The Delay-optimal Number of Hops in Poisson Multi-hop Networks

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***Abstract*—We study the delay and throughput in a wireless multihop network with sources that form a Poisson point process and relays which are placed equidistantly on the source-destination line. A combined TDMA/ALOHA MAC protocol with intra-route TDMA and inter-route ALOHA is employed. We give bounds on the delay-optimal number of hops and derive the asymptotic delay-throughput tradeoff as the source-destination distance R gets large. The delay includes both the service times and waiting times in the buffers of the typical route. One main finding is that when the transmission probability and number of hops are jointly optimized for minimum delay, the number of hops scales as $R^{2/3}$ while the delay scales as $R^{4/3}$.**

I. Introduction

End-to-end throughput and delay are two fundamental performance metrics in wireless networks. They have been studied intensely, in particular in the last decade, but relatively little work has been devoted to multi-hop networks with randomly placed nodes. In particular, to the best of our knowledge, there have been no throughput-delay studies that include random node placement, multi-hop routing, and queueing delays. The goal of this paper is to make a first step in this direction, by deriving concrete end-to-end delay results and an asymptotic throughput-delay tradeoff for a network model with Poisson distributed sources, each with its own destination at a (large) distance R and a dedicated set of $N-1$ equidistant relays on the line in between. A TDMA/ALOHA MAC scheme is employed, where, for each route, a TDMA token is passed from source to the first relay, the next relay, and so on, then back to the source, and the node with the token is allowed to transmit with a certain probability. First, we determine the number of hops N that minimizes the delay as a function of the transmit probability. We then jointly optimize the number of hops and the transmit probability as $R \to \infty$ and derive scaling laws for the delay and throughput.

While the network model with the desired number of dedicated relays per route is clearly idealized, we view it as a continuous relaxation of the "real" routing problem, where relays have to be chosen from a discrete set of points. Our analysis yields the optimum number and ideal locations of the relays. This information can then be fed into a routing algorithm that finds actual relay nodes, e.g., from a Poisson point process (PPP), that are close to the ideal locations.

Previous analyses of Poisson networks have mostly focused on the single-hop case [1]–[3]. Extensions to the multi-hop case are nontrivial and rare. In [4], a model similar to ours was used, but the focus was the transport capacity and queueing delays were ignored. Bounds on the propagation speed of prioritized packets in Poisson networks were obtained in [5] and [6], but this line of work also ignores queueing delays and does not provide routing guidelines.

II. System model

A. Network setting

The network consists of an infinite number of sources at locations $\{x_i\}$, each with a destination at a finite distance R_i, and a random orientation. The locations of the sources are drawn independently according to a homogeneous PPP $\Phi = \{x_i\} \subset \mathbb{R}^2$ of density λ. Packets are relayed from the source to its respective destination by $N_i - 1$ equidistantly placed relays, $N_i \in \mathbb{Z}^+$. Note that, henceforth, when referring to a typical source and its corresponding route, the index i is dropped for convenience.

The sources are backlogged, i.e., they always have packets to transmit. Each relay has an infinite buffer, where packets that are received from the previous node in the route can be stored in a first-in, first-out fashion. Time is divided into packet slots. Within a route of N hops, a TDMA/ALOHA protocol is observed: the source transmits with probability p, then each relay from 1 to $N-1$ is allowed[1], one at a time, to transmit with probability p_r, until the TDMA cycle is completed and the source is given its turn again. A packet is received successfully by a node, if the signal-to-interference-ratio (SIR) in that slot is above a target threshold θ. If it is not, the transmitting node is informed via an ideal feedback channel and the packet remains at the head of its queue until the node gets another opportunity to transmit.

Finally, we assume that routes are synchronized at the slot level but the TDMA schedules need not be aligned in any way, i.e., a source of one route and a given relay of another route might be scheduled in the same slot.

