Stochastic Geometry and Random Graphs for the Analysis and Design of Wireless Networks

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(Invited Paper)

***Abstract*—Wireless networks are fundamentally limited by the intensity of the received signals and by their interference. Since both of these quantities depend on the spatial location of the nodes, mathematical techniques have been developed in the last decade to provide communication-theoretic results accounting for the network's geometrical configuration. Often, the location of the nodes in the network can be modeled as random, following for example a Poisson point process. In this case, different techniques based on *stochastic geometry* and the *theory of random geometric graphs* – including point process theory, percolation theory, and probabilistic combinatorics – have led to results on the connectivity, the capacity, the outage probability, and other fundamental limits of wireless networks. This tutorial article surveys some of these techniques, discusses their application to model wireless networks, and presents some of the main results that have appeared in the literature. It also serves as an introduction to the field for the other papers in this special issue.**

***Index Terms*—Tutorial, wireless networks, stochastic geometry, random geometric graphs, interference, percolation**

I. Introduction

Emerging classes of large wireless systems such as ad hoc and sensor networks and cellular networks with multihop coverage extensions have been the subject of intense investigation over the last decade. Classical methods of communication theory are generally insufficient to analyze these new types of networks for the following reasons: (i) The performance-limiting metric is the signal-to-interference-plus-noise-ratio (SINR) rather than the signal-to-noise-ratio (SNR). (ii) The interference is a function of the *network geometry* on which the path loss and the fading characteristics are dependent upon. (iii) The amount of uncertainty present in large wireless networks far exceeds the one present in point-to-point systems: it is impossible for each node to know or predict the locations and channels of all but perhaps a few other nodes.

Two main tools have recently proved most helpful in circumventing the above difficulties: stochastic geometry [1] and random geometric graphs [2], [3]. Stochastic geometry allows to study the average behavior over many spatial realizations of a network whose nodes are placed according to some probability distribution. Random geometric graphs capture the distance-dependence and randomness in the connectivity of the nodes. This paper provides an introduction to these mathematical tools and discusses some recent results that were enabled by them, with the objective of being a first-hand tutorial for the researcher interested in the field.

A. The role of the geometry and the interference

Since Shannon's work [4], for the second half of the 20th century the SNR has been the main quantity of interest to communication engineers that determined the reliability and the maximum throughput that could be achieved in a communication system. In a wireless system, the SNR can vary among different users by as much as ten to hundreds of dBs due to differences in path loss, shadowing due to buildings and obstructions, fading due to constructive or destructive wave interference. The first-order contributor to this SNR variation is the path loss. In a cellular system for example, it is not uncommon for a user close to the base station to have a channel that is a million times (60 dB) stronger than the one of a user located near the cell's edge. If the receivers are randomly located in space, then the different SNRs due to the different path losses between them and the base station can be modeled as having a *spatial* distribution.

In a wireless network with many concurrent transmissions, the situation is even more complicated. In this case, the SINR becomes the relevant figure of merit for the system[1]. Not only the received signal power is random due to the random spatial distribution of the users, but also the interference power is governed by a number of stochastic processes including the random spatial distribution of the nodes, shadowing, and fading. The SINR for a receiver placed at the origin o in the 2 or 3-dimensional Euclidean space can be written as:

$$\mathsf{SINR} = \frac{S}{W+I}, \quad \text{where} \quad I = \sum_{i\in\mathcal{T}} P_i h_i \ell(\|x_i\|), \qquad (1)$$

where S, W, and I are the desired signal, noise, and interference powers, respectively. The summation for I is taken over the set of all interfering transmitters $\mathcal{T}$, P_i is the transmit power, h_i is a random variable that characterizes the cumulative effect of shadowing and fading, and ℓ is the path loss function, assumed to depend only on the distance $\|x_i\|$ from the origin of the interferer situated at position x_i in space. Often ℓ is modeled as a power law, $\ell(\|x_i\|) = k_0\|x_i\|^{-\alpha}$, or in environments where absorption is dominant, as an exponential law, $\ell(\|x_i\|) = k_0 \exp(-\gamma\|x_i\|)$, see [5], [6]. In a large system, the unknowns are $\mathcal{T}$, h_i, and x_i, and perhaps P_i, but it is the locations of the interfering nodes that most influence the SINR levels, and hence, the performance of the network. The

