Asymptotic Outage Analysis of General Motion-Invariant Ad Hoc Networks

Riccardo Giacomelli
Dept. of Control and Computer Eng.
Politecnico di Torino
Torino, Italy
E-mail: riccardo.giacomelli@polito.it

Radha Krishna Ganti
Dept. of Electrical and Computer Eng.
University of Texas at Austin
Austin, TX, USA
E-mail: rganti@austin.utexas.edu

Martin Haenggi
Dept. of Electrical Engineering
University of Notre Dame
Notre Dame, IN, USA
E-mail: mhaenggi@nd.edu

***Abstract*—The outage analysis of networks with randomly distributed nodes has been mostly restricted to the case of Poisson networks, where the node locations form a homogeneous Poisson point process. In this paper, we show that in great generality, the outage probability, as a function of the density of interfering nodes η, approaches $\gamma\eta^\kappa$ as η goes to zero, where γ and κ are the *spatial contention* and the *interference scaling exponent*, respectively. Interestingly, κ is restricted to a small range of possible values: $1 \leq \kappa \leq \alpha/2$ for a path loss exponent α. We also prove that for ALOHA, $\kappa = 1$ irrespective of the point process properties, and we demonstrate how the upper bound $\kappa = \alpha/2$ can be achieved.**

I. Introduction

The outage probability is the natural metric for large wireless systems, where it cannot be assumed that the transmitters are aware of the states of all the random processes governing the system. On of the main sources of uncertainty In many networks are the nodes' positions, which are then best modeled using a stochastic point process model whose points represent the locations of the nodes.

Previous work on outage characterization in networks with randomly placed nodes has mainly focused on the case of the homogeneous Poisson point process with ALOHA and Rayleigh fading [1], [2], for which a simple closed-form expression for the outage exists that valid for all network densities, thresholds, and path loss exponents. Extensions to models with dependence (node repulsion or attraction) are non-trivial. On the repulsion or hard-core side, where nodes have a guaranteed minimum distances, approximate expressions were derived in [3], [4]; on the attraction or clustered side, [5] gives an outage expression in the form of a multiple integral for the case of Poisson cluster processes.

Clearly, outage expressions for general networks would be highly desirable. In view of the difficulties of analyzing non-Poisson point processes, it cannot be expected that general closed-form expressions will be found. In this paper, we focus on Rayleigh fading and resort to the asymptotic regime, letting the density of interferers η go to zero. We will show the outage probability approaches is $\gamma\eta^\kappa$ as $\eta \to 0$, where γ is the network's *spatial contention* parameter [6], and κ is the *interference scaling exponent*. Interestingly, κ is confined to the range $1 \leq \kappa \leq \alpha/2$ for any reasonable[1] MAC scheme. While $\kappa = 1$ is the exponent for ALOHA, $\kappa = \alpha/2$ can be achieved with MAC schemes that impose a hard minimum distance between interferers that grows as η decreases.

We demonstrate both analytically and by means of simulations that the outage probabilities are related to the regularity of the network, *i.e.*, the more regular, the network the higher the probability of success. Furthermore the framework developed permits to determine the optimal MAC choice for different type of networks and outage probabilities.

We adopt the standard signal-to-interference-plus noise (SINR) model for link outages (aka the physical model [7]), where a transmission is successful if the instantaneous SINR exceeds a threshold θ. With Rayleigh fading, the success probability is known to factorize into a term that only depends on the noise and a term that only depends on the interference [1], [8], [9]:

$$\begin{aligned}\mathbb{P}(\mathsf{SINR} > \theta) &= \mathbb{P}(\mathsf{S} > \theta(I + W)) \\ &= \exp(-\theta W/P)\underbrace{\mathbb{E}\exp(-\theta I)}_{\mathsf{P}},\end{aligned}$$

where S is the received signal power, assumed exponential with mean 1 (unit link distance), W is the noise power, P the transmit power, and I the interference (the sum of the powers of all non-desired transmitters). The first term is the noise term, the second one is the Laplace transform of the interference, which does not depend on W or P. Since the first term is a pure point-to-point term, which does not depend on the interference or MAC scheme, it is less interesting, and we will focus on the second term, denoted by P throughput the paper.

