Interference in Ad Hoc Networks with General Motion-Invariant Node Distributions

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***Abstract*—In this paper we derive the tail properties of interference for any stationary and isotropic spatial distribution of transmitting nodes. Previously the properties of interference were known only when the nodes are distributed as a homogeneous Poisson point process on the plane. We show the effect of a singular path loss model on the tail distribution of the interference. When the path loss function has a singularity at the origin, the interference is shown to be a heavy-tailed distribution under very mild conditions. When the path loss is bounded, the distribution of the interference is predominantly dictated by the fading. We also provide asymptotically tight upper and lower bounds on the CDF of the interference, and discuss the effectiveness of using Gaussian approximation for modelling the interference.**

I. Introduction

Interference is one of the main performance limiting factors in ad hoc networks. Interference depends on the positions of the transmitting nodes, multiple access (MAC) scheme employed, power transmitted by each node and the fading distribution. In some sense, for a given set of nodes, the spatial distribution of the transmitting nodes at any instant is decided by the MAC protocol. The MAC protocol or the arrangement of nodes by themselves, may induce clustering, regular or a completely random arrangement of nodes. The spatial arrangement of nodes clearly affects the distribution of the interference. In this paper, we consider the decay of the tail of the interference for different path loss functions and spatial arrangement of nodes. We show the strong dependence of the distribution on fading when the path loss model is bounded. When the path loss model has a singularity at origin, the distribution depends only *weakly* on the fading process (see Table I)

Interference can be modeled as a shot noise process. Shot noise is very well studied topic [1], [2], but the caveat in ad hoc networks is that the *conditional* shot noise process needs to be studied, *i.e.*, the process given that there is a point at the origin (explained in more detail later). When the underlying node distribution is Poisson, one can replace the conditional shot noise process by the original shot noise process. Application of shot noise processes for the study of interference can be found in [3]–[7]. This paper generalizes our work in [8], where we study the properties of the interference in clustered ad hoc networks.

The paper is organized as follows. In Section II, we introduce the node distribution and fading models. In Section III, we derive the properties of the tail probability of interference for different path loss models. In Section IV, we consider concrete examples of spatial distribution of nodes and verify the results in Section III by simulations and analytical results.

II. System Model

The transmitters are modeled as a homogeneous (stationary and isotropic) point process ϕ of intensity λ on the plane. Every transmitter is assumed to transmit with unit power. The receiver under consideration is a point that does not belong to the point process ϕ and is located at $z = (R, 0)$. We assume that the fading is independent and identical for each transmit-receive pair. A transmitting node located at $x \in \phi$ can transmit to a node at z iff

$$S(x, z) = \frac{h_{xz} g(x - z)}{I_{\phi\setminus x}(z)} > \beta \tag{1}$$

where $g(x)$ represents the path loss model. h_{xz} denotes the square of the fading variable with a CDF F_h. We also use $f(h)$ to denote the PDF of the fading process. We assume $E[h] = 1$ and that the fading is iid per transmit-receive pair

$$I_\phi(z) = \sum_{y\in\phi} h_{yz} g(y - z) \tag{2}$$

We only consider[1] those point process ϕ for which, the second order density $\rho^{(2)}(x) = o(\log(\|x\|))$ for large x and $\rho^{(3)}(x, y) = o(\log(\|x\|)\log(\|y\|))$ for large values of x and y. See Section III for the definition of $\rho^{(2)}(x)$ and $\rho^{(3)}(x)$. In this paper we are interested in the properties of $I_{\phi\setminus\{0\}}(z)$. More precisely the characterization of the (complementary) CDF of I_ϕ given that there is a transmitting node at the origin. The importance of characterizing the conditional interference stems from (1), which in turn determines the performance of the system. We consider the following path loss models:

1) Singular model: $g(x) = \|x\|^{-\alpha}, \alpha > 2$.
2) Non-singular model: $g(x) = (1 + \|x\|^{\alpha})^{-1}, \alpha > 2$.

