Cellular Network Coverage with Inter-cell Interference Coordination and Intra-cell Diversity

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***Abstract*—Modeling cellular base stations (BSs) as a homogeneous Poisson point process (PPP), this paper provides exact expressions, in terms of a finite integral, for the coverage probability with inter-cell interference coordination (ICIC) and intra-cell diversity (ICD). Despite the fact that both ICIC and ICD can significantly improve the coverage probability, they improve coverage in drastically different ways in the high-reliability regime, where the user outage probability goes to zero. In particular, we show that ICD can provide *order* gain while ICIC only offers *linear* gain. This finding contrasts the recent result showing the absence of diversity gain in retransmission in *ad hoc* networks.**

I. INTRODUCTION

Inter-cell interference coordination (ICIC) and intra-cell diversity (ICD) can significantly improve the network coverage and thus play important roles in contemporary cellular systems. However, existing stochastic geometry-based cellular network analyses [1] largely ignore the effects of ICIC and ICD, resulting in overly pessimistic coverage estimates. To remedy this situation, this paper analyzes the benefits of ICIC and ICD under idealized assumptions.

Consider the case where a user is always served by the strongest (with shadowing but without fading) base station (BS). For ICD, we consider the case where each user is assigned M resource blocks (RBs) with independent fading and always decodes the packet from the RB with the best instantaneous signal-to-interference ratio (SIR) (selection combining). For ICIC, we assume under K-BS cooperation, the RBs that the user is assigned are silenced at the next $K-1$ strongest BSs. These abstractions hide the algorithmic details of complex ICIC [2], [3] and ICD [4]–[6] schemes, but allow an analytical coverage characterization based on the Poisson point process (PPP) cellular network model.

We show that while the coverage probability can be improved by both the ICIC and ICD, in the high-reliability regime, ICIC can only *linearly* affect the coverage probability but ICD can offer *order* gain. This finding is in sharp contrast with the recent discovery that retransmission does not result in diversity gain in *ad hoc* networks [7].

II. SYSTEM MODEL AND METRICS

A. System Model

Considering the typical user at the origin o, we use a homogeneous Poisson point process (PPP) $\Phi \subset \mathbb{R}^2$ with intensity λ to model the locations of BSs on the plane. To each element x of the ground process Φ, we add independent marks $S_x \in \mathbb{R}^+$ and $h_x^m \in \mathbb{R}^+$: S_x denotes the shadowing effect from BS x to o; h_x^m denotes the (Rayleigh) fading effect on the link from x to o at the m-th RB, where $m \in [M]$[1] and $M \in \mathbb{N}$. The combined (marked) PPP is written as $\hat{\Phi} = \{(x_i, S_{x_i}, (h_{x_i}^m)_{m=1}^M)\}$. In particular, under power law path loss, the received power at the typical user o at the m-th RB from a BS at $x \in \Phi$ is

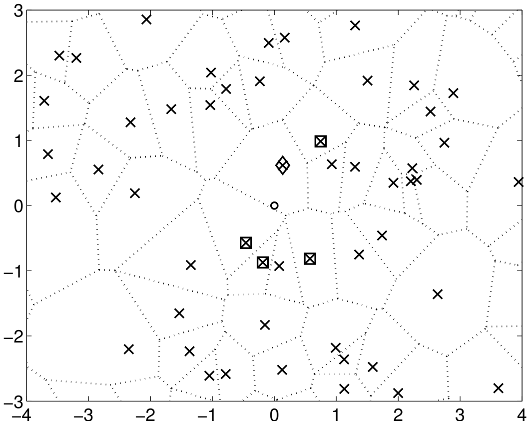

Fig. 1: A realization of the cellular network modeled by a homogeneous PPP Φ with K-BS ($K = 5$) coordination with lognormal shadowing. The typical user is denoted by ○, the BSs by ×, the serving BS by ◊ and the coordinated non-serving BS by □.

$$P_x = S_x h_x^m \|x\|^{-\alpha}, \tag{1}$$

where α is the path loss exponent. In this paper, we focus on Rayleigh fading, *i.e.,* h_x is exponentially distributed with unit mean but allow the shadowing distribution to be arbitrary with finite δ-th moment, *i.e.,* $\mathbb{E}[S_x^\delta] < \infty$, where $\delta = 2/\alpha$.