II. System Model

The location of the nodes (radios) is modeled as a stationary and isotropic point process Φ of density λ on the plane [10]–[12]. We assume that the time is slotted and that at every time instant, a subset of these nodes Φ_η, selected by the MAC protocol transmit. We constrain the MAC protocols to have the following properties:

[1]To be defined rigorously later.

- At every time instant the transmitting set $\Phi_\eta \subset \Phi$ is a stationary and isotropic point process of density $\lambda_{\rm t}$.
- The MAC protocol has some tuning parameter (for example the probability of transmission in ALOHA) so that the density $\lambda_{\rm t}$ can be tuned from 0 to λ.

We define a (normalized) tuning parameter $\eta \triangleq \lambda_{\rm t}/\lambda$ that denotes the fraction of nodes that transmit. The path-loss model is denoted by $\ell(x):\ \mathbb{R}^2 \setminus \{o\} \to \mathbb{R}^+$ is a continuous, positive, non-increasing function of $\|x\|$ and

$$\int_{\mathbb{R}^2 \setminus B(o,\epsilon)} \ell(x)\mathrm{d}x < \infty, \quad \forall \epsilon > 0\,, \tag{1}$$

where $B(o,r)$ denotes the ball of radius r around the origin o.

In this paper we assume $\ell(x)$ to be a power law in one of the forms:

1) Singular path loss model: $\|x\|^{-\alpha}$.
2) Bounded (non-singular) path loss model: $(1+\|x\|^\alpha)^{-1}$.

$\min\{1,\|x\|^{-\alpha}\}$ is also an example of a non-singular path-loss function. To satisfy the condition (1), we require $\alpha > 2$ in all the above models.

Select a node $y \in \Phi_\eta$ and let it be the receiver of a virtual transmitter z at a distance such that $\ell(y-z)=1$. Including the receiver y as part of the process Φ_η allows to study the success probability at the receiver rather than at the transmitter and accounts for the spacing of the transmitters. The success probability obtained is a good approximation for *transmitter-initiated* MACs if $\|y-z\|$ is short, since the interference power level at the receiver is approximately the same as the one at the transmitter if $\lambda_{\rm t}^{-1/2} \gg 1$. The analysis in the subsequent sections does not change much if the positions of the transmitter and the receiver are interchanged. Furthermore both the transmission power and the link distance are normalized to 1 so as to isolate the effect of η on the success probability. Let S be the received power from the intended transmitter; we assume that S is exponentially distributed with unit mean. Let $\mathtt{I}(y)$ denote the interference at the receiver

$$\mathtt{I}(y) = \sum_{\mathsf{x}\in\Phi_\eta} \mathsf{h}_\mathsf{x}\ell(\|\mathsf{x}-y\|), \tag{2}$$

where h_x is iid exponential fading with unit mean. Without loss of generality, we can assume that the virtual receiver is located at $y=0$ and hence the probability of success is given by

$$\mathsf{P}_\eta \triangleq \mathbb{P}^{!o}\left(\frac{\mathsf{S}}{\mathtt{I}(o)} \geq \theta\right), \quad \theta > 0, \tag{3}$$

where $\mathbb{P}^{!o}$ is the reduced Palm probability of Φ_η. The Palm probability of a point process is equivalent to conditional probability and $\mathbb{P}^{!o}$ denotes the probability conditioned on there being a point of the process at the origin but not counting the point. Since S is exponentially distributed, the success probability is given by

$$\mathsf{P}_\eta = \mathbb{E}^{!o}\exp\left(-\theta\mathtt{I}\right), \tag{4}$$

where for convenience we have used $\mathtt{I}$ to denote $\mathtt{I}(o)$.