[1]In many practical situations such as cellular networks and reasonably dense ad hoc networks, the noise is typically negligible and SIR can be used interchangeably with SINR without any appreciable loss of accuracy.

interference is a function of the underlying node distribution (and mobility pattern for mobile networks) and the channel access scheme.

A key objective of this tutorial and special issue is to show that the SINR in (1) can be characterized using stochastic techniques, and key related metrics such as outage probability, coverage, connectivity, and capacity can therefore be quantified.

B. Historical background

Although there has been growing recent interest in the use of stochastic geometry and random graph theory to describe wireless networks, some of the underlying approaches go back 100 years, or more. Wireless network performance is mostly interference-limited, and a large number of users contribute to the interference in vastly varying magnitudes, as described by the interference function stated in (1), which is a *shot noise process*. In one dimension, a shot noise process is defined as

$$I(x) = \sum_{i=-\infty}^{\infty} g(x - X_i), \tag{2}$$

where $\{X_i\}$ is a stationary Poisson process on $\mathbb{R}$ and $g(x)$ is the impulse response of a linear system. In its two-dimensional generalization, x represents a point on the plane and the Poisson point process is on $\mathbb{R}^2$. When $g(x)$ depends only on the Euclidean norm $\|x\|$, it can be identified with the path loss function $\ell(\|x\|)$, and $I(x)$ is the aggregate interference received at point x in a wireless network, without fading. The shot noise process has been studied at least dating back to Campbell in 1909 [7], who characterized its mean and variance, followed by Schottky in 1918 [8]. In addition, Rice performed extensive investigations on the distribution of $I(x)$ from 1940 through 1970 [9]. Power law shot noise – most relevant here – was considered by Lowen and Teich in 1990 [10].

Stochastic geometry has been used as a tool for characterizing interference in wireless networks at least as early as 1978 [11], and was further advanced by Sousa and Silvester in the early 1990s [12]–[14]. At that time, two useful mathematics texts were available to researchers: Stoyan et al.'s stochastic geometry first edition in 1987 (see [1] for the second edition) and Kingman's Poisson Processes [15]. In the late 1990's, Ilow and Hatzinakos [16] characterized the impact of random channel effects – fading, shadowing, path loss, and combinations thereof – on the aggregate co-channel interference, while Baccelli et al. also began developing tools primarily based on Poisson Voronoi tesselations and Delaunay triangulations for the optimization of hierarchical networks [17], [18] and of mobility management in cellular networks [19]. For a survey on this line of research, see the forthcoming book chapter by Zuyev [20].

In the past ten years, stochastic geometry and associated techniques have been applied and adapted to cellular systems [21]–[24], ultrawideband [25], cognitive radio [26]–[28], femtocells [29], [30], relay networks [31], and many other types of wireless systems. However, perhaps the largest impact has been in the area of ad hoc networks, which are fully distributed and in which all participating nodes – both transmitters and receivers – are randomly located. In such networks, it is impossible even with unlimited overhead to control the SINRs of all users, due to the coupling of interference: if one user raises its power, it causes an interference increase to all other communicating pairs. In this case, characterizing the (stochastic) geometry of the network is of utmost importance since it is the first-order determinant of the SINR.