Fig. 1 shows a realization of a PPP-modeled cellular network under K-BS coordination with lognormal shadowing. Due to shadowing, the K strongest BSs under coordination are not necessarily the K nearest BSs.

The BS locations and the shadowing S_x are constant over RBs, S_x is iid across space (*i.e.,* over x), and the small-scale fading random variables h_x^m are iid across both space and RBs (*i.e.,* over both x and m).

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[1] We use $[n]$ to denote the set $\{1, 2, \cdots, n\}$.

The user is assumed to be associated with the strongest (without fading) BS and is called *covered* (without ICIC) at the m-th RB iff

$$\mathrm{SIR}_m = \frac{S_{x_0} h^m_{x_0} \|x_0\|^{-\alpha}}{\sum_{y\in\Phi\setminus\{x_0\}} S_y h^m_y \|y\|^{-\alpha}} > \theta, \tag{2}$$

where $x_0 = \arg\max_{x\in\Phi} S_x\|x\|^{-\alpha}$ and SIR_m is the signal-to-interference ratio (SIR) at the m-th RB.

Definition 1 (The path loss process with shadowing (PLPS)). *The path loss process with shadowing (PLPS)* Ξ *is the point process on* $\mathbb{R}^+$ *mapped from* $\hat{\Phi}$*, where* $\Xi = \{\xi_i = \frac{\|x\|^\alpha}{S_x}, x \in \Phi\}$ *and the indices* $i \in \mathbb{N}$ *are introduced such that* $\xi_k < \xi_j$ *for all* $k < j$.

The PLPS captures both the node distribution and the shadowing effect; consequently, it also determines the BS association. Further, we have the following lemma which directly follows from the mapping theorem [8].

Lemma 1. *The PLPS* Ξ *is a one-dimensional PPP with intensity measure* $\Lambda((0,r]) = \lambda\pi r^\delta \mathbb{E}[S^\delta]$*, where* $\delta = 2/\alpha$*,* $S \overset{\mathrm{d}}{=} S_x$ *and* $\overset{\mathrm{d}}{=}$ *means equality in distribution.*

B. The Coverage Probability and Effective Load

Similar to the construction of $\hat{\Phi}$, We construct a marked PLPS $\hat{\Xi} = \{(\xi_i, (h^m_{\xi_i})^M_{m=1}, \chi_{\xi_i})\}$, where we put two marks on each element of the PLPS Ξ: $h^m_\xi = h^m_x$, $m \in [M]$, are the iid fading random variables directly mapped from $x \in \Phi$; $\chi_\xi \in \{0,1\}$ indicates whether a BS represented by ξ is transmitting at the RB(s) assigned to the typical user[2]. In the case where no ambiguity is introduced, we will use h^m_i as a short of $h^m_{\xi_i}$ and χ_i as a short of χ_{ξ_i}.

The value of χ_i is determined by the ICIC scheduling policy. Given χ_i, the coverage condition at the m-th RB under K-BS coordination can be written in terms of the marked PLPS as

$$\mathrm{SIR}_{K,m} = \frac{h^m_1 \xi_1^{-1}}{\sum_{i=2}^\infty \chi_i h^m_i \xi_i^{-1}} > \theta. \tag{3}$$

Under K-BS coordination, the $K-1$ strongest non-serving BSs of the typical user do not transmit at the RBs to which the user is assigned and thus we have $\chi_i = 0$, $\forall i \in [K]\setminus\{1\}$.[3] For $i > K$, the exact value of χ_i is hard to model since the BSs can either transmit to its own users in the RB(s) assigned to the typical user or reserve these RB(s) for users in nearby cells, and the muted BSs can effectively "coordinate" with multiple serving BSs at the same time. Therefore, the resulting density of the active BSs outside the K coordinating BSs is a complex function of the user distribution, (joint) scheduling algorithms and shadowing distribution.

