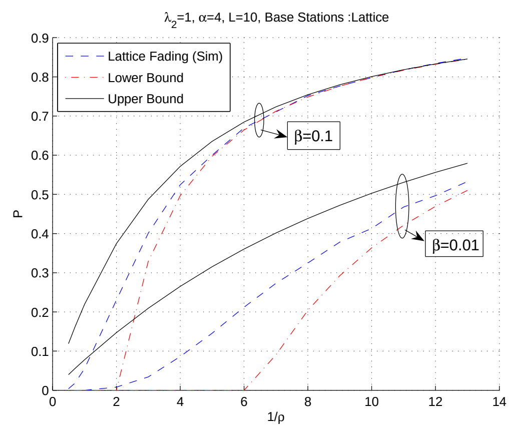

Fig. 6. P versus $1/\rho$. Comparison of simulation and theoretical bounds, when base stations are regular.

We observe from Fig 6, that the lower and upper bounds become tight for a lattice process when β and $1/\rho$ are large. In scaling scenarios, to maintain the same connectivity, both the intensities should scale in a similar fashion (with the same exponent). From Fig 7, we see that the connectivity is better when the base stations are regularly arranged. The gap in connectivity decreases when β becomes large. When $q \leq 1/\rho$, the gap between the upper bound of the isolation probability for random deployment of base stations (with fading) and the lower bound of isolation probabilty for lattice deployment (with fading) is $\rho^2 q^2/(1+\rho q) \to 0$ as $q \to 0$. So in contention-based systems like ALOHA, where $\beta > 1$, a regular arrangement of base stations offers no (specific) advantage. The above gap also goes to zero as $\alpha \to 2$. Also the number of base stations to which a sensor node can connect under fading, when the base stations are deployed randomly is approximately Poisson distributed with mean ρq. Also for a given ρ and β there exists an $\alpha \in (2, \infty)$ which maximizes the connectivity.

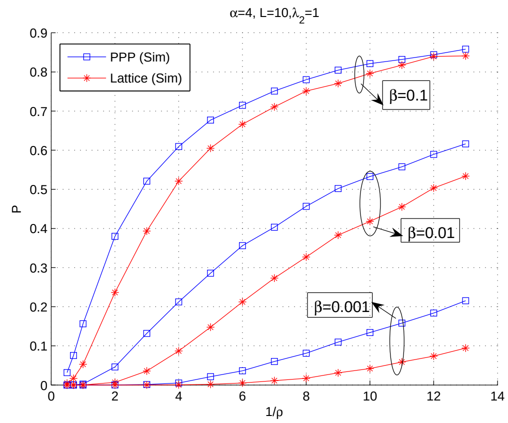

Fig. 7. P versus $1/\rho$. Comparison of regular and random arrangement of BS with fading.

V. Conclusion

In this paper, we have derived upper and lower bounds for the isolation probability for single-hop connectivity in an interference limited hybrid system. The cases when the base stations are deployed randomly and placed regularly in a lattice are considered. By simulations, the lower bound is shown to be very tight in the case when the base stations are randomly deployed. We show that the ratio of the number of sensor nodes to the base stations is a critical parameter for connectivity. We also show that the advantage of deploying base stations in a regular fashion decreases with increasing β (SIR threshold).

Appendix

A. Calculation of $\int_{\mathbb{R}^2} F(g(x)/\beta)dx$

Changing to polar coordinates, and using a change of variables

$$\frac{1}{2\pi}\int_{\mathbb{R}^2} F\Big(\frac{g(x)}{\beta}\Big)dx = \frac{\beta^{-2/\alpha}}{\alpha}\int_0^\infty F(y)y^{-2/\alpha-1}dy \overset{(a)}{=} \frac{\beta^{-2/\alpha}}{2}\int_0^\infty f(y)y^{-2/\alpha}dy \overset{(b)}{=} \Delta\int_0^\infty |\eta|^{2/\alpha-1}Re(\hat{f}(\eta))d\eta$$

where $\Delta = \frac{\Gamma(1/2-1/\alpha)\beta^{-2/\alpha}}{\Gamma(1/\alpha)\pi^{1/2-2/\alpha}}$. (a) follows from integration by parts and assuming $\lim_{y\to 0} F(y)y^{-2/\alpha} \to 0$. (b) follows from Folland [16, p.300]. Considering the real part from (8) and after integration, we have

$$\int_{\mathbb{R}^2} F\Big(\frac{g(x)}{\beta}\Big)dx = \frac{\beta^{-2/\alpha}\alpha\sin\left(\frac{2\pi}{\alpha}\right)}{2\pi\lambda_1} \quad (13)$$

B. Calculation of $P_1(x)$

$$\frac{1}{\lambda}\log\Big(\frac{1}{P_1(x)}\Big) \leq \int_{\mathbb{R}^2} F_f(\eta-x)\sum_{k\in\mathbb{Z}^2}\nu(\eta+k/\sqrt{\lambda_2})d\eta$$

$$\overset{(a)}{=} \lambda_2\int_{\mathbb{R}^2} F_f(\eta-x)\sum_{\xi\in\mathbb{Z}^2}\hat{\nu}(\sqrt{\lambda_2}\xi)e^{2\pi i\sqrt{\lambda_2}\langle\xi,\eta\rangle}d\eta$$

$$\overset{(b)}{=} \lambda_2\sum_{\xi\in\mathbb{Z}^2}\hat{\nu}(\sqrt{\lambda_2}\xi)e^{2\pi\sqrt{\lambda_2}i\langle\xi,x\rangle}\int_{\mathbb{R}^2} F_f(\eta)e^{2\pi i\sqrt{\lambda_2}\langle\xi,\eta\rangle}d\eta$$

$$= \lambda_2\sum_{\xi\in\mathbb{Z}^2}\hat{\nu}(\sqrt{\lambda_2}\xi)\cos(2\pi\sqrt{\lambda_2}\langle\xi,x\rangle)\widehat{F_f}(-\sqrt{\lambda_2}\xi)$$

$\langle\xi,\eta\rangle$ denotes the Euclidean inner product between ξ and η. $\widehat{F_f}$ denotes the Fourier transform of F_f. (a) follows from Poisson summation formula [16, p.254]. (b) follows from the translation property of the Fourier transform.

Acknowledgments

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