III. Outage Probability Scaling at High SIR

A. General result

In this section we show that for a wide range of MAC protocols,

$$\mathsf{P}_\eta \sim 1 - \gamma\eta^\kappa, \quad \eta \to 0. \tag{5}$$

While the spatial contention γ depends on θ, α, and the MAC scheme, the interference scaling exponent κ depends on α, and the MAC, but not on θ. For example when the node set Φ is a Poisson point process of unit density ($\lambda=1$), and ALOHA with parameter $\eta \leq 1$ is used as the MAC, the success probability [1] is

$$\mathsf{P}_\eta = \exp\left(-\eta\int_{\mathbb{R}^2}\frac{1}{1+\theta^{-1}\ell(x)^{-1}}\mathrm{d}x\right).$$

Hence for small η,

$$\mathsf{P}_\eta \sim 1 - \eta\underbrace{\int_{\mathbb{R}^2}\Delta(x)\mathrm{d}x}_{\gamma},$$

where

$$\Delta(x) = \frac{1}{1+\theta^{-1}\ell(x)^{-1}}.$$

Thus $\kappa = 1$ for Poisson distributed nodes with ALOHA as the MAC protocol. The parameter κ indicates the gain in performance of the network to a decrease in the density of transmitters. For $\kappa > 1$, the network can accommodate a certain density of interferers without affecting the outage, while for $\kappa = 1$, when increasing the density from 0 to $\mathrm{d}\eta$, the success probability decreases by $\gamma\mathrm{d}\eta$.

We begin by proving that the exponent κ cannot take arbitrary values. Let $\mathcal{K}_\eta(B)$, $B\subset\mathbb{R}^2$, denote the second-order reduced moment measure, defined as

$$\mathcal{K}_\eta(B) \triangleq (\eta\lambda)^{-1}\mathbb{E}^{!o}\sum_{\mathsf{x}\in\Phi_\eta}\mathbf{1}(\mathsf{x}\in B).$$

Alternatively, $\mathcal{K}_\eta$ can be expressed as

$$\mathcal{K}_\eta(B) = (\eta\lambda)^{-2}\int_B \rho^{(2)}(x)\mathrm{d}x,$$

where $\rho^{(2)}(x)$ is the second-order product density of the point process [10], [11]. For motion-invariant processes $\rho^{(2)}(x)$ is a function of only $\|x\|$, so we may use $\rho^{(2)}(r)$ instead. Intuitively $\rho^{(2)}(r)\mathrm{d}x\mathrm{d}y$ represents the probability of finding two points of the process located at x and y with $\|x-y\|=r$. The second-order measure $\mathcal{K}_\eta(B)$ is a positive and positive-definite (PPD) measure [11]. As a property of a PPD measure, we have

$$\mathcal{K}_\eta(B+x) < C_B(\eta), \quad \forall x\in\mathbb{R}^2,$$

whenever $\mathcal{K}_\eta(B)<\infty$, and where $C_B(\eta)<\infty$ is a constant that does not depend on x. Ripley's K-function, defined as $K_\eta(R) = \mathcal{K}_\eta(B(o,R))$, is the average number of points in a ball of radius R centered at the origin, normalized by the intensity of the point process, conditioned on there being a point at the origin but not counting it. For any stationary point process $K(R)\to\pi R^2$ as $R\to\infty$ [10].

Theorem 1 (Bounds on the interference scaling exponent κ)**.** *Any MAC protocol which results in a motion-invariant transmitter set of density $\lambda\eta$ such that*[2]*,*

$$\text{A.1} \qquad \lim_{\eta\to 0} \sup_{x\in\mathbb{R}^2} \mathcal{K}_\eta(S_1+x) < \infty, \quad \text{where } S_1=[0,1]^2,$$