In order to maintain tractability, we assume χ_i, $i > K$ are iid Bernoulli random variables with (transmitting) probability $1/\kappa$, $\kappa \in \mathbb{R}^+$. Such modeling is justified by the random deployment of the users and the shadowing effect [9]. Here, $\kappa \in [1, K]$ is called the *effective load* of ICIC. $\kappa = K$ implies all the coordinating BS clusters do not overlap while $\kappa = 1$ represents the scenario where all the users assigned to the same RB(s) in the network share the same $K-1$ muted BSs. The actual value of κ lies between these two extremes and is determined by the scheduling procedure which this paper does not explicitly study. However, we assume that κ is known.

Let $\mathbf{S}_{K,m} \triangleq \{\mathrm{SIR}_{K,m} > \theta\}$ be the event of coverage at the m-th RB. We consider the coverage probability with inter-cell interference coordination (ICIC) and intra-cell diversity (ICD) formally defined as follows.

Definition 2. *The coverage probability with* K*-BS coordination and* M*-RB selection combining is*

$$\mathsf{P}^{\mathrm{c}}_{K,M} = \mathsf{P}^{\cup\mathrm{c}}_{K,M} \triangleq \mathbb{P}(\cup_{m=1}^M \mathbf{S}_{K,m}).$$

Here, the superscript c denotes *coverage* and $\cup$ stresses that $\mathsf{P}^{\cup\mathrm{c}}_{K,M}$ is the probability of being covered in *at least one of the M RBs*. (If there is no possibility of confusion, we will use $\mathsf{P}^{\cup\mathrm{c}}_{K,M}$ and $\mathsf{P}^{\mathrm{c}}_{K,M}$ interchangeably.)

C. Diversity Gain and the High-Reliability Regime

Diversity is a classic metric that measures the reliability of wireless communication schemes under fading. The standard definition of the diversity is based on the high SNR analysis where the interference is ignored [4]. The following definition extends this notion to the case with interference.

Definition 3 (Diversity (order) gain in interference-limited networks). *The diversity (order) gain, or simply diversity, of interference-limited networks is*

$$d \triangleq \lim_{\theta\to 0} \frac{\log \mathbb{P}(\mathrm{SIR} < \theta)}{\log \theta}.$$

Def. 3 is consistent with the diversity gain defined in [7], where the authors showed, quite surprisingly, that in (interference-limited) *ad hoc* networks retransmission does not result in diversity gain. Interference correlation is the main contributor to the diversity loss [10]. In the rest of the paper, we will complement this finding by investigating how much diversity ICIC and ICD introduce in cellular networks, taking into account that interference is correlated.

III. INTERCELL INTERFERENCE COORDINATION (ICIC)

We first focus on the effect of ICIC on coverage probability. Since no ICD is considered, we will omit the superscript m on the fading random variable h^m_ξ, $\xi \in \Xi$, for simplicity.

A. Integral Form of Coverage Probability

Lemma 2. *For* $\hat{\Xi} = \{(\xi_i, h_i, \chi_i)\}$*, let* $X_k = \xi_1/\xi_k$ *and* $Y_k = \xi_k^{-1}/\tilde{I}_k$*, where* $\tilde{I}_k \triangleq \sum_{i=k+1}^\infty \chi_i h_i \xi_i^{-1}$*. For all* $k \in \mathbb{N}$*, the two random variables* X_k *and* Y_k *are independent. Further,* $\mathbb{P}(X_k > x) = (1 - x^\delta)^{k-1} 1_{[0,1]}(x)$*, for* $k \geq 2$.

Proof (sketch): First, if $k = 1$, the independence is obvious, since, in this case, $X_k \equiv 1$ (with a degenerate distribution) while Y_k has some non-degenerate distribution.

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[2] It is assumed that the RBs are grouped into chunks of size M, *i.e.,* each BS either transmits at all the M RBs or does not transmit at any of these RBs.

[3] By default $\chi_1 = 1$.