We require $\alpha > 2$ because we want $\int_{B(0,1)^c} g(x) < \infty$, where $B(a, r)$ denotes a ball of radius r centered around a.

III. CCDF bounds of Interference

We consider a transmitter and receiver pair, with the transmitter located at origin. i.e., $o \in \phi$ and the receiver located at z. So when calculating the outage probability for this pair, we have

$$P_o = P(h_{oz} g(z) < \beta I_{\phi\setminus o}(z) | \text{ transmitter at the origin}) \tag{3}$$

So all the probabilities are conditioned on the event that there is a transmitting node at the origin. In the theory of point processes, these probabilities are called the Palm probabilities

[1]This is not a restrictive assumption because $\rho^{(2)}(x) \to \lambda^2$, where λ is the intensity of the process.

Fading	$g(x)=\|x\|^{-\alpha}$ and $\rho^{(2)}(z)\neq 0$	$g(x)=(1+\|x\|^{\alpha})^{-1}$
$\bar{F}_h(y)\lesssim \exp(-\mu y)$	Heavy tailed with parameter $2/\alpha$.	$\bar{F}_{I_\phi(z)}(y)\lesssim \exp(-\mu y)$
$\bar{F}_h(y)\sim y^{-a}$, $a>1$	Heavy tailed with parameter $2/\alpha$.	$\bar{F}_{I_\phi(z)}(y)\sim y^{-a}$

Table I
OVERVIEW OF THE RESULTS

[9]–[11]. An equivalent and more convenient Palm probability representation is the reduced Palm probability and is denoted by $P^{!o}(.)$. This probability is the same as the Palm probability but one does not count the point at the origin. So to evaluate (3), we have to derive the properties of the interference $I_\phi(z)$ with respect to the Palm measures. Let $\mathcal{G}(v)$ denote the conditional generating functional of the point process ϕ, i.e.,

$$\mathcal{G}(v)=E_0^!\prod_{x\in\phi}v(x) \tag{4}$$

We will use a dot to indicate the variable on which the functional is acting on. For example $\mathcal{G}(v(.-y))=E_0^![\prod_{x\in\phi}v(x-y)]$. Let $\mathcal{L}_h(s)$ denote the Laplace transform of h_i. We have,

Lemma 1: The conditional Laplace transform of the interference is given by

$$\mathcal{L}_{I_\phi(z)}(s)=\mathcal{G}\left(\mathcal{L}_h(sg(.-z))\right) \tag{5}$$

Proof: From (2) we have

$$\begin{aligned}\mathcal{L}_{I_\phi(z)}(s) &= E_0^!\exp[-s\sum_{x_i\in\phi}h_i g(x_i-z)]\\ &\overset{(a)}{=} E_0^!\prod_{x_i\in\phi}\mathcal{L}_h(sg(x_i-z))\end{aligned}$$

where (a) follows from the independence of h_i and the result follows from (4). ■

Let $\mathcal{K}_n(B)$ denote the reduced n-th factorial moment measure [10], [11] of a point process ψ, and let $B=B_1\times\ldots\times B_{n-1},\quad B_i\in\mathbb{R}^2$. Then $\mathcal{K}_n(B)=E_0^!\left[\sum_{x_1,\ldots,x_{n-1}\in\psi}^{x_i\neq x_j}1_B(x_1,\ldots,x_{n-1})\right]$. $\mathcal{K}_2(B(0,R))$, for example denotes the average number of points inside a ball of radius R centered around the origin, given that a point exists at the origin. Also we observe that the interference distribution need not be the same for all receive points z (Palm distributions are not stationary in general). In a PPP, the distribution of $I_\phi(z)$ does not depend on z because of the stationarity of the Palm process for the PPP. But in general, $I_\phi(z)$ does not depend on the direction of z because of the isotropic property of Palm distribution for stationary process. First and second moments of the interference can be determined using the second and third order reduced factorial moments. When $\mathcal{K}_n(B)$ are absolutely continuous with respect to the Lebesgue measure, we denote the densities [10], [12] as $\rho^{(n)}$, the exact relation being[2] (in the stationary case)

$$\mathcal{K}_n(B)=\frac{1}{\lambda^n}\int_B\rho^{(n)}(x_{1,},x_2,\ldots,x_{n-1})dx$$