has the interference scaling exponent

$$1 \leq \kappa.$$

If the MAC protocol is such that there exists a $R>0$ such that

$$\text{B.1} \qquad \lim_{\eta\to 0} \eta K_\eta(R\eta^{-1/2}) > 0\,,$$

then

$$\kappa \leq \alpha/2.$$

Proof: Part 1: We first prove that $\kappa \geq 1$. We will show that $\forall\epsilon>0$,

$$\lim_{\eta\to 0}\frac{1-\mathsf{P}_\eta}{\eta^{1-\epsilon}} = 0\,,$$

which implies the result. From (4) we have

$$\begin{aligned} \mathsf{P}_\eta &= \mathbb{E}^{!o}\exp\left(-\theta\sum_{\mathsf{x}\in\Phi_\eta}\mathsf{h}_\mathsf{x}\ell(\mathsf{x})\right) && (6)\\ &\overset{(a)}{=} \mathbb{E}^{!o}\left[\prod_{\mathsf{x}\in\Phi_\eta}\frac{1}{1+\theta\ell(\mathsf{x})}\right] && (7)\\ &= \mathbb{E}^{!o}\left[\prod_{\mathsf{x}\in\Phi_\eta}1-\Delta(\mathsf{x})\right], && (8)\end{aligned}$$

where (a) follows from the independence and exponential distribution of the h_x. Using the inequality

$$\prod(1-y_i)\geq 1-\sum y_i, \quad y_i<1,$$

we obtain

$$\mathsf{P}_\eta \;\geq\; 1-\mathbb{E}^{!o}\sum_{\mathsf{x}\in\Phi_\eta}\Delta(\mathsf{x}). \qquad (9)$$

Hence

$$\begin{aligned}\frac{1-\mathsf{P}_\eta}{\eta^{1-\epsilon}} &\leq \eta^{\epsilon-1}\mathbb{E}^{!o}\sum_{\mathsf{x}\in\Phi_\eta}\Delta(\mathsf{x}) && (10)\\ &= \eta^{\epsilon}\int_{\mathbb{R}^2}\eta^{-2}\rho^{(2)}_\eta(x)\Delta(x)\mathrm{d}x, && (11)\end{aligned}$$

where $\rho^{(2)}_\eta(x)$ is the second-order product density of Φ_η. We have

$$\int_{\mathbb{R}^2}\eta^{-2}\rho^{(2)}_\eta(x)\Delta(x)\mathrm{d}x \;=\; \eta^{-2}\sum_{(k,j)\in\mathbb{Z}^2}\int_{S_{kj}}\rho^{(2)}_\eta(x)\Delta(x)\mathrm{d}x,$$

where $S_{kj}=[k,\ k+1]\times[j,\ j+1]$. Let $\Delta_{kj}\triangleq\Delta(\inf\{\|x\|, x\in S_{kj}\})$. Since $\Delta(x)$ is a decreasing function of $\|x\|$, we have

$$\begin{aligned}\int_{\mathbb{R}^2}\eta^{-2}\rho^{(2)}_\eta(x)\Delta(x)\mathrm{d}x &< \eta^{-2}\sum_{(k,j)\in\mathbb{Z}^2}\Delta_{kj}\int_{S_{kj}}\rho^{(2)}_\eta(x)\mathrm{d}x && (12)\\ &= \sum_{(k,j)\in\mathbb{Z}^2}\Delta_{kj}\mathcal{K}_\eta(S_{kj}) && (13)\\ &\overset{(a)}{<} C_{S_1}\sum_{(k,j)\in\mathbb{Z}^2}\Delta_{kj} && (14)\\ &\overset{(b)}{<} \infty, && (15)\end{aligned}$$

where (a) follows from the transitive boundedness property of a PPD measure and (b) follows since for $\alpha>2$, $\sum_{(k,j)\in\mathbb{Z}^2}\Delta_{kj}<\infty$ and Condition A.1. Hence it follows from (9) that

$$\lim_{\eta\to 0}\frac{1-\mathsf{P}_\eta}{\eta^{1-\epsilon}} = 0.$$

Part 2: Next we prove that $\kappa\leq\alpha/2$. We will show that $\forall\epsilon>0$,