The idea of modeling wireless networks using random graphs dates back to Gilbert [32], whose paper marks the starting point for continuum percolation theory. He considered a random network formed by connecting points of a Poisson point process that are sufficiently close to each other. Using this model, he proved the existence of a critical connection distance above which an infinite chain of connected nodes forms and below which, in contrast, any connected component is bounded. His investigations were based on prior work on the discrete percolation model of Broadbent and Hammersley [33] and on the theory of branching processes [34]. Gilbert's original model has undergone many generalizations, and continuum percolation theory is now a rich mathematical field, see [2], [35]. Extensions most relevant to us consider graphs which account for interference-limited communication. In this case, the graph connectivity depends on the SINR at different nodes. These are studied in [36] and [37]. Another relevant generalization is the random connection model [38], which is a random graph that can account for random connections due to shadow fading effects; as well as nearest neighbor network models [39]. Finally, a coverage model for wireless networks based on percolation theory was introduced in [40]. A compendium of random graph results related to wireless communications appears in [3]. Elements of random graphs have also proven to be useful to characterize the scaling behavior of the capacity of wireless networks [41].

C. Paper overview and organization

In §II, we provide an overview of the key mathematical tools, including point processes, random geometric graphs, and percolation. In §III, the interference is characterized, and it is shown that the outage probability is a natural metric to study in a spatially random wireless network. We then move to other performance metrics, such as the connectivity of the network and its coverage in §IV, and the data-carrying capacity and area spectral efficiency of the network in §V. We conclude by providing some additional applications of these results, including epidemic models, wireless security, and transmission protocol design. The notation and symbols used in the paper are listed in Table I.

II. Mathematical Preliminaries

Stochastic geometry [1] is a rich branch of applied probability which allows the study of random phenomena on the plane or in higher dimensions. It is intrinsically related to the theory of point processes [42]. Initially its development was stimulated by applications in biology, astronomy and material sciences. Nowadays, it is also used in image analysis and in the context of communication networks.

Symbol	Definition/explanation
$[k]$	the set $\{1, 2, \ldots, k\}$
$\mathbb{Z}, \mathbb{N}$	integers, positive integers
$\mathbb{R}, \mathbb{R}^+$	real numbers, positive real numbers
$\mathbf{1}_A(x)$	indicator function
$u(x)$	$\triangleq \mathbf{1}_{\{x \geqslant 0\}}(x)$ (unit step function)
d	number of dimensions of the network
$\#A$	cardinality of A
$\mathbb{P}(A)$	probability of event A
$\mathbb{E}(X)$	expectation of random variable X
$\mathcal{L}_X(s) = \mathbb{E}(e^{-sX})$	Laplace transform of random variable X
$\lvert \cdot \rvert$	Lebesgue measure
$\lVert \cdot \rVert$	Euclidean norm
o	origin in $\mathbb{R}^d$
B	a Borel subset of $\mathbb{R}$ or $\mathbb{R}^d$
$B(x; r)$	ball of radius r centered at x
c_d	$\triangleq \pi^{d/2}/\Gamma(1 + d/2)$ (volume of the d-dim. unit ball)
α	path loss exponent
$F_X(x) = \mathbb{P}(X \leqslant x)$	distribution of random variable X (cdf)
$\Phi = \{X_i\} \subset \mathbb{R}^d$	point process
Λ, λ	counting measure and density for Φ
p_c, λ_c	critical probability, density for percolation
h	(power) fading random variable ($\mathbb{E}(h) = 1$)
$T \in \mathbb{R}^+$	SIR or SINR threshold for successful communication
$\epsilon \in (0, 1)$	target outage probability for transmission capacity
$\ell : \mathbb{R} \to \mathbb{R}^+$	(isotropic) large-scale path loss function
W	thermal noise

TABLE I
NOTATION AND SYMBOLS USED IN THE PAPER.

A. Point processes

The most basic objects studied in stochastic geometry are point processes. Visually, a point process can be depicted as a random collection of points in space. More formally, a point process (PP) is a measurable mapping Φ from some probability space to the space of point measures (a point measure is a measure which is locally finite and which takes only integer values) on some space E. Each such measure can be represented as a discrete sum of Dirac measures on E:

$$\Phi = \sum_i \delta_{X_i}.$$

The random variables $\{X_i\}$, which take their values in E are the points of Φ. Most often, the space E is the Euclidean space $\mathbb{R}^d$ of dimension $d \geq 1$. The *intensity measure* Λ of Φ is defined as $\Lambda(B) = \mathbb{E}\Phi(B)$ for Borel B, where $\Phi(B)$ denotes the number of points in $\Phi \cap B$.