[2]Intuitively, $\rho^{(2)}(x)$ is the probability that there are two points separated by $\|x\|$. For PPP, it is $\rho^{(2)}(x)=\lambda^2$ independent of x. Also the second order product density is a function of two arguments i.e., $\rho^{(2)}(x_{1,},x_2)$. But when the process ϕ is stationary, $\rho^{(2)}$ depends only on the difference of its arguments i.e., $\rho^{(2)}(x_1,x_2)=\nu(x_1-x_2)$ for all $x_1,x_2\in R^2$. Furthermore if ϕ is motion-invariant, i.e., stationary and isotropic, then ν depends only on $\|x_1-x_2\|$ [10, pg 112].

One can easily show that the average interference is given by $E_0^![I_\phi(z)]=\frac{E[h]}{\lambda}\int_{R^2}g(x-z)\rho^{(2)}(x)dx$. We cite the following theorem from [8].

Theorem 1: When the transmitters are distributed as a homogeneous point process, the CCDF $\bar{F}_I(y)$ of the interference at location z, conditioned on a transmitter present at the origin[3] is lower bounded by $\bar{F}_I^l(y)$ and upper bounded by $\bar{F}_I^u(y)$, where

$$\bar{F}_I^l(y)=1-\mathcal{G}\left(F_h\left(\frac{y}{g(.-z)}\right)\right) \tag{6}$$

$$\bar{F}_I^u(y)=1-(1-\varphi(y))\mathcal{G}\left(F_h\left(\frac{y}{g(.-z)}\right)\right) \tag{7}$$

where $F_{h^2}(x)$ denotes the CDF of the square of the fading coefficient h and

$$\varphi(y)=\frac{1}{y\lambda}\int_{R^2}g(x-z)\rho^{(2)}(x)\int_0^{y/g(x-z)}\nu dF_h(\nu)dx. \tag{8}$$

If $E_0^![I_\phi^p]<\infty$, we can also use a loose $\varphi(y)=E_0^![I_\phi^p]y^{-p},\quad p\geq 1$.

Proof: See [8] ■

Lemma 2: $\varphi(y)\to 0$ as $y\to\infty$. Also if $g(x)=\|x\|^{-\alpha},\alpha>2$, then $\varphi(y)\sim y^{-2/\alpha}$.

Proof: From (8), increasing the limits of integration, we have $\varphi(y)<\frac{1}{y\lambda}\int_{R^2}g(x-z)\rho^{(2)}(x)dx$. This clearly tends to zero as y increases to infinity. When $g(x)=\|x\|^{-\alpha},\alpha>2$, we have

$$\begin{aligned}&\varphi(y)\\ =\ &\frac{1}{y\lambda}\int_0^\infty\nu dF_h\ (\nu)\int\|x\|^{-\alpha}1_{\|x\|^\alpha>\nu y^{-1}}\rho^{(2)}(x+z)dx\\ \sim\ &\frac{2\pi y^{-2/\alpha}}{\alpha-2}\rho^{(2)}(z)\int\nu^{2/\alpha}dF_h\ (\nu)\end{aligned}$$

■

The above lemma also indicate the asymptotic tightness of the bounds (6) and (7).

Theorem 2: When $g(x)=\|x\|^{-\alpha},\alpha>2$, factorial moments of ϕ exist and $\rho^{(2)}(z)\neq 0$ and continuous around a small nbhd of z, then $I_\phi(z)$ is heavy tailed distributed with parameter $2/\alpha$.