$$\lim_{\eta\to 0}\frac{1-\mathsf{P}_\eta}{\eta^{\alpha/2+\epsilon}} = \infty\,, \qquad (16)$$

which implies the result. The success probability is

$$\begin{aligned}\mathsf{P}_\eta &= \mathbb{E}^{!o}\left[\prod_{\mathsf{x}\in\Phi_\eta}\frac{1}{1+\theta\ell(\mathsf{x})}\right]\\ &\leq \mathbb{E}^{!o}\left[\prod_{\mathsf{x}\in\Phi_\eta\cap B(o,R\eta^{-1/2})}\frac{1}{1+\theta\ell(\mathsf{x})}\right]\\ &\leq \mathbb{E}^{!o}\left[\frac{1}{1+\theta\ell(R\eta^{-1/2})}\right]^{\Phi_\eta(B(o,R\eta^{-1/2}))}\end{aligned}$$

As $\eta\to 0$, $\ell(R\eta^{-1/2})\sim R^{-\alpha}\eta^{\alpha/2}$, and using the identity $(1+x)^{-k}\sim 1-kx$ for small x we obtain

$$\begin{aligned}\lim_{\eta\to 0}\frac{1-\mathsf{P}_\eta}{\eta^{\alpha/2}} &\geq \lim_{\eta\to 0}\mathbb{E}^{!o}[\Phi_\eta(B(o,R\eta^{-1/2}))]\theta R^{-\alpha}\\ &= \lim_{\eta\to 0}\eta K_\eta(R\eta^{-1/2})\theta R^{-\alpha}\\ &\overset{(a)}{\geq} C>0,\end{aligned}$$

where (a) follows from Condition B.1. Hence (16) follows. ■

Discussion of the conditions:

1) Let

$$C_\eta = \sup_{x\in\mathbb{R}^2}\mathcal{K}_\eta(x+S_1)\,.$$

Then from a similar argument as in the proof, it is easy to observe that

$$\mathbb{E}^{!o}[\Phi(B(o,R))] = \lambda\eta\mathcal{K}(B(o,R)) < \lambda\eta\lceil\pi R^2\rceil C_\eta.$$

Condition A.1 can be expressed as $\lim_{\eta\to 0}C_\eta<\infty$, which implies that

$$\mathbb{E}^{!o}[\Phi(B(o,\eta^{-a})]\to 0 \qquad \text{for } a<1/2\,.$$

[2]See the discussion after the proof.

So Condition A.1 implies that the average number of points in a ball of radius $R_\eta = \eta^{-a}, a < 1/2$, is zero as the density tends to zero. This condition is violated when the average nearest-interferer distance remains constant with decreasing density η. For example, consider a clustered point process with cluster density η and each cluster having a fixed number of points on average. In this case, Condition A.1 is violated as $\eta \to 0$.

2) $\lambda\eta K_\eta(R\eta^{-1/2})$ is equal to the average number of points in a ball of radius $R\eta^{-1/2}$ and hence condition B.1 requires the number of points inside a ball of radius $R\eta^{-1/2}$ to be greater than zero. By the sphere-packing argument, in any stationary point process of density λ, the probability that the nearest neighbor is within a distance $\sqrt{2/\sqrt{3}}\lambda^{-1/2}$ is greater than zero. In other words, the probability of the event that all the nearest neighbors are further away than $1.075\eta^{-1/2}$ is zero. Hence $\eta K_\eta(R\eta^{-1/2}) > 0$, where $R = \sqrt{\frac{2}{\sqrt{3}}}$. But this does not strictly satisfy Condition B.1 which requires the limit to be greater than zero. Except for pathological cases, this condition is generally valid since the nearest-neighbor distance scales like $\Theta(\eta^{-1/2})$ when the point process is of density $\eta\lambda$. So, while Condition A.1 requires the nearest interferer distance to increase with decreasing η, Condition B.1 requires an interferer to be present at a distance $\Theta(\eta^{-1/2})$. Essentially any MAC which schedules the nearest interferer only at an average distance that scales with $\eta^{-1/2}$ satisfies these two conditions, and in this case, $1 \le \kappa \le \alpha/2$.