B. Physical layer

The channel between two nodes at distance r includes Rayleigh fading - with a coherence time of one slot - and path-loss according to the law r^{-b}, where $b > 2$ is the path-loss exponent. We consider an interference-limited setting, where

[1] The source *transmits* with probability p since it always has a packet to transmit. A relay *is allowed to transmit* with probability p_r, since its queue may be empty.

thermal noise at the receiver is assumed to be negligible in comparison to the transmit power, which is normalized to one[2]. The SIR over a given hop is defined as

$$\mathrm{SIR} = \frac{A(R/N)^{-b}}{\sum_{z\in\Pi\setminus\{y\}} e_z t_z B_z d_z^{-b}} \quad (1)$$

where

- A is the fading coefficient between the transmitting node at location y and its receiving node; it is exponentially distributed with unit mean.
- Π is the point process of nodes which are scheduled in the given slot.
- $e_z = 1$ when the node at location z is allowed to transmit and $e_z = 0$ otherwise. If the node is a source, then $\mathbb{P}(e_z = 1) = p$; if it is a relay, then $\mathbb{P}(e_z = 1) = p_r$. The random variables $\{e_z\}$ are independent.
- $t_z = 1$ when the node at location z has packets in its queue and $t_z = 0$ otherwise. If the node is a source, then $\mathbb{P}(t_z = 1) = 1$.
- d_z is the distance between z and the location of the receiver; B_z is the respective fading coefficient, exponentially distributed with unit mean.

C. Metrics

We define the mean end-to-end delay D corresponding to the typical source as the mean total time (in slots) that it takes a packet to travel from the head of the source queue to its destination. Assuming negligible propagation times, D is the sum of the mean *waiting times* and *service times* along the queues of the route. The waiting time at a node is measured starting from the moment a packet arrives at that node's queue until it becomes the head-of-line packet, i.e., all packets in front of it have been successfully transmitted to the next node. The service time is measured from the moment a packet reaches the head of the queue until it is successfully received by the next node and includes the access delay associated with the MAC protocol.

We also define the route throughput (RT) as the expected number of packets successfully delivered to the destination per slot.

III. Evaluation of the Delay and Throughput

The objective of this section is the evaluation of D and RT. We begin by making the following assumptions.

- Packet successes are independent across hops of the same route.
- The network reaches a stationary regime, i.e., it is dynamically stable.

The first assumption is based on the observation that, for sufficiently small p and p_r, the point processes of nodes which are allowed to transmit can be considered approximately independent across slots. As a result, the SIRs across hops, hence the corresponding outcomes of packet transmissions, can be considered independent. The effect is also accentuated by fading [8].

A necessary condition for the second assumption to hold is that $p < p_r$. We can see this as follows: Denote the hop success probability in a route by p_s (due to symmetry, the success probability is the same across hops of the same route). According to [9], the relay queues will be stable provided that the packet arrival probability to the first relay, pp_s, is smaller than the packet departure probability from the first and all subsequent relays, $p_r p_s$, or $p < p_r$. Under this condition, packet arrivals to all relays are iid geometric with parameter pp_s. Moreover, the probability that a relay at location z transmits a packet is simply $\mathbb{P}(t_z = 1, e_z = 1) = p$.

Since only one node is allowed to transmit per route and Φ is a PPP, it follows from the displacement theorem [7] that the point process of potential interferers Π is a PPP. Based on our previous assumptions, the point process of actual interferers $\Pi' = \{z : e_z = 1, t_z = 1\}$ is thus also a PPP with density λp. By Corollary 3.2 in [2], the hop success probability is therefore

$$p_s = \mathrm{e}^{-\lambda p c (R/N)^2}, \quad (2)$$

where $c = \Gamma(1+2/b)\Gamma(1-2/b)\pi\theta^{2/b}$ is the *spatial contention parameter* [3] and $\Gamma(x),\ x > 0,$ is the gamma function.

Following the analysis in [10], the service time for the head-of-line packet at the source is

$$H = \frac{N}{pp_s} - N + 1,$$

and, similarly, the service time for the head-of-line packet at a relay is

$$H_r = \frac{N}{p_r p_s} - N + 1.$$

Moreover, the waiting time at the queue of a relay is

$$Q_r = N\frac{p}{p_r}\frac{1 - p_r p_s}{(p_r - p)p_s}.$$

The end-to-end delay is therefore given by

$$\begin{aligned} D &= H + (N-1)(H_r + Q_r) \\ &= \frac{N}{pp_s} + N(N-1)\frac{1 - p_r p_s}{(p_r - p)p_s}. \end{aligned} \quad (3)$$

Given that for small values of p_r, $1 - p_r p_s \approx 1$, the simpler expression

$$\bar{D} = \frac{N}{pp_s} + \frac{N(N-1)}{(p_r - p)p_s} \quad (4)$$

provides a satisfactorily tight upper bound.