Proof: Due to space constraints we just provide the basic idea of the proof. The basic idea is to use Tauberian theorem to relate the decay of the CCDF at infinity with the behavior of the Laplace transform of the interference at $s=0$. To evaluate the behaviour of Laplace transform at $s=0$,we show and use the expansion of the probability generating functional of the point process. ■

Observe that when $g(x)=\|x\|^{-\alpha}$ and $\rho^{(2)}(z)\neq 0$, that $I_\phi(z)$ is always heavy tailed distribution and depends only on the

[3]We do not include the contribution of the transmitter at origin in the interference. This is because the transmitter at the origin is the intended transmitter which we would be focusing on.

$2/\alpha$ th moment of the fading process.

Theorem 3: Let $g(x) = 1/(1 + \|x\|^\alpha)$. Then

1) If the fading has at-most an exponential tail, i.e., $\bar{F}_h < \exp(-ah)$ for large h, then the interference tail is bounded by an exponential.
2) If the fading has a heavy tail with parameter a, i.e., $\bar{F}_h \sim h^{-a}$, the tail of the interference $P(I_\phi(z) > y)$ decays like y^{-a}.

Proof: See Appendix. ■

We now show that the distribution of interference decays exponentially fast at origin. The basic idea is that there is some contribution from some point of the process, however small it is.

Theorem 4: The CDF of the interference decays exponentially at the origin, i,e $\forall n \in \mathbb{N}$,

$$P(I_\phi(z) < y) = o(y^n)$$

as $y \to 0$.

Proof: Let $F_h(y) < y^a$ for some $a > 0$ and y small. Let k be chosen such that $ak > n$. From Theorem 1, we have $P(I_\phi(z) < y)$

$$\begin{aligned} &< \mathcal{G}\left(F_h\left(\frac{y}{g(.-z)}\right)\right) \\ &= E^{!0}\left[\prod_{x\in\phi} F_h\left(\frac{y}{g(x-z)}\right) |\phi \text{ has at least k points}\right] \end{aligned}$$

So we can multiply both sides by y^{-n} and take the limit. On the RHS we can take the limit inside by dominated convergence theorem. Since there are atleast k points almost surely on the plane (because ϕ is stationary), we have that the limit on the right goes to zero by our choice of k. ■

IV. Examples and Simulation results

In this section we give examples of interference for different point processes and fading distributions. We specifically concentrate on three different point process: The PPP, the Thomas cluster process and the Matern Hard core (minimum distance) process. We always consider stationary and isotropic point processes of intensity λ on the plane.

Poisson point process: PPPs are point processes which exhibit complete spatial randomness. PPP is generally used to model the node locations in a ad hoc network. The independence of the node locations makes this process easier to analyze. For PPP, $P^{!0} = P$ [10]. We also have $\rho^{(2)}(x) = \lambda^2$ and $\mathcal{G}(f) = \exp\left(-\lambda \int (1 - f(x))dx\right)$ [10].

1) $g(x) = \|x\|^{-\alpha}$: In this case the Laplace transform of the interference is given by, $\mathcal{L}_{I_\phi}(s) = \exp(-\lambda\pi s^{2/\alpha} E[h^{2/\alpha}]\Gamma(\alpha/2 - 1))$. Observe that the Laplace transform is independent of z and hence the distribution is independent of the location z. Also $\mathcal{L}_{I_\phi}(s)$ is the Laplace transform of a stable random variable with parameter $2/\alpha$ and hence heavy tailed. We can observe [13] that in this singular case, the interference depends only on the $2/\alpha$ th moment of power fading.