3) Indeed, if Condition A.1 is violated,

$$\lim_{\eta\to 0} \eta K_\eta(R) > 0 \quad \text{for some } R > 0,$$

and it follows that

$$\lim_{\eta\to 0} \mathsf{P}_\eta < 1.$$

Based on this discussion, we can define the class of *reasonable* MAC schemes:

Definition 1. *A* reasonable *MAC scheme is a MAC scheme for which Conditions A.1 and B.1 hold.*

Theorem 1 implies that for all reasonable MAC schemes, $1 \le \kappa \le \alpha/2$. A MAC scheme for which $\lim_{\eta\to 0} \mathsf{P}_\eta < 1$ would clearly be unreasonable—it would defeat the purpose of achieving high reliability as the density of interferers is decreased.

B. Achieving the boundary points $\kappa = 1$ and $\kappa = \alpha/2$

In this section, we provide exact conditions on the MAC protocols which achieve the boundary points $\kappa = 1$ and $\kappa = \alpha/2$. ALOHA is a simple MAC protocol, and its fully distributed nature makes it very appealing. As shown before, a Poisson distribution of transmitters with ALOHA as the MAC protocol results in $\kappa = 1$. In a stationary point process of density λ, the average nearest-neighbor distance scales like $1/\sqrt{\lambda}$. Using ALOHA with parameter η results in a thinning of Φ, the resultant process has an average nearest-neighbor distance of $1/\sqrt{\lambda\eta}$. Independent thinning of a point process does not guarantee that there is no receiver within a distance $R = c/\sqrt{\lambda\eta}$, $c < 1$ as η goes to zero. If suppose there are n points originally in the ball $B(o, R)$, the probability that none of the points are selected by ALOHA is $(1-\eta)^n$. So although ALOHA with parameter η would guarantee an average nearest-neighbor distance $\eta^{-1/2}$, there is a finite probability $1-(1-\eta)^n$ that the ball $B(o, R)$ is not empty. So ALOHA leads to a *soft* minimum distance proportional to $\eta^{-1/2}$, and as we state in the following theorem, results in $\kappa = 1$ for any distribution of nodes which uses ALOHA as MAC. The theorem is presented without proof.

Theorem 2 (ALOHA)**.** *When ALOHA is used as the MAC protocol with transmit probability η and*

$$\int_{\mathbb{R}^2}\int_{\mathbb{R}^2} \rho^{(3)}(x,y)\Delta(x)\Delta(y)\mathrm{d}x\mathrm{d}y < \infty, \tag{17}$$

the outage probability satisfies

$$\mathsf{P}_\eta \sim 1 - \gamma\eta, \quad \eta \to 0, \tag{18}$$

where

$$\gamma = \frac{1}{\lambda}\int_{\mathbb{R}^2} \rho^{(2)}(x)\Delta(x)\mathrm{d}x.$$

$\rho^{(2)}(x)$ and $\rho^{(3)}(x,y)$ denote the second- and third-order product densities [10], [11], [13] of the point process Φ, respectively.