Since a packet is received by the destination every N slots with probability pp_s, the route throughput is $\mathrm{RT} = pp_s/N$.

IV. Delay Optimization

In this section, we first optimize $\bar{D}$ over N keeping p fixed, and then over N and p jointly. We first relax the requirement $N \in \mathbb{Z}^+$ and let $N \in (0, +\infty)$.

[2]Thermal noise can be incorporated in the analysis. See [7, eq. (9)].

A. Fixed source transmission probability p

Proposition 1 *Let $p < \frac{p_r}{2}$. The number of hops N_o that minimizes $\bar{D}$ satisfies the inequality*

$$\sqrt{\lambda pc}R < N_o < \sqrt{2\lambda pc}R. \quad (5)$$

Moreover, for $p = \frac{p_r}{2}$, $N_o = \sqrt{\lambda pc R^2}$.

Proof: We rewrite (4) as

$$\bar{D} = \left(\frac{1}{p} - \frac{1}{p_r - p}\right) N e^{\lambda pc\left(\frac{R}{N}\right)^2} + \frac{1}{p_r - p} N^2 e^{\lambda pc\left(\frac{R}{N}\right)^2}. \quad (6)$$

The functions $f(N) \triangleq N e^{\lambda pc\left(\frac{R}{N}\right)^2}$ and $g(N) \triangleq N^2 e^{\lambda pc\left(\frac{R}{N}\right)^2}$ are continuous, differentiable and strictly convex in $(0, +\infty)$. $\bar{D}(N)$ is a linear combination of $f(N)$ and $g(N)$; for $p < \frac{p_r}{2}$, the coefficients of this linear combination are positive, hence $\bar{D}(N)$ is also strictly convex. Moreover, for $N \to 0$ or $N \to \infty$, $\bar{D} \to \infty$. This implies that $\bar{D}$ has a unique global minimum in $(0, +\infty)$, which satisfies $\bar{D}'(N_o) = 0$, or, after some algebra,

$$N_o(N_o^2 - \lambda pc R^2) + \left(\frac{p_r}{2p} - 1\right)\left(N_o^2 - 2\lambda pc R^2\right) = 0. \quad (7)$$

For $\frac{p_r}{2p} > 1$, it follows that N_o must satisfy (5). For $\frac{p_r}{2p} = 1$, we obtain $N_o = \sqrt{\lambda pc}R$. ■

Proposition 2 *Let $p \in \left(\frac{p_r}{2}, p_r\right)$. There exists $R_o > 0$, such that, for $R > R_o$, the number of hops N_o that minimizes $\bar{D}$ satisfies*

$$\sqrt{\left(1 - \frac{p}{p_r}\right)\lambda pc} R < N_o < \sqrt{\lambda pc}R.$$

Proof: $\bar{D}(N)$ is twice-differentiable in $(0, +\infty)$; after some algebra, we find that

$$\begin{aligned}\frac{\partial^2 \bar{D}}{\partial N^2} &= e^{\frac{\lambda pc R^2}{N^2}}\left[\frac{4(\lambda pc R^2)^2}{N^4}\left(\frac{1 - 1/N}{p_r - p} + \frac{1}{Np}\right)\right. \\ &- \left.\frac{2(\lambda pc R^2)}{N^2}\left(\frac{1 - 1/N}{p_r - p} + \frac{1}{Np}\right) + \frac{2}{p_r - p}\right].\end{aligned}$$

For $p \in (p_r/2, p_r)$ and $N \geq 1$, the coefficient of $(\lambda pc R^2)^2$ is positive, which means that there exists R'_o such that (s.t.), for $R > R'_o$, $\bar{D}''(N) > 0$, i.e., $\bar{D}(N)$ is strictly convex in $[1, \infty)$. Moreover, we can show that there exists $R''_o > 0$ s.t., for $R > R''_o$, $\bar{D}'(1) < 0$. As a result, taking $R_o = \max\{R'_o, R''_o\}$, there exists R_o, s.t., for $R > R_o$, $\bar{D}(N)$ has a unique global minimum $N_o > 1$ in $[1, \infty)$ which is given by the solution of (7).