2) $g(x) = (1 + \|x\|^\alpha)^{-1}$: We first consider the case of $f(h) = \mu \exp(-\mu h)$. In this case,

$$\mathcal{L}_{I_\phi}(s) = \exp\left(-\lambda 2\pi^2 \csc(2\pi/\alpha)\alpha^{-1} \frac{s}{(\mu + s)^{1-2/\alpha}}\right)$$

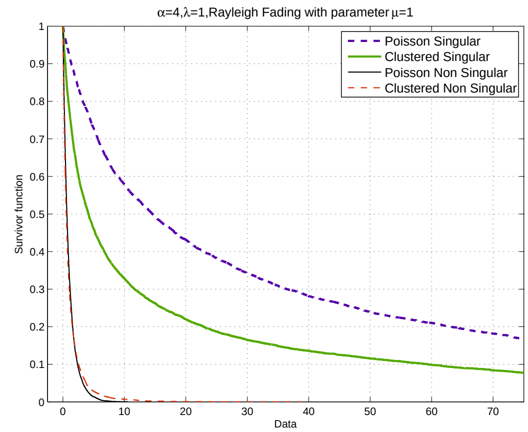

Figure 1. CCDF of the interference for exponential power fading and path loss $\alpha = 4$, $z = (3, 0)$

Observe that $\mathcal{L}_{I_\phi}(s)$ is well defined for $s > -\mu$. Let $\nu_{I_\phi}(x)$ denote the pdf of interference, we then have by the final value theorem, $\lim_{x\to\infty} e^{\mu_1 x}\nu_{I_\phi}(x) = \lim_{s\to 0} \mathcal{L}_{I_\phi}(s-\mu_1) < \infty$ for all $\mu_1 < \mu$. So the pdf is a combination of many decaying exponentials. In Figure 1, we plot the CCDF of Poisson interference with Rayleigh fading. We observe the heavy tailed distribution for $g(x) = \|x\|^{-\alpha}$ and the exponential decay when $g(x) = (1 + \|x\|^\alpha)^{-1}$.

Thomas Cluster processes: Thomas cluster processes [10], [11], are a class of clustered point process on the plane build on a PPP. Each point of the PPP is independently replaced by a cluster of points. This process can be used to model clustering (as the name suggests) for points stationary distributed on the plane. In [8], we have derived the properties of interference for this distribution of nodes. We show that (Lemma 3) that the interference has a stable distribution for $g(x) = \|x\|^{-\alpha}$ with parameter $2/\alpha$. We also provide an expression of the Laplace transform of the interference in [8]. So using a technique similar to the PPP case, we can show exponential decay of tail probability when $g(x)$ is bounded and the fading is exponentially dominated.

Hard Core process: We consider the underlying process ϕ to be a randomly shifted and rotated lattice. More precisely $\phi = \mathbb{Z}^2 e^{i\theta} + u$ where $\theta \sim U(0, 2\pi)$ and $u \sim U([0, 1]^2)$. This is a stationary and isotropic point process. The Palm process is the randomly rotated $\mathbb{Z}^2$. The interference results are verified by simulation. In Figure 2, we plot the CCDF for different values of z and with Rayleigh fading. We observe that the tail properties depend heavily on z. When $\|z\| < 1$, we have that $\rho^{(2)}(z) = 0$ (actually the associated measure $\mathcal{K}_2(A) = 0,\ \forall A \subset B(0, 1)$). So here the effective path loss model is bounded and hence the interference tail follows that of the fading. When $z = (1, 0)$, there is a positive probability that a transmitting node can be arbitrarily close to z and hence the interference follows a heavy tail distribution.

Approximation of the distribution of interference: We have the following observations from Theorems 1-3.

1) The CDF $F(y)$ of the interference decays exponentially fast near $y = 0$.
2) When $g(x) = (1 + \|x\|^\alpha)^{-1}, \alpha > 2$, the mean interference is finite, and the CCDF tail decays like that of the fading distribution.
3) When $g(x) = \|x\|^{-\alpha}$, the mean diverges and CDF has

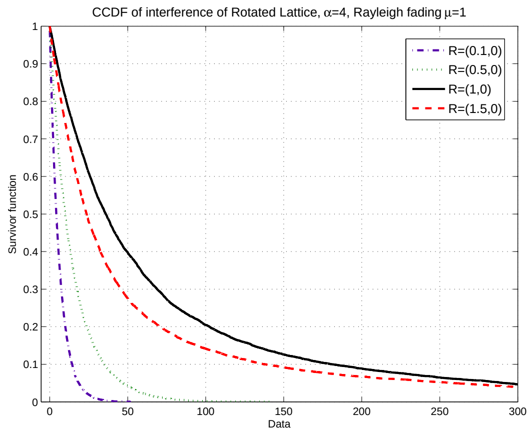

Figure 2. CCDF of the interference for exponential power fading and path loss $\alpha = 4$ when the point process is the lattice process.

