The Aggregate Throughput in Random Wireless Networks with Successive Interference Cancellation

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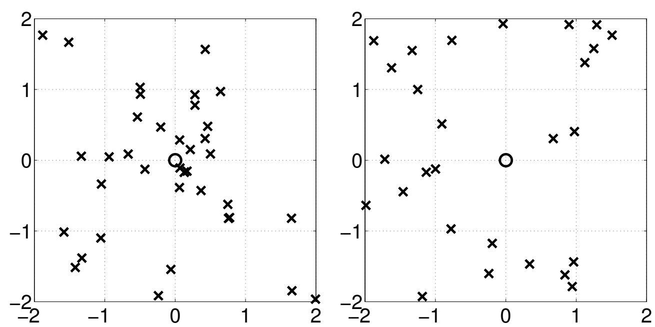
***Abstract*—The feasibility of successive interference cancellation (SIC) depends on the received power ordering from different users, which, in turn, depends on the fading distribution, path loss function and network geometry. Using a framework based on stochastic geometry, this paper studies the aggregate throughput in d-dimensional random wireless networks with SIC capability. We consider networks with arbitrary fading distribution, power-law path loss; the network geometry is governed by a non-uniform Poisson point process (PPP). Our results demonstrate how the performance of SIC changes as a function of the network geometry, fading distribution, and the path loss law. An important observation is that, in interference-limited networks, lower per-user information rate always results in higher aggregate throughput, while in noisy networks, there exists a positive optimal per-user rate at which the aggregate throughput is maximized.**

I. Introduction

Successive interference cancellation (SIC) is a promising technique to significantly improve the efficiency of wireless networks. While known to be suboptimal in general, SIC is more amenable to implementation compared with the capacity-achieving scheme (joint decoding) [1]–[3]. However, in a network without centralized power control, the use of SIC hinges on the ordering of the received power from different users (active transmitters) [4], which further depends on the spatial distribution of the users as well as many other network parameters.

Focusing on the aggregate throughput and assuming that all the transmitters use the same rate and the same power, this paper investigates the performance of SIC in d-dimensional random wireless networks with general fading distribution, power law path loss and Poisson distributed users. We provide upper and lower bounds on the aggregate throughput. Our results suggest that, in interference-limited networks, the aggregate throughput always increases as the per-user rate decreases. We also derive a closed-form upper bound on the asymptotic throughput which is shown by simulation to be tight. On the other hand, with noise, there exists an optimal positive per-user rate that maximizes the aggregate throughput.

Existing works studying SIC in similar contexts, *e.g.,* [1], are typically based on a *guard zone* approximation and consider exclusively uniform network and Rayleigh fading. In contrast, this paper uses an exact approach to tackle the problem in a more general type of networks with arbitrary fading distribution.



(a) $b = -1$ (b) $b = 1$

Fig. 1: Realizations of two non-uniform PPP with intensity function $\lambda(x) = 3\|x\|^b$ with different b, where × denotes an active transmitter and o denotes the receiver at the origin.

II. System Model and the Aggregate Throughput

A. The Power-law Poisson Network with Fading (PPNF)

Let the receiver be at the origin o and the active transmitters be represented by a marked Poisson point process (PPP) $\hat{\Phi} = \{(x_i, h_{x_i})\} \subset \mathbb{R}^d \times \mathbb{R}^+$, where x is the location of the users, h_x is the iid fading coefficient associated with the link from x to o, and d is the number of dimensions of the space. When the density function of the ground process $\Phi \subset \mathbb{R}^d$ is $\lambda(x) = a\|x\|^b,\ a > 0,\ b \in (-d, \alpha - d)$, where $\|x\|$ is the distance from $x \in \mathbb{R}^d$ to the origin and α is the path-loss exponent, we refer this network as a *power-law Poisson network with fading (PPNF)*. The condition $b \in (-d, \alpha - d)$ is needed in order to maintain a finite received power at o.

Fig. 1 shows realizations of two 2-d PPNFs with different b; Fig. 1a represents a network clustered around o whereas the network in Fig. 1b is sparse around the receiver at o. In general, the smaller b, the more clustered the network is at the origin, and $b = 0$ refers to uniform networks.