Since $\frac{p_r}{2p} < 1$, it follows that, either $N_o < \sqrt{\lambda pc}R$ or $N_o > \sqrt{2\lambda pc}R$. We prove via contradiction that the latter is not possible. Setting $N_o = \sqrt{a\lambda pc}R$, $a > 2$, in (7)

$$\sqrt{a\lambda pc}R(a - 1) - \left(1 - \frac{p_r}{2p}\right)(a - 2) = 0. \quad (8)$$

Since $\sqrt{a\lambda pc}R > 1$ and $\frac{p_r}{2p} \in (0, 1)$, the left hand side (LHS) is greater than $a - 1 - a + 2 = 1$, hence there is no $a > 1$ s.t. (8) is satisfied. Now assume that $a \leq 1 - \frac{p}{p_r}$. The LHS of (8) is smaller than $\left(1 - \frac{p_r}{2p}\right)(2 - a) - 1 + a \leq 0$, hence there is no $a \leq 1 - \frac{p}{p_r}$ s.t. (8) is satisfied. ■

Remarks on Propositions 1 and 2:

- It is straightforward to show that setting $N = \sqrt{2\lambda pc}R$ maximizes RT. Hence, the delay-optimal number of hops is always smaller than the throughput-optimal number of hops.
- $\bar{D} = \Theta(R^2)$. The square at the exponent of R is a result of the fact that there are approximately N relays, which are allowed to transmit only every N slots.
- Asymptotic results can also be derived by keeping R fixed and letting $\lambda \to \infty$. In fact, the scaling laws are the same with $\sqrt{\lambda}$ in place of R, i.e., $N_o = \Theta(\sqrt{\lambda})$ and $D = \Theta(\lambda)$. Note that such a limiting regime results in an interference-limited network so the high-SNR assumption of Section II is not required.

We now prove the following exact scaling.

Proposition 3 *For $R \to \infty$, $N_o \to \sqrt{\lambda pc}R$.*

Proof: Note that (7) is a cubic equation over N. We rewrite it as follows

$$N_o^3 + \left(\frac{p_r}{2p} - 1\right) N_o^2 - \lambda pc R^2 N - 2\lambda pc R^2 \left(\frac{p_r}{2p} - 1\right). \quad (9)$$

Denoting the coefficients of each power as $a_0, \ldots, a_3 (= 1)$, the discriminant of the polynomial is given by $\Delta = \beta^3 + \gamma^2$, where $\beta = (3a_1 - a_2^2)/9$ and $\gamma = (9a_1 a_2 - 2a_2^3 - 27a_0)/54$. For $R \to \infty$, $\beta \to -\lambda pc R^2/3$ and $\gamma \to \frac{45}{54}\left(\frac{p_r}{2p} - 1\right)\lambda pc R^2$, hence, $\Delta \to -(\lambda pc R^2/3)^3$. Since Δ is negative, (9) has three real roots. The only positive root is given by the formula

$$N_o = 2\sqrt{-\beta}\cos\left(\frac{1}{3}\cos^{-1}\left(\frac{\gamma}{\sqrt{-\beta^3}}\right)\right) - \frac{a_2}{3}.$$

For $R \to \infty$, $\frac{\gamma}{\sqrt{-\beta^3}} \to 0$, therefore the argument of the cosine is $\pi/6$, which leads to $\lim_{R\to\infty} N_o = \sqrt{\lambda pc}R$. ■

Remarks on Proposition 3:

- An intuitive explanation for the limiting behavior of N_o is that, as R grows larger, the second term in (6) dominates the delay, thus the optimal number of hops tends to $\sqrt{\lambda pc}R$. The convergence to $\sqrt{\lambda pc}R$ is slower for smaller values of p as the first term has a substantial contribution to the delay for a wider range of R.
- The limit of the optimal success probability is $p_s \to e^{-1}$.
- The limit of the optimal distance per hop is $\frac{R}{N_o} \to \frac{2}{\sqrt{c}}\bar{R}$, where $\bar{R} = 1/(2\sqrt{\lambda p})$ is the expected closest neighbor distance in the PPP Π'.