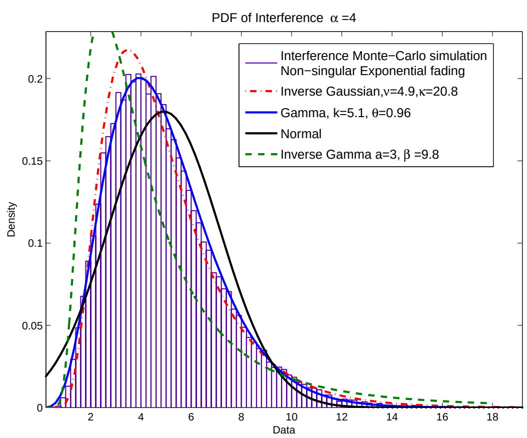

Figure 3. PDF of Interference and the corresponding fits for $g(x) = (1 + \|x\|^4)^{-1}$ and the square of fading as exponential with parameter 1.

a heavy tail.

Observation 1 eliminates the use of Gaussian distribution to model the interference except when the mean $\mu = E[I_\phi]$ is very large (but finite), so that $\exp(-\mu^2/2\sigma^2)$ is small. We choose three probability distributions which have these properties. The gamma distribution, and the inverse Gaussian distribution.

1) Gamma distribution: $f(x) = x^{k-1}\exp(-x/\theta)/\Gamma(k)\theta^k$. Mean : $k\theta$, Variance: $k\theta^2$.
2) Inverse Gaussian :
$$f(x) = \left[\frac{\nu}{2\pi x^3}\right]^{1/2} \exp\left(-\frac{\kappa(x-\nu)^2}{2\nu^2 x}\right)$$
Mean: ν, Variance: ν^3/κ.
3) Inverse Gamma :
$$f(x) = \beta^a x^{-a-1}\exp\left(-\beta/x\right)\Gamma(a)^{-1}$$
Mean: $\beta/(a-1)$, Variance: $\beta^2/((a-1)^2(a-2))$.

Observe that in the inverse Gaussian distribution, the mean and variance can be chosen independently of each other. Observe that the gamma distribution only has a $k-1$th order of decay at origin and has an exponential tail. On the other hand the inverse Gaussian distribution has exponential decay at origin and a slightly super exponential tail.

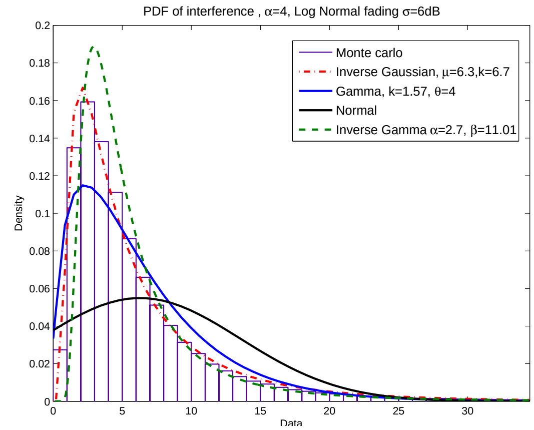

Figure 4. PDF of Interference and the corresponding fits for $g(x) = (1 + \|x\|^4)^{-1}$ and the fading as log-normal shadowing with $\sigma = 6$dB.