A few dichotomies concerning point processes on Euclidean space $\mathbb{R}^d$ are as follows:

- A PP can be simple or not. It is simple if the multiplicity of a point is at most one (no two points are at the same location).
- A PP can be *stationary* or not. Stationarity holds if the law of the point process is invariant by translation.
- A PP can be Poisson or not. A formal definition of the Poisson point process (PPP) is given in the following subsection. A PPP offers a handy computational framework for different network quantities of interest.
  - A PPP can be homogeneous or not. In the homogeneous case, the density of the points is constant across space.
  - The homogeneous PPP is stationary and simple. This may be considered as the simplest (and most natural) point process.
  - The framework for non-homogeneous PPPs is also well developed, although more technical than that of the homogeneous case. They can be used to model distributions of users which are not uniform across space.
  - There is also a comprehensive computational framework for stationary point processes which are not Poisson. This is Palm calculus (see below).
- A point process can be *isotropic* or not. Isotropy holds if the law of the PP is invariant to rotation. The homogeneous PPP is isotropic. If a PP is isotropic and stationary, it is called *motion-invariant*.
- A PP can be marked or not; marks assign labels to the points of the process, and they are typically independent of the PP and i.i.d. The study of marked point processes may require the handling of Palm calculus.

1) Poisson point processes: Let Λ be a locally finite measure on some metric space E. A point processes Φ is Poisson on E if

- For all disjoint subsets $A_1, \cdots, A_n$ of E, the random variables $\Phi(A_i)$ are independent;
- For all sets A of E, the random variables $\Phi(A)$ are Poisson.

A key property states that conditionally on the fact that $\Phi(A) = n$, these n points are independently (and uniformly for homogeneous PPP) located in A. This leads to an explicit representation of the Laplace functional of Φ. If $E = \mathbb{R}^d$, this Laplace functional is defined for general point processes Φ by

$$\mathcal{L}_\Phi(f) \triangleq \mathbb{E}\left[e^{-\int_{\mathbb{R}^d} f(x)\,\Phi(\mathrm{d}x)}\right] = \mathbb{E}\left[e^{-\sum_{X\in\Phi} f(X)}\right],$$

where f is a non-negative function on $\mathbb{R}^d$. In the Poisson case,

$$\mathcal{L}_\Phi(f) = \exp\left(-\int_{\mathbb{R}^d}\left(1 - e^{-f(x)}\right)\,\Lambda(\mathrm{d}x)\right). \qquad (3)$$

This is the basis for a large number of formulas like, *e.g.*, those on the Laplace transform of the shot noise (or interference), see §III. Other appealing features of PPPs are their invariance to a large number of key operations. In particular,

- The superposition of two or more independent PPPs (which is defined as the sum of the associated point measures) is again a PPP; this can be extended to denumerable sums under some conditions.
- The independent thinning of a PPP is again a PPP; this can be extended to the case of location-dependent thinning, where a point is retained or not depending on its location.
- The point process obtained by displacing point X_i independently of everything else according to some Markov kernel $K(X_i, \cdot)$ that defines the distribution of the displaced position of the point X_i yields another PPP; this result is often referred to as the displacement theorem.

In each case, the intensity of the resulting PPP can be obtained in closed form from that of the initial PPP and the involved transformations (*e.g.*, the thinning probability or the kernel K).

If $\rho(x,\cdot)$ is the probability density pertaining to the Markov kernel applied to a PPP of intensity $\lambda(x)$ on $\mathbb{R}^d$, the displaced points form a PPP of intensity

$$\lambda'(y) = \int_{\mathbb{R}^d} \lambda(x)\rho(x,y)\mathrm{d}x\,.$$

In particular, if $\lambda(x)$ is constant λ and $\rho(x,y)$ is a function of $y-x$ only, then $\lambda'(y)=\lambda$ for all y.