B. SIC Model and the Aggregate Throughput

Consider the case where all the nodes (users) transmit with unit power. Then, with an SINR model, a particular user at $x \in \Phi$ can be successfully decoded (without SIC) iff

$$\mathrm{SINR}_x = \frac{h_x\|x\|^{-\alpha}}{\sum_{y\in\Phi\setminus\{x\}} h_y\|y\|^{-\alpha} + W} > \theta,$$

where W is the noise power and θ is the SINR decoding threshold.

In the case of perfect interference cancellation, once a user is successfully decoded, its signal component can be completely subtracted from the received signal. Assuming the decoding order is always from the stronger users to the weaker users[1], we can generalize the SINR model above to the case with SIC. More precisely, a user x can be decoded if all the users in $\mathcal{I}_c = \{y \in \Phi : h_y\|y\|^{-\alpha} > h_x\|x\|^{-\alpha}\}$ are successfully decoded and

$$\frac{h_x\|x\|^{-\alpha}}{\sum_{y\in\Phi\setminus\{x\}\setminus\mathcal{I}_c} h_y\|y\|^{-\alpha} + W} > \theta.$$

Consequently, consider the ordering of all nodes in Φ such that $h_{x_i}\|x_i\|^{-\alpha} > h_{x_j}\|x_j\|^{-\alpha}$, $\forall i < j$. The number of users that can be successively decoded is N iff $h_{x_i}\|x_i\|^{-\alpha} > \theta\sum_{j=i+1}^{\infty} h_{x_j}\|x_j\|^{-\alpha} + \theta W$, $\forall i \le N$ and $h_{x_{N+1}}\|x_{N+1}\|^{-\alpha} \le \theta\sum_{j=N+2}^{\infty} h_{x_j}\|x_j\|^{-\alpha} + \theta W$.

The aggregate throughput (or, sum rate) is the total information rate received at the receiver o. Since all the users in the system transmit at the same rate $\log(1+\theta)$, the sum rate is

$$R = \mathbb{E}[\log(1+\theta)N] = \log(1+\theta)\mathbb{E}[N]. \quad (1)$$

The goal of this paper is to evaluate R as a function of different system parameters.

C. The Path Loss Process with Fading (PLPF)

In order to address the randomness from fading and random location of the nodes more concisely, we use the unified framework introduced in [5] which has been shown to be quite convenient in understanding the behavior of SIC in [6]. In particular, we define the path loss process with fading (PLPF) as $\Xi \triangleq \{\xi_i = \frac{\|x\|^\alpha}{h_x}\}$, where the index i is introduced in the way such that $\xi_i < \xi_j$ for all $i < j$. Then, we have the following lemma whose proof is provided in [6].

Lemma 1. *The PLPF Ξ, corresponding to a PPNF, is a one-dimensional PPP on $\mathbb{R}^+$ with intensity measure $\Lambda([0,r]) = a\delta c_d r^\beta \mathbb{E}[h^\beta]/\beta$, where $\delta \triangleq d/\alpha$, $\beta \triangleq \delta + b/\alpha \in (0,1)$ and h is a fading coefficient.*

Since ξ_i^{-1} is, by definition, the received power from the ith strongest users in the network, it suffices to only consider the PLPF Ξ to study the aggregate throughput.

III. Interference-limited Networks

In interference-limited networks, the noise power is negligible in comparison with interference and thus can be ignored, *i.e.,* $W = 0$. In this case, to study the statistics of N (and thus R), it suffices to consider only the standard PLPF (SPLPF) Ξ_β, *i.e.,* the PLPF with normalized intensity measure $\Lambda([0,r]) = r^\beta$ [6, Fact 1].

[1]It is straightforward to show that this stronger-to-weaker decoding order maximizes the aggregate throughput despite the fact that it is not necessarily the *only* optimal decoding order.

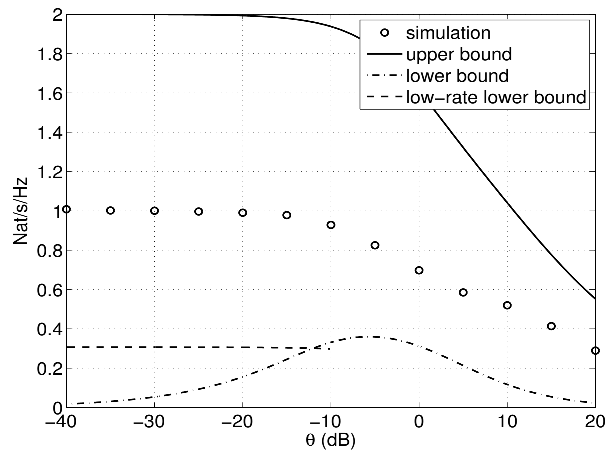


Fig. 2: Aggregate throughput at o in a 2-d uniform network with with path loss exponent $\alpha = 4$, *i.e.,* $\beta = \delta = 2/\alpha = 1/2$. The upper bound, lower bound and low-rate lower bound are based on the bounds Propositions 4, 2 and 3 in [6], respectively.