For $k \geq 2$, the proof is supported by the (somewhat surprising) observation that X_k is independent from ξ_k. Formally, for all $x \in [0,1]$ and $y \in \mathbb{R}^+$, the joint ccdf of ξ_1/ξ_k and $\xi_k/\tilde{I}_k$ can be expressed as $\mathbb{P}(X_k > x, Y_k > y)$

$$\begin{aligned} &\overset{\text{(a)}}{=} \mathbb{E}_{\xi_k}\left[\mathbb{P}\left(\frac{\xi_1}{\xi_k} > x\right)\mathbb{P}\left(\frac{\xi_k^{-1}}{\tilde{I}_k} > y\right) \mid \xi_k\right] \\ &\overset{\text{(b)}}{=} \mathbb{P}\left(\frac{\xi_1}{\xi_k} > x\right)\mathbb{E}_{\xi_k}\left[\mathbb{P}\left(\frac{\xi_k^{-1}}{\tilde{I}_k} > y\right) \mid \xi_k\right] \\ &= \mathbb{P}\left(X_k > x\right)\mathbb{P}\left(Y_k > y\right), \end{aligned}$$

where (a) is due to the fact that $\{\xi_i, i < k\}$ and $\{\xi_i, i > k\}$ are conditionally independent given ξ_k by the Poisson property and $\{h_i\}$, $\{\chi_i\}$ are iid and independent from Ξ. (b) holds since, thanks to the Poisson property, conditioned on ξ_k, it can be shown that ξ_1/ξ_k follows the same distribution as that of the minimum of $k-1$ iid random variables with cdf $x^\delta 1_{[0,1]}(x)$. Since the resulting conditional distribution of ξ_1/ξ_k does not depend on ξ_k, this distribution is also the marginal distribution of ξ_1/ξ_k as is stated in the lemma. ∎

Lemma 3. *The Laplace transform of* $\xi_k\tilde{I}_k$ *is* $\mathcal{L}_{\xi_k\tilde{I}_k}(s) = (C_\kappa(s,1))^{-k}$, *where* $C_\kappa(s,m) = \frac{\kappa-1}{\kappa} + \frac{1}{\kappa}{}_2F_1(m,-\delta;1-\delta;-s)$ *and* ${}_2F_1(a,b;c;z)$ *is the Gauss hypergeometric function.*

The proof of Lemma 3 can be found in [11, Lemma 5].

Theorem 1 (K-BS coordination). *The coverage probability for a typical user under K-BS coordination ($K > 1$) is*

$$\mathsf{P}^{\mathrm{c}}_{K,1} = (K-1)\int_0^1 \frac{(1-x^\delta)^{K-2}\delta x^{\delta-1}}{(C_\kappa(\theta x,1))^K}\mathrm{d}x, \tag{4}$$

where $C_\kappa(s,m) = \frac{\kappa-1}{\kappa} + \frac{1}{\kappa}{}_2F_1(m,-\delta;1-\delta;-s)$.

Proof: The coverage probability can be written in terms of the PLPS as

$$\mathsf{P}^{\mathrm{c}}_{K,1} = \mathbb{P}(h_1\xi_1^{-1} > \theta\tilde{I}_K) = \mathbb{P}\left(\frac{h_1\xi_K^{-1}}{\tilde{I}_K} > \theta\frac{\xi_1}{\xi_K}\right), \tag{5}$$

where h_1 is exponentially distributed with mean 1, and thus $\mathbb{P}(\frac{h_1\xi_K^{-1}}{\tilde{I}_K} > x) = \mathcal{L}_{\xi_K\tilde{I}_K}(s)|_{s=x}$. Since $h_1\xi_K^{-1}/\tilde{I}_K$ and ξ_1/ξ_K are statistically independent (Lemma 2), we can calculate the coverage probability by

$$\mathsf{P}^{\mathrm{c}}_{K,1} = \int_0^1 \mathcal{L}_{\xi_K\tilde{I}_K}(\theta x)\mathrm{d}F_{\xi_1/\xi_K}(x), \tag{6}$$

where $\mathcal{L}_{\xi_K\tilde{I}_K}(\cdot)$ is given by Lemma 3 and $F_{\xi_1/\xi_K}(x) = 1 - (1-x^\delta)^{K-1}$ is the cdf of ξ_1/ξ_K given by Lemma 2. The theorem is thus proved by change of variables. ∎