A striking property of PPPs is Slivnyak's theorem which states that the law of $\Phi-\delta_x$ conditional on the fact that Φ has a point at x is the same as the law of Φ. In mathematical terms, the reduced Palm probability $P^{x!}$ of a PPP is the distribution of this Poisson point process itself. This is usually expressed as $P^{x!}=P$, for all points $x\in\Phi$. This means that the properties seen from a point $x\in\mathbb{R}^d$ are the same whether we condition on having a point $x\in\Phi$ or not — if the point at x is not considered. For example, we have for the mean number of points within distance r of x:

$$\begin{aligned}\mathbb{E}\Phi(B(x;r)\setminus\{x\}) &= \mathbb{E}\Big(\Phi(B(x;r)\setminus\{x\}) \mid x\in\Phi\Big)\\ &= \Lambda(B(x;r))\,,\end{aligned}$$

where $B(x;r)$ is the ball of radius r centered at x.

2) Stationary point processes: The theory of stationary point processes is based on the concept of marks and on the Matthes definition of Palm probability [42], [43].

Roughly speaking, a mark of some point of a stationary point process is a quantity that "follows this point" when the collection of points is transported by a global translation operation. For instance, the local configuration of neighbors of point X, which is defined as the collection of points in a ball of radius R centered at X, is a mark of this point. If R is infinity, this mark is the universal mark of X, namely 'the point process seen from X'.

The Palm probability P^o of a stationary point process is the law of this universal mark, which can be shown to be the same for all points. It can be understood as the law of the point process given that it has a point at the origin. As defined here P^o is a probability on the space of point measures.

Campbell's formula for stationary point processes, which is a direct consequence of the last definition, states that when denoting by $\Phi-x$ the global translation of all points of Φ by the vector x, then

$$\mathbb{E}\left[\sum_{X\in\Phi} f(X,\Phi-X)\right] = \int_{\mathcal{N}}\int_{\mathbb{R}^d} f(x,\phi)\lambda\mathrm{d}x P^o(\mathrm{d}\phi), \quad (4)$$

where $\mathcal{N}$ is the space of simple point sequences, for all positive functions $f(x,\phi)$ of $x\in\mathbb{R}^d$ and ϕ a point measure on $\mathbb{R}^d$. Here λ is the intensity of Φ, that is the mean number of points per unit space. The last formula is the key tool for computing the mean values of sums on the points of a stationary point process. Applied to PPPs, they are particularly simple. Let Φ be a stationary PPP of intensity λ on $\mathbb{R}^d$. Then for non-negative f,

$$\mathbb{E}\left(\sum_{X\in\Phi} f(X)\right) = \lambda\int_{\mathbb{R}^d} f(x)\mathrm{d}x \quad (5)$$

and

$$\mathrm{var}\left(\sum_{X\in\Phi} f(X)\right) = \lambda\int_{\mathbb{R}^d} f^2(x)\mathrm{d}x\,. \quad (6)$$

These expressions can be used, for example, to calculate the mean and variance of the interference in a network or to determine the mean node degree.

Important examples of stationary point processes that lead to nice computational results include

- point processes with repulsion, *e.g.*, Matérn hard core point processes or determinantal point processes;
- point processes with attraction, *e.g.*, Neyman-Scott cluster processes, permanental point processes [44] or Hawkes point processes [45].

For more on this, see [1] and [42].

B. Boolean models and random geometric graphs

1) The germ–grain model: The most celebrated model of stochastic geometry and the basic model of continuum percolation is the Boolean or germ-grain model. In the simplest setting, the Boolean model is based on a Poisson point process Φ, the points of which, $\{X_i\}$, are also called *germs*, and on an independent sequence of i.i.d. compact sets $\{K_i\}$ called the *grains*. Formally, the Boolean model is

$$\Xi = \bigcup_i (X_i + K_i)$$

where $X_i+K_i=\{X_i+y,\ y\in K_i\}$.