A. Bounds on the Aggregate Throughput

Due to the definition of the aggregate throughput in (1), we can directly estimate R by the bounds on $\mathbb{E}[N]$ provided in [6]. More precisely, based on the bounds on $\mathbb{E}[N]$ in Propositions 2, 3 and 4 of [6], we can produce the corresponding bounds on R and plot these bounds in Fig. 2. Here, we only show the low-rate lower bound [6, Proposition 3] for $\theta < -10$dB as it is only informative in the small θ regime. From the figure, we see that, just like the upper bound, the lower bound of the aggregate throughput becomes a non-zero constant when $\theta \to 0$. This indicates that while the aggregate throughput diminishes when $\theta \to \infty$, it converges to a finite positive constant when $\theta \to 0$. Furthermore, Fig. 2 suggests that when $W = 0$, the aggregate throughput is a monotonically decreasing function of θ, which is also verified by other simulations.

B. The Asymptotic Aggregate Throughput

Since the small θ regime is the regime where SIC is particularly useful [1], [3], [6], it is of interest to estimate the asymptotic aggregate throughput as $\theta \to 0$. One way of doing this is to use Proposition 4 of [6] and let $\theta \to 0$. This gives us $\frac{2}{\beta} - 2$ as an upper bound which turns out to be loose.

Fortunately, it is possible to construct a better bound which improves (reduces) the bound by a factor of 2 and is numerically shown to be tight. To show this better bound, we need the following lemma.

Lemma 2. *The Laplace transform of $\xi_k I_k$ is*

$$\mathcal{L}_{\xi_k I_k}(s) = \frac{1}{(c(s)+1)^k}, \quad (2)$$

where $I_k \triangleq \sum_{j=k+1}^{\infty} \xi_j^{-1}$ and $c(s) = s^\beta\gamma(1-\beta,s) - 1 + e^{-s}$ and $\gamma(\cdot,\cdot)$ is the lower *incomplete gamma function.*

Proof: For a non-fading 1-d network, the Laplace transform of the total interference from $[\rho,\infty)$ can be calculated by the probability generating functional (PGFL) of the PPP [7]. Similarly, the Laplace transform of $I_\rho \triangleq I_k \mid \{\xi_k = \rho\}$ is

$$\mathcal{L}_{I_\rho}(s) = \exp\left(-\int_\rho^\infty (1 - e^{-sr^{-1}})\Lambda(\mathrm{d}r)\right)$$
$$= \exp\left(-\left(s^\beta \int_0^{s\rho^{-1}} r^{-\beta} e^r \mathrm{d}r - \rho^\beta (1 - e^{-s\rho^{-1}})\right)\right), \quad (3)$$

where $\Lambda([0, r]) = r^\beta$ is the intensity measure of the SPLPF Ξ_β.

Then, considering the random variable $\rho I_\rho \triangleq \xi_k I_k \mid \{\xi_k = \rho\}$, we have

$$\mathcal{L}_{\rho I_\rho}(s) = \mathbb{E}[e^{-s\xi_k I_k} \mid \xi_k = \rho]$$
$$= \mathcal{L}_{I_\rho}(s\rho) = \exp(-c(s)\rho^\beta),$$

where $c(s) = s^\beta \gamma(1-\beta, s) - 1 + e^{-s}$. Using the results in [6, Lemma 3], we can calculate the Laplace transform of $\xi_k I_k$,

$$\mathcal{L}_{\xi_k I_k(s)} = \mathbb{E}_{\xi_k}[\mathcal{L}_{\rho I_\rho}(s) \mid \xi_k = \rho]$$
$$= \int_0^\infty \frac{\beta x^{k\beta-1}}{\Gamma(k)} e^{-(1+c(s))x^\beta} \mathrm{d}x = \frac{1}{(1 + c(s))^k}.$$

■

Then, we have the following bound on the asymptotic aggregate throughput, which is numerically shown to be tight.