B. ICIC in the High-Reliability Regime

Proposition 1. *Let* $\mathsf{P}^{\mathrm{o}}_{K,1} = 1 - \mathsf{P}^{\mathrm{c}}_{K,1}$ *be the outage probability of the typical user for* $K \in \mathbb{N}$. *Then,*

$$\mathsf{P}^{\mathrm{o}}_{K,1} \sim a_K\theta, \text{ as } \theta \to 0, \tag{7}$$

where $a_K = \frac{1}{\kappa}\frac{K!}{(1+\delta^{-1})_{K-1}}\frac{\delta}{1-\delta}$ *and* $(x)_n = \prod_{i=0}^{n-1}(x+i)$ *is the (Pochhammer) rising factorial.*

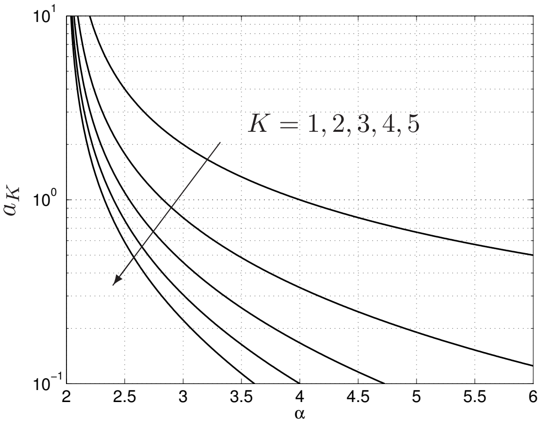

Fig. 2: The asymptotic coverage probability coefficient a_K from Prop. 1 as a function of the path loss exponent α under K-cell coordination (for $K = 1,2,3,4,5$, upper to lower). Here, $\kappa = K$.

The proof of Prop. 1 can be found in [11, App. A]. Prop. 1 shows that for pure ICIC schemes, the number of coordinating BSs only linearly affects the outage probability in the high-reliability regime. Hence *there is no diversity gain resulting from ICIC*, regardless of the effective load κ.

In Fig. 2, we plot the coefficient a_K for $K = 1,2,3,4,5$ as a function of the path loss exponent α assuming $\kappa = K$ in all the cases. The difference (in ratio) between a_K for different K indicates the usefulness of ICIC. This figure shows that ICIC is more useful when the path loss exponent α is large. This is consistent with intuition, since the smaller the path loss exponent, the more the interference depends on the far interferers and thus the less useful the local interference coordination. For other κ values, *e.g.,* $\kappa \equiv 1$, the same trend is observed.

IV. Intra-cell Diversity (ICD)

This section focuses on the case of ICD (only). Since it is a special case of the more general results discussed in Sec. V, we defer most of the proofs and only focus on discussing the implications of the result.

A. Coverage under ICD

Theorem 2. *The joint success probability of transmission over M RBs (without ICIC) is*

$$\mathsf{P}^{\cap\mathrm{c}}_{1,M} = \mathbb{P}(\bigcap_{m=1}^{M}\mathbf{S}_{1,m}) = \frac{1}{C_1(\theta,M)},$$

where $C_1(\theta,m) = {}_2F_1(m,-\delta;1-\delta;-\theta)$ *(as in Thm. 1).*

Due to the inclusion and exclusion principle, we have the coverage probability with selection combining over M RBs:

Corollary 1 (M-RB selection combining). *The coverage probability over M RBs without BS-coordination is*

$$\mathsf{P}^{\cup\mathrm{c}}_{1,M} = \sum_{m=1}^{M}(-1)^{m+1}\binom{M}{m}\mathsf{P}^{\cap\mathrm{c}}_{1,m},$$