The Poisson set of germs and the independence of the grains make the Boolean model analytically tractable. It is often considered as the null hypothesis in stochastic geometry modeling.

Among the key results on this model, let us quote (from [1]):

- Coverage: the simplest coverage question is the distribution of the number of grains that intersect a given compact set, for instance a given location of the space. This distribution is Poisson.
- Volume fraction: in the homogeneous case, what is the fraction of a big ball which is covered by grains? This can be derived using the analysis of coverage and an ergodic theorem.
- Contact distribution: given that a location is not covered, what is the radius of the bigest ball (resp. length of the longest segment of orientation θ) centered at this location that does not intersect any grain of the Boolean model?

These questions extend to many other models, either with non-Poisson germs, or with Poisson germs but grains that are not i.i.d. An interesting application in the context of networks is considered in [46], where the grains are SINR cells, namely the region of the space around a transmitter where the SINR with respect to this transmitter exceeds a given threshold. Here the interference is the field created by the other points of the Poisson point process.

2) Gilbert's random disk graph: This is a model for wireless networks that is a special case of the Boolean model described above, and is due to Gilbert [32]. Assume that the compact sets described in the last subsection are all balls of radius $r/2$. We define the random disk graph of a Poisson point Φ process of intensity λ with range r, denoted as $G_{\lambda,r}$ as the graph with nodes the points of Φ and with edges between X and Y if the two grains touch, *i.e.*, if $\|X-Y\| \leq r$. This is the most basic random geometric graph and a central object in random graph theory. The following questions are of particular interest within this setting:

- Does this random graph has an infinite component? This is of course equivalent to Ξ having an infinite component. This property is referred to as *percolation*. A striking result is that there is a deterministic critical value $0 < r_c < \infty$ such that when $r < r_c$, there is no such infinite component with probability 1 (*i.e.*, there is no percolation for all realizations of Φ), whereas when $r > r_c$, there is an infinite component with probability 1 (*i.e.*, there is percolation for all realizations of Φ). This is proven in §IV-C.
- In case percolation occurs, what is the fraction of nodes included in the infinite component? In case of it does not occurr, what is the typical size of a component?

The main tool for addressing these continuum percolation questions is a reduction to discrete bond or site percolation. For more on the matter, see [3], [35], [47].

C. Voronoi tessellation

By definition, a tessellation is a collection of open, pairwise disjoint polyhedra (polygons in the case of $\mathbb{R}^2$) whose closures cover the space, and which is locally finite (i.e., the number of polyhedra intersecting any given compact set is finite).

Given a simple point measure (or point sequence) ϕ on $\mathbb{R}^d$ and a point $x \in \mathbb{R}^d$, we define the *Voronoi cell* $\mathcal{C}_x(\phi)$ of the point $x \in \mathbb{R}^d$ w.r.t. ϕ to be the set

$$\mathcal{C}_x(\phi) = \{y \in \mathbb{R}^d : \|y-x\| < \inf_{x_i \in \phi, x_i \neq x} \|y - x_i\|\}. \quad (7)$$

For a simple point process $\Phi = \sum_i \delta_{X_i}$ on $\mathbb{R}^d$, we define the *Voronoi tessellation* or *mosaic* generated by Φ to be the marked point process

$$\mathcal{V} = \sum_i \delta_{(X_i, \mathcal{C}_{X_i}(\Phi) - X_i)},$$

whose marks are the Voronoi cells shifted to the origin.

The *Delaunay triangulation* generated by a simple point measure ϕ is a graph with the set of vertices ϕ and edges connecting each $y \in \phi$ to any of its Voronoi neighbors.

The Delaunay triangulation of a Poisson point process is an object of central importance in communications. In a regular periodic (say hexagonal or triangular) grid it is obvious how to define the neighbors of a given vertex. However, for irregular patterns of points like a realization of a PPP, which is often used to model set of nodes in mobile ad hoc networks, this notion is much less evident. The Delaunay triangulation offers some purely geometric definition of neighborhood in such patterns.