Proposition 1. *The aggregate throughput* $R = \log(1+\theta)\mathbb{E}[N]$ *is (asymptotically) upper bounded by*

$$\lim_{\theta\to 0} R \le \frac{1}{\beta} - 1.$$

Proof: First, letting $p_k = \mathbb{P}(N \ge k) = \mathbb{P}(\xi_i^{-1} > \theta I_i, \forall i \le k)$, we have

$$\mathbb{E}[N] = \sum_{k=1}^\infty p_k \le \sum_{k=1}^\infty \mathbb{P}(\xi_k I_k < 1/\theta) \quad (4)$$
$$= \sum_{k=1}^\infty \int_0^{1/\theta} f_{\xi_k I_k}(x)\mathrm{d}x = \int_0^{1/\theta} \sum_{k=1}^\infty f_{\xi_k I_k}(x)\mathrm{d}x. \quad (5)$$

In general, the RHS of (5) is not available in closed form since $f_{\xi_k I_k}$, the pdf of $\xi_k I_k$, is unknown. However, when $\theta \to 0$, this quantity can be evaluated in the Laplace domain. To see this, consider a sequence of functions $(f_n)_{n=1}^\infty$, where $f_n(x) = \frac{1}{n}\sum_{k=1}^n f_{\xi_k I_k}(x),\ \forall x > 0$ and, obviously, $\int_0^\infty f_n(x)\mathrm{d}x = 1$ for all n. Thus, $\forall n \in \mathbb{N}$, we have

$$1 = \lim_{\theta\to 0} \frac{\int_0^{1/\theta} f_n(x)\mathrm{d}x}{\int_0^\infty e^{-\theta x} f_n(x)\mathrm{d}x} = \lim_{\theta\to 0} \frac{\int_0^{1/\theta} \sum_{k=1}^\infty f_{\xi_k I_k}(x)\mathrm{d}x}{\int_0^\infty e^{-\theta x} \sum_{k=1}^\infty f_{\xi_k I_k}(x)\mathrm{d}x}, \quad (6)$$

where

$$\int_0^\infty e^{-\theta x} \sum_{k=1}^\infty f_{\xi_k I_k}(x)\mathrm{d}x = \sum_{k=1}^\infty \mathcal{L}_{\xi_k I_k}(s)|_{s=\theta}.$$

Comparing (5) and (6) yields that

$$\lim_{\theta\to 0} \frac{\mathbb{E}[N]}{\sum_{k=1}^\infty \mathcal{L}_{\xi_k I_k}(s)|_{s=\theta}} \le 1,$$

where $\mathcal{L}_{\xi_k I_k}(s)$ is given by Lemma 2. Therefore, we have

$$\lim_{\theta\to 0} \log(1+\theta)\mathbb{E}[N] \le \lim_{\theta\to 0} \theta \sum_{k=1}^\infty \mathcal{L}_{\xi_k I_k}(\theta) = \lim_{\theta\to 0} \frac{\theta}{c(\theta)}.$$

The proof is completed by noticing that $\lim_{\theta\to 0} \frac{\theta}{c(\theta)} = \frac{1-\beta}{\beta}$. ■

In the example considered in Fig. 2, we see the bound in Proposition 1 matches the simulation exactly. Along with this example, we tested $\beta = 1/3$ and $\beta = 2/3$, and the bound is tight in both cases. The tightness of the bound is not surprising. Because, in the proof of Proposition 1, the only slackness we introduced while deriving the bound is due to putting $\mathbb{P}(\xi_k^{-1} > \theta I_k)$ in the place of p_k, and it is conceivable that, for every given k, this slackness diminishes in the limit, since $\lim_{\theta\to 0} \mathbb{P}(\xi_k^{-1} > \theta I_k) = \lim_{\theta\to 0} p_k = 1$. Thus, estimating $\mathbb{E}[N]$ by $\sum_{k=1}^\infty \mathbb{P}(\xi_k^{-1} > \theta I_k)$ is exact in the limit.

As many simulation results (including the one in Fig. 2) suggest that the aggregate throughput monotonically increases with decreasing θ, Proposition 1 provides an upper bound on the aggregate throughput in the network for all θ. We also conjecture that this upper bound is tight and thus can be achieved by driving the code rate at every user to 0, which is also backed by simulations (*e.g.,* see Fig. 2).