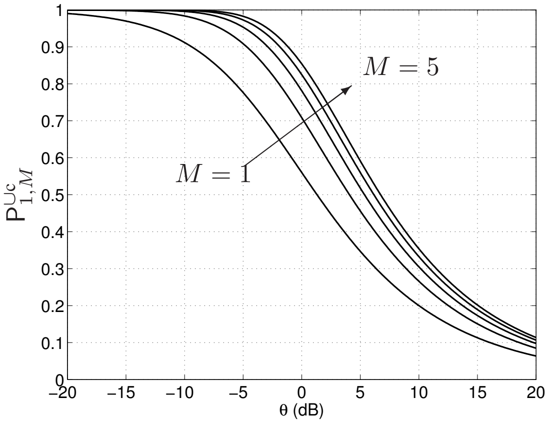

Fig. 3: The coverage probability with selection combining over M RBs without ICIC for $M=1,2,3,4,5$. Here, $\alpha=4$.

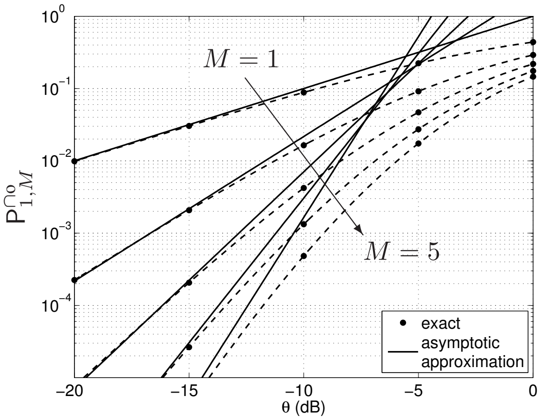

Fig. 4: Asymptotic behavior (and approximation) of the outage probability $\mathsf{P}^{\cap o}_{1,M}$ with M-RB joint transmission for $M=1,2,3,4,5$ (upper to lower). Here, $\alpha=4$.

where $\mathsf{P}^{\cap c}_{1,m}$ *is given by Thm. 2.*

Fig. 3 compares the coverage probability under M-RB selection combining, $\mathsf{P}^{\cup c}_{1,M}$ for $M=1,\cdots,5$. As expected, the more RBs assigned to the users, the higher the coverage probability and the marginal gain in coverage probability due to ICD diminishes with M.

B. ICD in the High-Reliability Regime

Proposition 2. *Let* $\mathsf{P}^{\cap o}_{1,M}=1-\mathsf{P}^{\cup c}_{1,M}$ *be the outage probability of a typical user under* M*-RB selection combining. We have*

$$\mathsf{P}^{\cap o}_{1,M}\sim a_M\theta^M,\ \text{as}\ \theta\to 0,$$

where $a_M=\mathsf{D}^M_x\left({}_1F_1(-\delta;1-\delta;x)\right)^{-1}\Big|_{x=0}$, ${}_1F_1(a;b;z)$ *is the confluent hypergeometric function of the first kind, and* $\mathsf{D}^n_x=\frac{\partial^n}{\partial x^n}$, $n\in\mathbb{N}$, *is the partial differential operator.*

The proof of Prop. 2 can be found in [11, App. B]. Prop. 2 clearly shows that a diversity gain can be obtained by selection combining, in stark contrast with the results presented in [7], where the authors show that there is no such gain in retransmission. The reason of this difference lies in the different association assumptions. [7] considers the case where the desired transmitter is at a fixed distance to the receiver which is independent from the locations of the interferers. However, this paper assumes that the user is associated with the strongest BS (on average). In other words, the signal strength from the desired transmitter and the interference are correlated. Prop. 2 together with [7] demonstrate that this correlation is critical in terms of the time/frequency diversity.

Fig. 4 compares the asymptotic approximation, *i.e.,* $a_M\theta^M$, with the exact expression provided in Cor. 1. A reasonably close match can be found when $\theta<-10$dB. Thus, despite the fact that main purpose of Prop. 2 was to indicate the qualitative behavior of ICD, the analytical tractability of a_M also provides useful approximations in applications with small coding rate, *e.g.,* spread spectrum/ultra-wide band communication, node discovery, *etc*.