Example: Consider a network with the parameter values $\lambda = 10^{-4}$, $p_r = 0.05$, $p = 0.01$, $\theta = 6$ dB and $b = 4$. In Fig. 1,

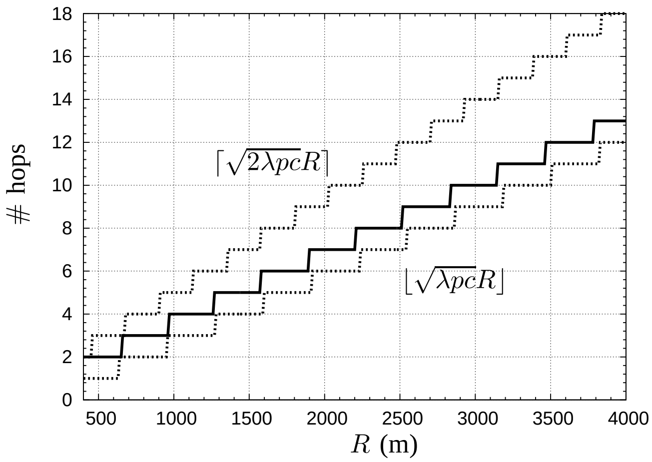

Fig. 1. Delay-optimal number of hops vs. R with related upper and lower bounds. ($\lambda = 10^{-4}$, $p_r = 0.05$, $p = 0.01$, $\theta = 6$ dB and $b = 4$)

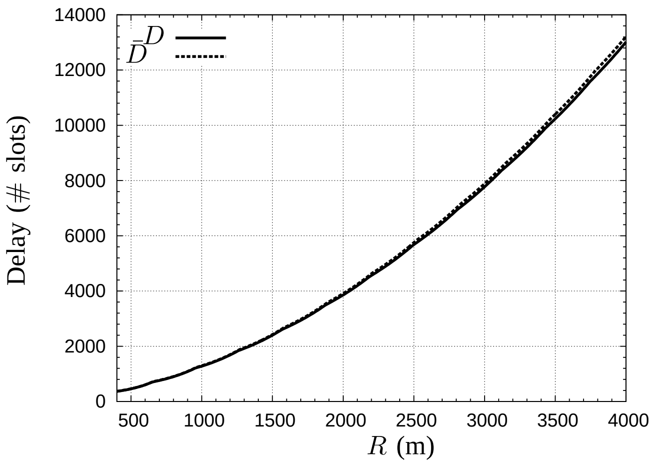

Fig. 2. Optimal end-to-end delay D and upper-bound $\bar{D}$ vs. R. The two curves are virtually identical. ($\lambda = 10^{-4}$, $p_r = 0.05$, $p = 0.01$, $\theta = 6$ dB and $b = 4$)

the number of hops that minimizes $\bar{D}$ is plotted vs. R and we can see that it lies between the predicted bounds $\lfloor\sqrt{\lambda pc}R\rfloor$ and $\lceil\sqrt{\lambda pc}R\rceil$. The optimized D and $\bar{D}$ are plotted in Fig. 2.

B. Variable source transmission probability p

We now explore the scenario where, for a given route distance, N and $p \in (0, p_r)$ are both optimization parameters. We have the following result.

Proposition 4 *The jointly delay-optimal N and p follow the scaling laws $N_o = \Theta(R^{2/3})$ and $p_o = \Theta(R^{-2/3})$.*

Proof: $\bar{D}$ is continuous, differentiable and strictly convex over $p \in (0, p_r)$ and $\lim_{p\to 0} \bar{D} = \lim_{p \to p_r} \bar{D} = \infty$. As a result, $\bar{D}$ has a unique global minimum $p_o \in (0, p_r)$, given by the solution of $\bar{D}'(p_o) = 0$, or

$$\frac{\lambda c R^2}{N_o^2}\left(\frac{N_o}{p_o} + \frac{N_o(N_o-1)}{p_r - p_o}\right) = \frac{N_o}{p_o^2} - \frac{N_o(N_o-1)}{(p_r-p_o)^2}. \quad (10)$$