Since the upper bound is a monotonically decreasing function of β we can design system parameters to maximize the maximum achievable aggregate throughput provided that we can manipulate β to some extent. For example, since $\beta = \delta + b/\alpha$ and $\delta = d/\alpha$, one can try to reduce b to increase the upper bound. Note that b is a part of the density function of the *active* transmitters in the network and can be changed by independent thinning of the transmitter process [8], and a smaller b means the active transmitters are more clustered around the receiver. This shows that MAC schemes which introduce clustering have the potential to achieve higher aggregate throughput in the presence of SIC.

C. A Laplace-transform Based Approximation

Lemma 2 gives the Laplace transform of $\xi_k I_k$, which completely characterizes $\mathbb{P}(\xi_k^{-1} > \theta I_k)$, an important quantity in bounding p_k, $\mathbb{E}[N]$ and thus R. As analytically inverting (2) seems hopeless, a numerical inverse Laplace transform naturally becomes an interesting alternative to provide more accurate system performance estimate. However, the numerical inverse Laplace transform (numerical integration in complex domain) is generally difficult to interpret and offers limited insights on the system performance.

On the other hand, $\mathcal{L}_{\xi_k I_k}(s)|_{s=\theta} = \mathbb{P}(H > \theta\xi_k I_k)$, for an independent unit-mean exponential random variable H. This suggests to use $\mathcal{L}_{\xi_k I_k}(s)|_{s=\theta}$ to approximate $\mathbb{P}(\xi_k^{-1} > \theta I_k)$, and we would expect such an approximation to work for (at least) small θ. Because, first, it is obvious that for each k, this approximation is exact as $\theta \to 0$ since in that case both probabilities go to 1; second and more importantly, Proposition 1 shows that the approximated R based on this idea is asymptotically exact.

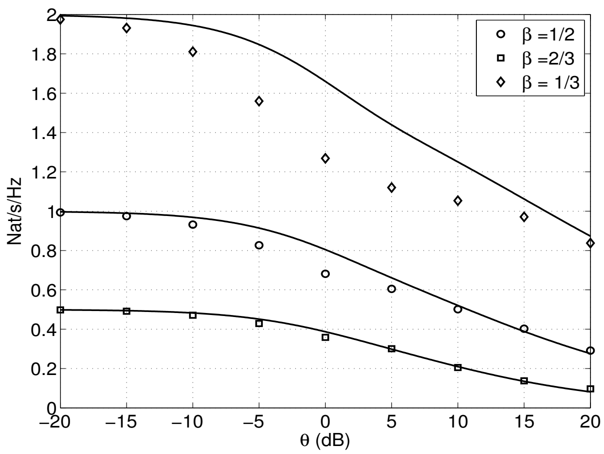


Fig. 3: Simulated and approximated aggregate throughput at o in a 2-d uniform network.

According to this approximation, we have

$$R \approx \frac{\log(1+\theta)}{c(\theta)} = \frac{\log(1+\theta)}{\theta^\beta \gamma(1-\beta,\theta) - 1 + e^{-\theta}}.$$

This approximation is compared with simulation results in Fig. 3, where we consider $\beta = \frac{1}{3}$, $\frac{1}{2}$ and $\frac{2}{3}$. As shown in the figure, the approximation performs quite well from -20dB to 20dB which covers the typical values of θ. Also, as expected, the approximation is most accurate in the small θ regime[2], which is known to be the regime where SIC is most useful [1], [3], [6].

IV. The Effect of Noise

In many wireless network outage analyses, the consideration of noise is neglected mainly due to the argument that most networks are interference-limited (without SIC). However, this is not necessarily the case for a receiver with SIC capability, especially when a large number of transmitters are expected to be successively decoded. Since the users to be decoded in the later stages have significantly weaker signal power than the users decoded earlier, even if for the first a few users interference dominates noise, after decoding a number of users, the effect of noise cannot be neglected.

In this section, taking an aggregate throughput perspective and considering non-zero noise power, we show quite different phenomena from the ones shown in Section III.

Defining $p_k^W \triangleq \mathbb{P}(N \geq k)$ to be the probability of successively decoding at least k users in the presence of noise of power W, we can rewrite p_k^W as

$$p_k^W \triangleq \mathbb{P}\left(\xi_i^{-1} > \theta(I_i + W), \forall i \leq k\right),$$

and we have the following lemma.