V. COMBINED ICIC AND ICD

A. Coverage with both ICIC and ICD

In order to derive the coverage probability in the case with both ICIC and ICD, we first generalize Lemma 3 beyond Rayleigh fading. For a generic fading random variable H and PLPS $\Xi=\{\xi_i\}$, let $\tilde{I}^H_k$ be the interference from the BSs weaker (without fading) than the k-th strongest BS, *i.e.,* $\tilde{I}^H_k=\sum_{i>k}\chi_iH_i\xi_i^{-1}$, where $H_i\overset{\text{d}}{=}H$, $\forall i\in\mathbb{N}$, are iid and $\chi_i,\ i>k$ are iid. Then, we have the following lemma whose proof is analogous to that of Lemma 3 and is thus omitted.

Lemma 4. *For* $m\in\mathbb{N}$, *if* H *is a gamma random variable with pdf* $f_H(x)=\frac{1}{\Gamma(m)}x^{m-1}e^{-x}$, $\mathcal{L}_{\xi_k\tilde{I}^H_k}(s)=(C_\kappa(s,m))^{-k}$.

Theorem 3. *For all* $M\in\mathbb{N}$ *and* $K\in\mathbb{N}\setminus\{1\}$, *the joint coverage probability over* M*-RBs with* K*-cell coordination is*

$$\mathsf{P}^{\cap c}_{K,M}=\mathbb{P}(\bigcap_{m=1}^{M}\mathbf{S}_{K,m})=(K-1)\int_0^1\frac{(1-x^\delta)^{K-2}\delta x^{\delta-1}}{(C_\kappa(\theta x,M))^K}\mathrm{d}x.$$

Proof: Let h^m_i be the fading coefficient from the i-th strongest (on average) BS at RB m for $m\in[M]$. By definition, we have

$$\begin{aligned}\mathsf{P}^{\cap c}_{K,M}&=\mathbb{E}_\Xi\mathbb{P}\left(h^m_1\xi_1^{-1}>\theta\sum\nolimits_{i>K}\chi_ih^m_i\xi_i^{-1},\ \forall m\in[M]\right)\\&=\mathbb{E}_\Xi\prod_{m=1}^{M}\mathbb{P}(h^m_1>\theta\xi_1\sum\nolimits_{i>K}\chi_ih^m_i\xi_i^{-1})\\&=\mathbb{E}_\Xi\mathbb{E}\prod_{m=1}^{M}\exp(-\theta\xi_1\sum\nolimits_{i>K}\chi_ih^m_i\xi_i^{-1}),\qquad(8)\\&=\mathbb{E}_\Xi\mathbb{E}\exp(-\theta\xi_1\sum\nolimits_{i>K}\chi_iH_i\xi_i^{-1}),\end{aligned}$$

where the inner expectation in (8) is taken over h^m_i for $m\in[M]$ and $i\in\mathbb{N}$, and $H_i\triangleq\sum_{m=1}^{M}h^m_i$ are iid gamma distributed with pdf $\frac{1}{\Gamma(M)}x^{M-1}e^{-x}$ due to the independence (across m and i) and (exponential) distribution of h^m_i.

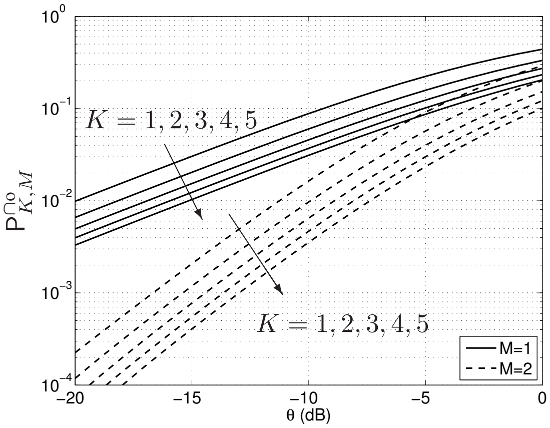

Fig. 5: The outage probability $\mathsf{P}^{\cap\mathrm{o}}_{K,M}$ under K-BS coordination over M RBs for $K = 1, 2, 3, 4, 5$ and $M = 1, 2$. Here, $\kappa \equiv 1$.