In Figure 3, we have plotted[4] the PDF of the interference using Monte-Carlo simulation when the underlying node distribution is PPP and the fading is Rayleigh with a non singular path loss model. We observe that the Normal fit performs the worst. Both the gamma and Inverse Gaussian give us a good fit. Also the inverse gamma pdf is a bad fit since it has a fourth order decaying tail, while the fading is exponentially decaying. In Figure 4, we have plotted the PDF of the interference with log-normal shadowing ($\sigma = 6$dB) and $g(x) = (1+\|x\|^4)^{-1}$. Since log normal has a tail which decays polynomially, we have from Theorem 3 that the pdf should also decay polynomially. This is indeed true and hence inverse gamma function gives the best fit. We also observe that inverse Gaussian also gives a good fit. This is because the variance can be increased independently and the exponential term in the pdf becomes small. In finite networks, where the number of nodes are finite and fixed and are distributed on bounded subset of the Euclidean plane, the interference does not decay infinitely fast at the origin, but only goes to zero like y^{na} where n is the number of transmitters and a the decay of the fading at origin.

V. Conclusion

In this paper, we have shown that the interference in an ad hoc network depends heavily on the path loss model chosen. When the path loss model is singular, the interference has a heavy tail, irrespective of the fading distribution. When the path loss model is bounded, the interference tail follows the tail of fading distribution. We also illustrate that using a Gaussian distribution to model interference is a bad approximation at least at low density of nodes. Inverse gamma and inverse Gaussian seem to approximate the interference distribution much better than the Gaussian distribution. The choice of distribution depends on the dominating fading distribution when the path loss model is non-singular.

VI. Appendix

Proof of Theorem 3:

Proof: Case 1):$\bar{F}_h \sim \exp(-ah)$. We will first show that the Laplace transform (5) of the interference converges for

[4]We have used a square of size 40×40 for simulation and averaged over 200000 instances.

$s < \sigma$, $\sigma < 0$ and diverges for $s > \sigma$. The real part of σ is also called the abscissa of convergence. From Lemma 1, we have $\mathcal{L}_{I_\phi(z)}(s) = E^{!0}[\prod_{x\in\phi} k(s,x)]$ where $k(s,x) = \mathcal{L}_h(sg(.-z))$. We have that $\mathcal{L}_{I_\phi(z)}(s)$ is finite if and only if $\eta(s) = E^{!0}[\sum_{x\in\phi} |\log k(s,x)|] < \infty$. So now show that the abscissa of convergence σ of $\mathcal{L}_{I_\phi(z)}(s)$ is strictly less than zero. Let $\delta(s,x,h) = \exp\left(\frac{-sh}{(1+\|x-z\|^\alpha)}\right)$. We have

$$\begin{aligned}\eta(s) &= E^{!0}\sum_{x\in\phi}|\log k(s,x)| \\ &= \int \left|\log\left(\int_0^\infty \delta(s,x,h)dF(h)\right)\right| \rho^{(2)}(x)dx\end{aligned}$$

We have $\bar{F}_h \sim \exp(-ah)$. So let the pdf $f(h) = a\exp(-ah), h > R$, for some large R. We have $k(s,x) =$

$$\begin{aligned}&= \int_0^R \delta(s,x,h)dF(h) \\ &\quad + a\int_R^\infty \exp\left(-h\left[a + \frac{s}{(1+\|x-z\|^\alpha)}\right]\right)dh \quad (9)\end{aligned}$$

So by dominated convergence theorem $k(s,x)$ is well defined[5] for all x and $s > -a$. Also $k(s,x) > 1$ for $s \in (-a,0)$. Let $b =\in (-a,0)$. We now prove that $\eta(b) < \infty$. Since $k(b,x) > 1$, we have $\log(k(b,x)) \leq k(b,x) - 1$. So if we show $\int_{B(0,\delta)^c}(k(b,x)-1)\rho^{(2)}(x)dx < \infty$ for large δ, we are done. We also have that for large $\|x\|$, $\rho^{(2)}(x) \to \lambda^2$. Choose δ such that for all $\|x\| > \delta$, $\rho^{(2)}(x)$ is very close to λ^2. So if we prove $\int_{B(0,\delta)^c}(k(b,x)-1)dx < \infty$, we are done. We have $\int_{B(0,\delta)^c}(k(b,x)-1)dx$

$$\begin{aligned}&= \underbrace{\int_{B(0,\delta)^c}\int_0^R \left[\delta(|b|,x,h)-1\right]f(h)}_{I} \\ &\quad + \underbrace{\int_{B(0,\delta)^c}\int_R^\infty \left[\delta(|b|,x,h)-1\right]f(h)}_{II}\end{aligned}$$