Lemma 3. *In a PPNF, the probability of successively decoding at least k users is bounded as follows:*

- $p_k^W \geq (1+\theta)^{-\frac{\beta k(k-1)}{2}} \mathbb{P}(\xi_k^{-1} > \theta\,(I_k + W))$
- $p_k^W \leq \theta^{-\frac{\beta k(k-1)}{2}} \mathbb{P}(\xi_k^{-1} > \theta(I_k + W))$

where $\Xi_\beta = \{\xi_i\}$ is the corresponding SPLPF and $I_k \triangleq \sum_{j=k+1}^{\infty} \xi_j^{-1}$.

Proof: The proof is analogous to the proof of Lemma 2 in [6] with two major distinctions: First, we need to redefine the event A_i to be $\{\xi_i^{-1} > \theta(I_i + W)\}$. Second, we need to consider the (original) PLPF instead of the SPLPF as the scale-invariance property [6, Proposition 1] does not hold in the noisy case. However, this does not introduce any difference in the order statistics of the $k-1$ smallest elements in Ξ conditioned on the ξ_k, and thus the proof follows exactly the same as that of Lemma 2 in [6] otherwise. ■

Thanks to Lemma 3, bounding p_k^W reduces to bounding $\mathbb{P}(\xi_k^{-1} > \theta\,(I_k + W))$. Ideally, we can bound $\mathbb{P}(\xi_k^{-1} > \theta\,(I_k + W))$ by reusing the bounds we have on $\mathbb{P}(\xi_k^{-1} > \theta I_k)$. Yet, this method does not yield a closed-form expression. Thus, we turn to a very simple bound which can still illustrate the distinction between the noisy case and the noiseless case.

Lemma 4. *In a noisy PPNF, we have*

$$\mathbb{P}(\xi_k^{-1} > \theta\,(I_k + W)) \leq \frac{\gamma(k, \frac{\bar{a}}{\theta^\beta W^\beta})}{\Gamma(k)}, \tag{7}$$

where $\bar{a} = a\delta c_d \mathbb{E}[h^\beta]/\beta$, $\beta = \delta + b/\alpha$, and $\delta = d/\alpha$.

Proof: First, note that $\mathbb{P}(\xi_k^{-1} > \theta\,(I_k + W)) \leq \mathbb{P}(\xi_k < \frac{1}{\theta W})$ which equals the probability that there are no fewer than k elements of the PLPF smaller than $1/\theta W$. By Lemma 1, the number of elements of the PLPF in $(0, 1/\theta W)$ is Poisson distributed with mean $\bar{a}/\theta^\beta W^\beta$, and the result follows. ■

Although being a very simple bound, Lemma 4 directly leads to the following proposition which contrasts what we observed in the interference-limited networks.

Proposition 2. *In a noisy PPNF, the aggregate throughput goes to 0 as $\theta \to 0$.*

Proof: Combining Lemma 3 and Lemma 4, we have

$$\begin{aligned}\mathbb{E}[N] &= \sum_{k=1}^{\infty} p_k^W \leq \sum_{k=1}^{\infty} \mathbb{P}(\xi_k^{-1} > \theta\,(I_k + W)) \\ &\leq \sum_{k=1}^{\infty} \frac{\gamma(k, \frac{\bar{a}}{\theta^\beta W^\beta})}{\Gamma(k)} = \bar{a}/\theta^\beta W^\beta.\end{aligned}$$

In other words, $\mathbb{E}[N]$ is upper bounded by the mean number of elements of the PLPF in $(0, 1/\theta W)$. Then, it is straightforward to show that $\lim_{\theta\to 0} R \leq \lim_{\theta \to 0} \bar{a}\theta^{1-\beta}/W^\beta$, and the RHS equals zero since $\beta \in (0,1)$. ■

Since it is obvious that $\lim_{\theta\to\infty} R = 0$, we immediately obtain the following corollary.

Corollary 1. *There exists at least one optimal $\theta > 0$ that maximizes the aggregate throughput in a noisy PPNF.*

As is shown in the proof of Proposition 2, $\bar{a}/\theta^\beta W^\beta$ is an upper bound on $\mathbb{E}[N]$. We can obtain an upper bound on the aggregate throughput by taking the minimum of

[2] The fact that the approximation seems also accurate for very large θ is more of a coincidence, as the construction of the approximation ignores ordering requirement within the strongest (decodable) k users and is expected to be fairly inaccurate when $\theta \to \infty$ [6, Lemma 2].