For quantitative properties of Poisson–Voronoi tessellations (mean cell size, mean number of sides of the cell, etc.) and Poisson–Delaunay graphs (mean degree, mean length of a typical edge, mean size of a typical triangle) see, *e.g.*, [48]. In [17], [18], [49], Voronoi tessellation-based models of cellular access network were considered to derive closed-form expressions for the mean number of users in a cell, the mean length of connections, and the total power received at the base station.

III. Interference Characterization and Outage

In this section we apply some of the techniques introduced above to study the interference in large ad hoc networks and the outage probability of any given link.

A. Interference

We start with the general question: what can be said of the total interference power measured at a point x in the network, given by

$$I(x) \triangleq \sum_{Y \in \Phi_t} \ell(\|x-Y\|),$$

where Φ_t is a point process of transmitters (assumed to be interferers) on $\mathbb{R}^2$? Φ_t is typically a subset of a larger point process Φ since it constitutes the nodes selected by the MAC scheme to transmit concurrently. For example, if nodes in a homogeneous PPP of intensity 1 transmit independently and randomly with probability p (slotted ALOHA), Φ_t is a PPP with intensity p. Due to Slivnyak's result (see §II-A1), $I(x)$ does not depend on the given location where interference is measured; in particular, it does not matter whether x is part of the underlying point process Φ or not (as long as its contribution to I is not considered if Φ_t is conditioned on having a point at x).

As mentioned in §I-B, researchers have followed an analogy to *shot noise processes* to analyze the distributional properties of $I(x)$ [9], [16], [50]. This analogy can be used to derive the Laplace transform of the interference as follows.

Let $\Phi \triangleq \{R_i = \|X_i\|\}$ be the distances of the points of a d-dimensional uniform PPP of intensity μ from an arbitrary origin o. Then Φ is an inhomogeneous PPP with intensity function $\lambda(r) = \mu c_d d r^{d-1}$, where $c_d = |B(o,1)|$ is the volume of the d-dimensional unit ball. Considering the interference as a shot noise process (2), and also accounting for the fading terms, we can identify $h_r \ell(r) = h_r r^{-\alpha}$ for i.i.d. fading h with the impulse response of the shot noise process. We would then like to calculate the Laplace transform

$$\mathcal{L}_I(s) \triangleq \mathbb{E}[e^{-sI}] = \mathbb{E}\left[\prod_{R \in \Phi} \exp\left(-s h_R R^{-\alpha}\right)\right]$$

of the interference. This is a Laplace functional with $f(r) = s\ell(r) = s h_r r^{-\alpha}$. The expectation is to be taken over both Φ and h, but since the fading is assumed independent of the point process, the expectation over h can be moved inside the

product, so we have from (3) (see also [61, Eqn. (3)])

$$\mathcal{L}_I(s) = \exp\left\{ -\int_0^\infty \mathbb{E}_h[1-e^{-shr^{-\alpha}}]\lambda(r)\mathrm{d}r \right\} = \exp\Big(-\mu c_d \mathbb{E}[h^\delta]\Gamma(1-\delta)s^\delta \Big), \quad (8)$$

where $\delta \triangleq d/\alpha$. Note that this expression is only valid for $\delta < 1$. So:

- For $\alpha \leqslant d$, we have $I = \infty$ a.s. This is a consequence of the cumulated interference from the many far transmitters whose signal powers do not decay fast enough to keep the interference power finite. For a finite network, the interference would be finite.
- For $\alpha > d$ we have $I < \infty$ a.s. but $\mathbb{E}(I) = \infty$ due to the singularity of the path loss law at the origin. Even if we consider only the nearest interferer, $\mathbb{E}(I)$ is infinite. If a bounded path loss law is used, all moments exist.