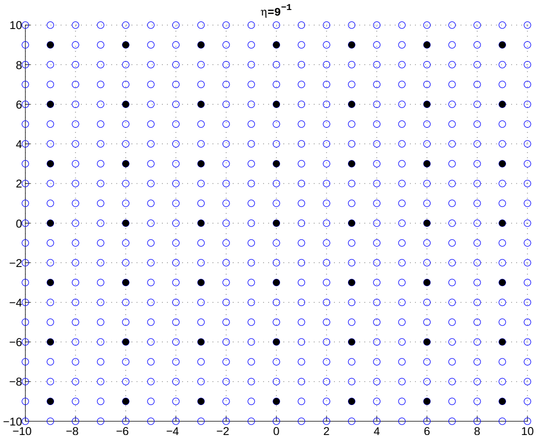
In this theorem, we characterize the scaling law with ALOHA as the MAC protocol in which only the average distance to the nearest interferer scales like $\eta^{-1/2}$. We now consider the MAC protocols in which the nearest interferer distance scales like $\eta^{-1/2}$ a.s. For example, a TDMA scheme in which the distance between the nearest transmitters scale like $\eta^{-1/2}$ falls into this category. In Fig. 1, TDMA on a lattice network is illustrated. An alternative version of TDMA is illustrated in Fig. 2. From Fig. 1, we also observe that $\rho^{(2)}_\eta(x/\sqrt{\eta}) = 0$ for $x < 1$, while this is not the case in the modified TDMA scheme in Fig. 2. More precisely, it is easy to observe that the *minimum distance criterion*

$$\lim_{\eta\to 0}\int_0^\infty \eta^{-2}\rho^{(2)}_\eta(r\eta^{-1/2})r^{1-\alpha}\mathrm{d}r < \infty \tag{19}$$

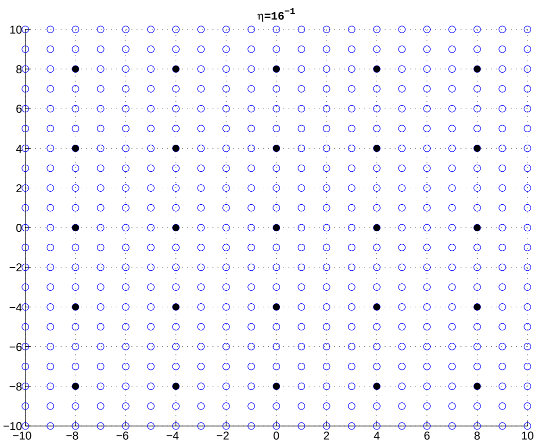
holds in the TDMA scheme illustrated in Fig. 1 but not in the alternative *unreasonable* TDMA version in Fig. 2. We require the multiplying factor η^{-2} in front of the second-order product density since $\rho^{(2)}_\eta(x)$ scales like η^2. In the first TDMA we also observe that the resulting transmitter process is self-similar if the spatial axes are scaled by $\eta^{-1/2}$. It can be proven that for all MAC schemes that satisfy (19) and a few more constraints, $\kappa = \alpha/2$.

In the reasonable TDMA case shown in Fig. 1, tight bounds on the success probability can be derived following the procedure in [6]:

$$e^{-\gamma_{\mathrm{TDMA}}/m^\alpha} \lessapprox \mathsf{P}_\eta \lessapprox \frac{1}{1+\gamma_{\mathrm{TDMA}}/m^\alpha}, \tag{20}$$