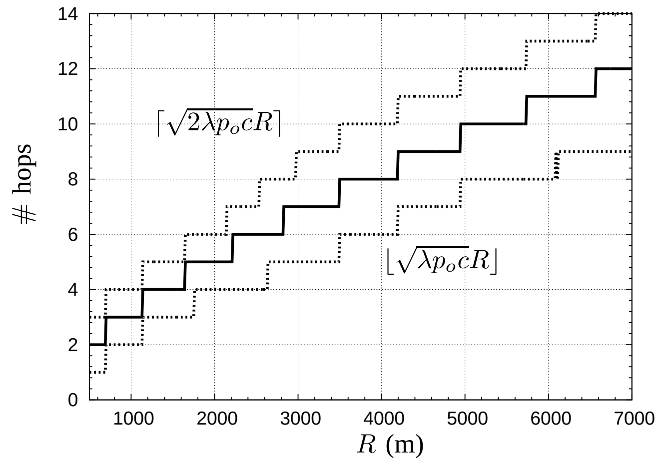

Fig. 3. Delay-optimal number of hops vs. R with related upper and lower bounds. The scaling law $R^{2/3}$ can be verified. ($\lambda = 10^{-4}$, $p_r = 0.05$, $\theta = 6$ dB, $b = 4$, variable p)

Note that we can rewrite (7) as

$$\frac{\lambda c R^2}{N_o^2}\left(\frac{N_o}{p_o} + \frac{N_o(N_o-1)}{p_r - p_o}\right) = \frac{N_o}{2p_o}\left(\frac{1}{p_o} + \frac{2N_o - 1}{p_r - p_o}\right). \quad (11)$$

Combining (10) and (11), we obtain

$$\frac{2N_o - 1}{\frac{p_r}{p_o} - 1} + \frac{2N_o - 2}{(\frac{p_r}{p_o} - 1)^2} = 1. \quad (12)$$

Note that, for $R \to \infty$, (12) requires that $p_o \to 0$, since, by Propositions 1 and 2, $N = \Theta(\sqrt{p_o(R)}R)$. As a result, in the limit of large R, (12) results in

$$\Theta(\sqrt{p_o}R)(p_o + p_o^2) = \Theta(1), \quad (13)$$

which implies that $p_o = \Theta\left(R^{-2/3}\right)$, and, consequently, $N_o = \Theta(R^{2/3})$. ■

Remarks on Proposition 4:

- In contrast to the fixed-p scenario, the optimal number of hops scales sublinearly with R.
- Since $p_o \to 0$, Proposition 1 requires that, asymptotically, N_o satisfies (5), and thus the optimal success probability is bounded as $\mathrm{e}^{-1} < p_s < \mathrm{e}^{-1/2}$.
- $\bar{D} = \Theta(R^{4/3})$, i.e., the delay scales superlinearly in R with exponent $4/3$. Moreover, $R/N = R^{1/3}$, i.e., the optimal hopping distance increases with R, which is a consequence of the absence of noise.
- As in the fixed-p case, these scaling laws may be expressed in terms of λ for a fixed R, i.e., $p_o = \Theta(\lambda^{-1/3})$, $\mathrm{RT} = \Theta(\lambda^{-2/3})$ and $\bar{D} = \Theta(\lambda^{2/3})$. Note that the scaling of RT is worse than $\Theta(\lambda^{-1/2})$, as N and p are optimized for minimum delay and not for maximum throughput.

Example: Consider a network with the same parameter values as in Section IV-A and $p \in (0, 0.05)$. Fig. 3 demonstrates the sublinear scaling of N_o with R and Fig. 4 shows the jointly optimal p_o. The optimal delay is plotted in Fig. 5.