Further, writing ξ_1 as $\frac{\xi_1}{\xi_K}\xi_K$ and letting $\Xi_K = \{\xi_i : \xi_i \in \Xi, i > K\}$, we obtain the following expression by taking advantage of the statistical independence shown in Lemma 2:

$$\mathsf{P}^{\cap\mathrm{c}}_{K,M} = \mathbb{E}_{\frac{\xi_1}{\xi_K}} \mathcal{L}_{\xi_k \tilde{I}^H_K}\left(\theta \frac{\xi_1}{\xi_K}\right),$$

where $\mathcal{L}_{\xi_K \tilde{I}^H_K}(\cdot)$ is gvien in Lemma 4. The proof is completed by applying the distribution of ξ_1/ξ_K given in Lemma 2. ■

Similar to Cor. 1, the following corollary follows directly from the inclusion and exclusion principle.

Corollary 2 (K-BS coordination and M-RB selection combining). *The coverage probability over M RBs with K BS-coordination is*

$$\mathsf{P}^{\cup\mathrm{c}}_{K,M} = \sum_{m=1}^{M} (-1)^{m+1} \binom{M}{m} \mathsf{P}^{\cap\mathrm{c}}_{K,m}, \tag{9}$$

where $\mathsf{P}^{\cap\mathrm{c}}_{K,m}$ is given by Thm. 3.

B. The High-Reliability Regime

Proposition 3. *Let $\mathsf{P}^{\cap\mathrm{o}}_{K,M} = 1 - \mathsf{P}^{\cup\mathrm{c}}_{K,M}$ be the outage probability of a typical user under M-RB selection combining and K-BS coordination. We have*

$$\mathsf{P}^{\cap\mathrm{o}}_{K,M} \sim a_\kappa(K, M)\theta^M, \text{ as } \theta \to 0,$$

where $a_\kappa(K, M) > 0$, $\forall K, M \in \mathbb{N}$.

Prop. 3 combines Props. 1 and 2. Its proof is analogous to that of Prop. 2 and is thus omitted from the paper. It shows, as expected, the diversity gain for a ICIC-ICD combined scheme *only* comes from ICD.

In Fig. 5, we plot the outage probability for different numbers of coordinated cell $K = 1, 2, 3, 4, 5$ and RBs for selection combining $M = 1, 2$, assuming $\kappa \equiv 1$, and observe the consistency with Prop. 3.

VI. CONCLUSIONS

This paper analyzes the cellular network coverage using a PPP-based model, incorporating inter-cell interference coordination (ICIC) and intra-cell diversity (ICD). We show that while ICIC reduces the interference by muting nearby interferers, the number of coordinated BSs only affects the outage probability by the coefficient and does not change the fact that $\mathsf{P}^{\mathrm{o}}_{K,1} = \Theta(\theta)$ as $\theta \to 0$. In contrast, ICD affects the outage probability by both the coefficient and the exponent, resulting in diversity *order* gain in the network coverage. This result contrasts the recent discovery that retransmission does not provide diversity in *ad hoc* networks [10].

We emphasize that although ICIC and ICD are fundamentally different strategies, they both improve the network coverage by introducing extra load in the network: ICIC at the nearby BSs and ICD at the serving BS. By ergodicity, it is easy to show that with K-BS coordination and M-RB selection combining, the mean load at each BS is κM times the load in the case without ICIC and ICD, where $\kappa \in [1, K]$ is the *effective load* of ICIC and depends on the scheduling implementation. Thus, the result of this paper suggests that ICD is a more effective approach to provide coverage in the high-reliability regime. However, in order to achieve better throughput, carefully choosing ICIC-ICD combined schemes is necessary but beyond the scope of this paper.

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