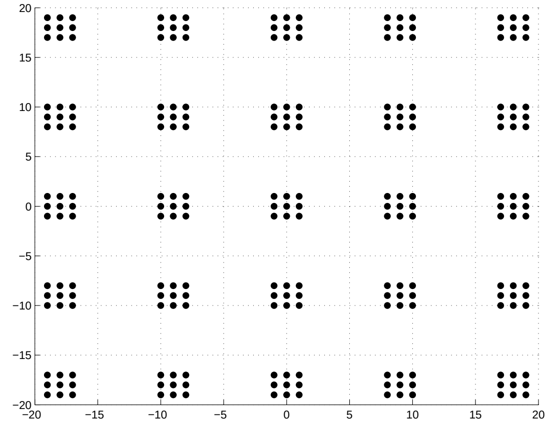


(a) $\eta = 1/9$

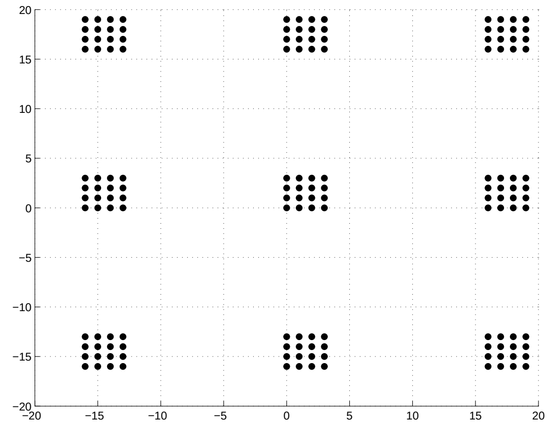


(b) $\eta = 1/16$

Fig. 1. *Reasonable* TDMA on a two-dimensional lattice $\mathbb{Z}^2$ for $\eta = 1/9$ and $\eta = 1/16$. In this arrangement, the nearest interferer is at distance $\eta^{-1/2}$.



(a) $\eta = 1/9$



(b) $\eta = 1/16$

Fig. 2. *Unreasonable* TDMA on a two-dimensional lattice $\mathbb{Z}^2$ for $\eta = 1/9$ and $\eta = 1/16$. In this case, the nearest interferer is at distance 1 — irrespective of η.

where $m = \eta^{-1/2}$ is the distance between nearest transmitters, $\gamma_{\mathrm{TDMA}} = 4\zeta(\alpha/2)\beta(\alpha/2)\theta$ with ζ the Riemann zeta and β the Dirichlet beta functions. If follows from (20) that

$$\mathsf{P} \sim 1 - \gamma_{\mathrm{TDMA}}\eta^{\alpha/2} \,,$$

as expected. The results in Fig. 3 show the simulation result together with the two bounds. The plot confirms that the slope at $\eta = 0$ is indeed 0. In this case, since $\alpha = 4$, the success probability is quadratically decreasing near $\eta = 0$.

IV. Conclusions

In this paper we provide asymptotics of the outage probability in the high SIR regime for essentially all MAC protocols. The asymptotic results are of the form $\mathsf{P}_\eta \sim 1 - \gamma\eta^\kappa$, $\eta \to 0$, where η is the fraction of nodes selected to transmit by the MAC. The two parameters κ and γ are related to the network and to the MAC: γ the intrinsic *spatial contention* of the network and κ the *interference scaling exponent* that quantifies *coordination level* achieved by the MAC. We studied P_η under the signal-to-interference ratio (SIR) model, Rayleigh fading and power law path loss, explaining how the parameters

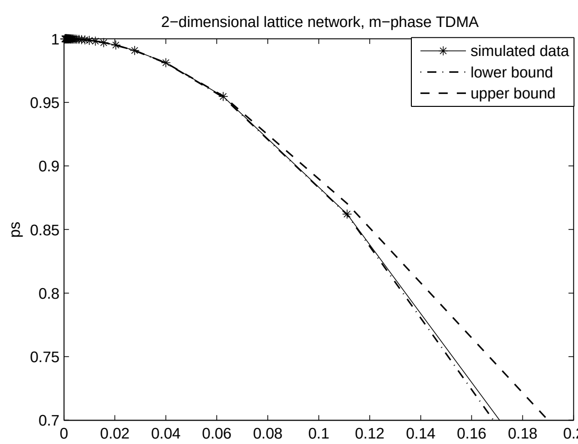


Fig. 3. Success probability as a function of the transmitter density η for *reasonable* TDMA on a two-dimensional lattice for $\alpha = 4$. Simulation results and the bounds are shown.

depends on models parameters in use. We prove that any *reasonable* MAC protocol results in $\kappa \in [1, \alpha/2]$, where α is the path-loss exponent. For ALOHA, we show that $\mathsf{P}_\eta \sim 1 - \gamma\eta$, *i.e.*, $\kappa = 1$, for all motion-invariant networks. If the MAC protocol is such that the nearest interferer distance scales like $\eta^{-1/2}$, the exponent $\kappa = \alpha/2$, a value achieved by using TDMA or CSMA.

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