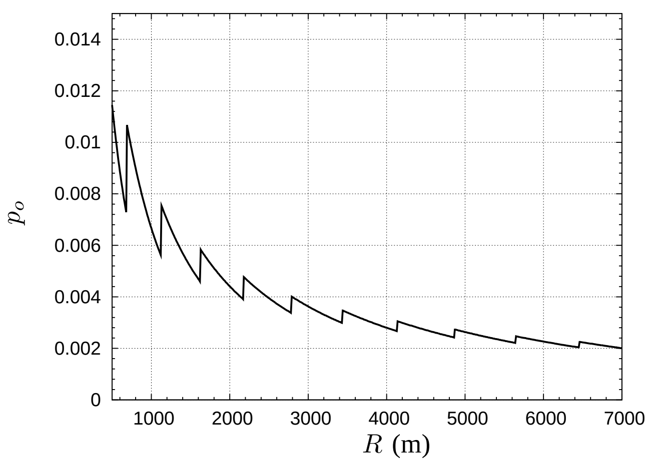

Fig. 4. Delay-optimal source transmission probability p vs. R. The scaling law $R^{-2/3}$ can be verified. ($\lambda = 10^{-4}$, $p_r = 0.05$, $\theta = 6$ dB, $b = 4$)

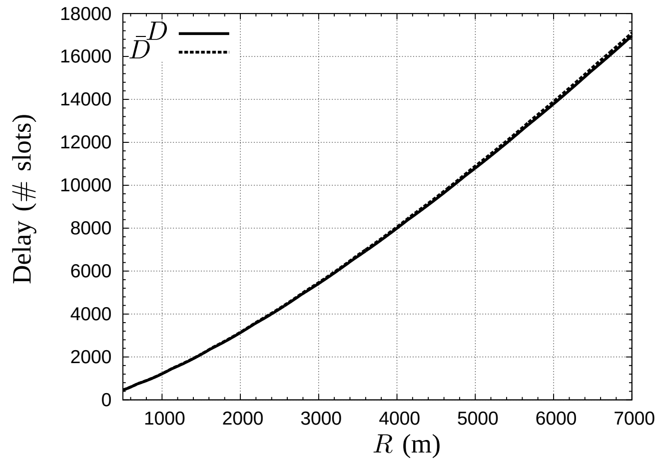

Fig. 5. Optimal end-to-end delay D and upper-bound $\bar{D}$ vs. R. As in the fixed-p case, the two curves are virtually identical. ($\lambda = 10^{-4}$, $p_r = 0.05$, $\theta = 6$ dB, $b = 4$, variable p)

V. DELAY-THROUGHPUT TRADE-OFF

The relation between the end-to-end delay and throughput merits special consideration. Let $p = qR^{-\kappa}$, $q \in (0, p_r)$ and $\kappa \geq 0$ and define the *delay and throughput exponents*

$$\delta = \lim_{R\to\infty} \frac{\log D(R)}{\log R}$$
$$\tau = \lim_{R\to\infty} \frac{\log \mathrm{RT}(R)}{\log R}.$$

Since $N = \Theta(\sqrt{p(R)}R) = \Theta(R^{1-\kappa/2})$, from the definitions of D in (3), and RT, it follows that $\delta(\kappa) = \max\{1 + \kappa/2, 2 - \kappa\}$ and $\tau(\kappa) = -1 - \kappa/2$. The plots of these exponents as functions of κ are shown in Fig. 6. For $\kappa = 0$, i.e., constant p, the delay scales as $\Theta(R^2)$ and the throughput as $\Theta(R^{-1})$. As κ increases, both $\delta(\kappa)$ and $\tau(\kappa)$ decrease, indicating that reducing p improves the performance in terms of delay but results in a throughput penalty. Increasing κ beyond the delay-optimal value $2/3$ - derived in Section IV-B - results in a delay penalty as $\delta(\kappa)$ starts to increase.

An interesting open question is how the scaling laws derived in this paper are modified when intra-route spatial reuse is allowed.

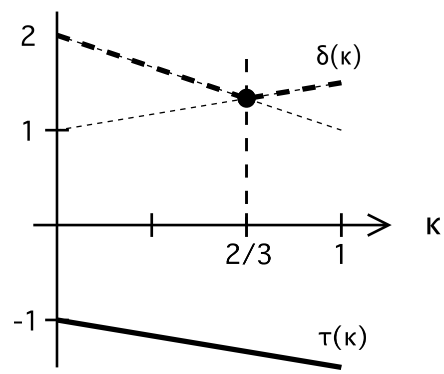

Fig. 6. Delay-throughput trade-off. $\kappa = 2/3$ is the delay-optimal exponent.

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