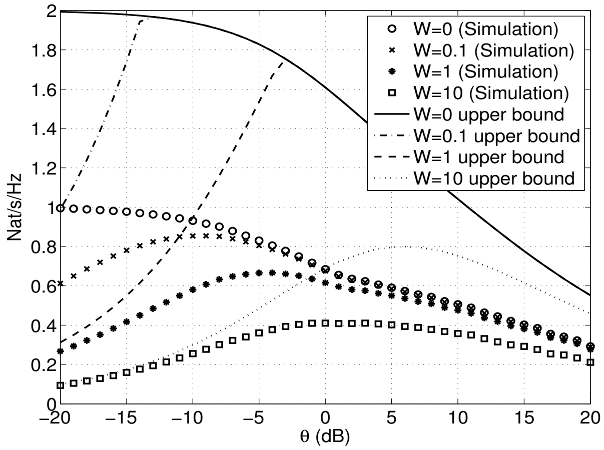


Fig. 4: Aggregate throughput at o in a 2-d uniform network with noise. where $\alpha = 4$, $a = 1$, $b = 0$ and fading is not (explictly) simulated. Three levels of noise are considered: $W = 0.1$, $W = 1$ and $W = 10$. $W = 0$ refers to the interference-limited case.

$\bar{a}\log(1+\theta)/\theta^{\beta}W^{\beta}$ and the upper bound shown in Fig. 2. Fig. 4 compares the upper bounds with simulation results, considering different noise power levels. This figure shows that the noisy bound becomes tighter and the interference bound becomes looser as $\theta \to 0$. This is because as θ decreases the receiver is expected to successively decode a larger number of users. The large amount of interference canceled makes the residual interference (and thus the aggregate throughput) dominated by noise. In this sense, the optimal per-user rate mentioned in Corollary 1 provides the right *balance* between interference and noise in noisy networks.

In [6], we showed that the absolute density of the network does not affect the performance of SIC in interference-limited networks. Thanks to Lemma 4, we see that the same result clearly does not hold for noisy networks. Nevertheless, there is still a monotonicity property in noisy networks, analogous to the scale-invariance property in noiseless networks, as stated by the following proposition.

Proposition 3 (Scale-monotonicity)**.** *For two PLPF Ξ and $\bar{\Xi}$ with intensity measure $\Lambda_1([0,r]) = a_1 r^{\beta}$ and $\Lambda_2([0,r]) = a_2 r^{\beta}$, where a_1 and a_2 are positive real numbers and $a_1 \leq a_2$, we have $p_k^W(\Xi) \leq p_k^W(\bar{\Xi}),\ \forall k \in \mathbb{N}$.*

Due to the space limitation, the proof of Proposition 3 is omitted from the paper. Combining Lemma 1 and Proposition 3 yields the following corollary since $\mathbb{E}[h^{\beta}] \leq 1$ given that $\mathbb{E}[h] = 1$.

Corollary 2. *In a noisy PPNF, fading reduces p_k^W, the mean number of users that can be successively decoded and the aggregate throughput.*

Since random power control, *i.e.,* randomly varying the transmit power at each transmitter under some mean and peak power constraint [9], can be considered as a way of manipulating the fading distribution, Corollary 2 also indicates that (distributed) random power control cannot increase the network throughput in a noisy PPNF.

V. Conclusions

This paper investigates the aggregate throughput of SIC in d-dimensional power-law Poisson networks with arbitrary fading distribution. We observe that, in interference-limited networks, the aggregate throughput (or, sum rate) is a monotonically decreasing function of the per-user information rate. This suggests low-rate/wideband transmission has the potential to improve the aggregate throughput given the SIC capability at the receiver.

Furthermore, the asymptotic sum rate is shown to be $\frac{1}{\beta} - 1$ as the per-user information rate goes to 0, where $\beta = \frac{b+d}{\alpha}$, α is the pathloss exponent and b determines the network geometry (clustering). Since b can be manipulated by distance-dependent access control or power control, the result shows that properly designed MAC or power control schemes can significantly increase the network performance when combined with low rate codes and SIC.

On the other hand, in noisy networks, there exists at least one positive optimal per-user rate which maximizes the aggregate throughput. Moreover, different from interference-limited networks where fading does not affect the performance of SIC [6], we proved fading to be harmful in noisy networks. This suggests communication schemes that eliminate (average out) the channel randomness are desirable in noisy networks with SIC capability.

Acknowledgment

The partial support of the NSF (grants CCF 728763 and CNS 1016742) is gratefully acknowledged.

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