We first consider I. We can always increase δ such that $\exp\left(\frac{-|b|h}{(1+\|x-z\|^\alpha)}\right) \approx 1 + \frac{|b|h}{(1+\|x-z\|^\alpha)}$ (This can be done since $h < R$). We also have that $I = \int_{B(0,\delta)^c}\int_0^R \frac{|b|h}{(1+\|x-z\|^\alpha)}f(h)$ is finite. Considering the second integral, we have II

$$\begin{aligned}&= e^{-aR}\int_{B(0,\delta)^c}\frac{a(1+\|x-z\|^\alpha)\left(\delta(|b|,x,h)-1\right)}{a(1+\|x-z\|^\alpha)-b} \\ &\quad + \frac{|b|}{a(1+\|x-z\|^\alpha)-b}dx\end{aligned}$$

If we use the fact that $\exp(x) \approx 1 + x$ for small x, we immediately observe that $II < \infty$. So we have shown that $\eta(b) < \infty$ for all $b \in (-a,\infty)$. We also observe that $\eta(s) = \infty$ for $s < -a$. So the abscissa is equal to $-a < 0$. Using Theorem 3 in [14], we have that the tail falls exponentially.

Case 2: $\bar{F}_h \sim h^{-a}$ is a heavy tailed distribution: In this case we observe from (9), that $k(s,x) = \infty$ for all $s < 0$. So Theorem 3 in [14] cannot be applied. We will use Theorem 1 and provide upper and lower bounds. We first evaluate $\mathcal{G}\left(F_h\left(\frac{y}{g(.-z)}\right)\right)$. We have for large y

$$\begin{aligned}&\mathcal{G}\left(F_h\left(\frac{y}{g(.-z)}\right)\right) \\ &= \mathcal{G}\left(1 - \left[1 - F_h\left(y(1+\|x-z\|^\alpha)\right)\right]\right) \\ &\overset{(a)}{\sim} \mathcal{G}\left(1 - \left[y(1+\|x-z\|^\alpha)\right]^{-a}\right) \\ &\overset{(b)}{\sim} 1 - y^{-a}\int\left[(1+\|x-z\|^\alpha)\right]^{-\alpha}\rho^{(2)}(x)dx\end{aligned}$$

where (a) follows by continuity of $\mathcal{G}$ and large y (The continuity follows from the above argument). (b) follows from an argument similar to Theorem 2. We also have from the footnote in Theorem 1, that $\varphi(y) = y^{-a}E^{!0}[I_\phi(z)^a]$. Here $E^{!0}[I_\phi(z)^a] < \infty$ because of the bounded nature of $g(x)$ and its fast decaying tail. So from 1, we have that

$$\begin{aligned}&P(I_\phi(z) > y) \\ &< y^{-a}\left[\int\left[(1+\|x-z\|^\alpha)\right]^{-\alpha}\rho^{(2)}(x)dx + E^{!0}[I_\phi(z)^a]\right]\end{aligned}$$

and also

$$P(I_\phi(z) > y) > y^{-a}\left[(1+\|x-z\|^\alpha)\right]^{-\alpha}\rho^{(2)}(x)dx$$

So the tail decays like y^{-a}. ■

[5] Observe the importance of 1 in the denominator of the second term. If the one wasn't present, then $\forall s < 0$, $k(s,x)$ would be become undefined on an open neighborhood of z.

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