In the important case of Rayleigh fading, $\mathbb{E}[h^\delta] = \Gamma(1+\delta)$, so, using the properties of the gamma function, we obtain the closed-form result

$$\mathcal{L}_I(s) = \exp\Big(-\mu c_d s^\delta \frac{\pi\delta}{\sin(\pi\delta)} \Big).$$

So the interference has a *stable distribution* with characteristic exponent δ and dispersion $\mu c_d \mathbb{E}[h^\delta]\Gamma(1-\delta)$. Since $\delta < 1$, I does not have any finite moments.

A closed-form expression for the interference distribution only exists for $\delta = 1/2$; this is the inverse Gaussian or Lévy distribution, as has been established in [51].

Using the distribution of the distances to the n-th nearest neighbor [52], the distributions of the interference (without fading) from the n-th nearest neighbor are easily found. The tail probabilities do not depend on the presence or type of fading and are given by

$$\mathbb{P}(I_n > x) \sim \frac{1}{n!}(\lambda c_d)^n x^{-n\delta}, \qquad x \to \infty.$$

This means that $\mathbb{E}(I_n^p)$ exists for $p < n\delta$. For example, if interference-canceling techniques are used and the interference from the k nearest interferers can be cancelled, we need $k > \alpha$ in two-dimensional networks to have a finite second moment.

If the non-singular path loss model $\ell(x) = (1+\|x\|)^{-\alpha}$ is used, the tail probability of the interference reflects the tail probability of the fading process: If the fading has an exponential or power-law tail, so does the interference. This holds for general motion-invariant processes [53]. Other approaches to the singularity issue are given in this issue [54], [55].

B. Outage

An *outage* of a wireless link is said to occur when a packet transmission fails. In many situations, it is justified to equate the outage event to the event that $\mathsf{SINR} < T$ for some threshold T that depends on the physical layer parameters such as rate of transmission, modulation, and coding. The complementary probability is the success probability $p_s \triangleq \mathbb{P}(\mathsf{SINR} \geqslant T)$.

In the case where the desired signal S is subject to Rayleigh fading, we obtain for the success probability over a link of distance R

$$p_s = \mathbb{P}(S > T(W+I)) = \exp\left(-\frac{TR^\alpha W}{P}\right)\mathbb{E}(e^{-TR^\alpha I}), \quad (9)$$

where P is the transmit power. The first term only depends on the noise (or SNR), while the second only depends on the interference (or SIR). Focusing on this interference term we notice that this is the Laplace transform of the interference evaluated at $s = TR^\alpha$. So, in a d-dim. interference-limited network whose nodes are distributed as a uniform PPP of intensity λ with ALOHA channel access with probability p, the success probability is given by (8), replacing s by TR^α:

$$p_s = \exp\Big(-\lambda p c_d R^d \mathbb{E}[h^\delta]\Gamma(1-\delta)T^\delta \Big) \quad (10)$$

Here we have used the fact that ALOHA channel access performs independent thinning of the PPP, which results in a PPP of lower intensity. The interferers' channels may be subject to a different type of fading (or no fading), all that matters is $\mathbb{E}[h^\delta]$.

The equivalence of Laplace transforms and success probabilities has been pointed out in [56]–[58], and in [58] several generalizations can be found.

C. Throughput

The transmission success probability in the previous subsection is derived assuming that the desired transmitter transmits while the received listens. To optimize the network parameters, such as the ALOHA transmit probability p, the unconditioned success probabilities must be considered. In the case of ALOHA and half-duplex transceivers, the *spatial throughput* is $p(1-p)p_s(p)$. Finding the optimum p means finding the optimum trade-off between spatial reuse (a larger p results in a higher density of concurrent transmissions) and success probabilities (a larger p results in higher interference and thus a lower success probability). In [58], a related metric, the *spatial density of success* pp_s is optimized in function of p. In some cases, the optimal value, which is known in closed form, does not depend on the intensity λ of the underlying point process.

The same framework can also be used to find the optimum value for the SINR threshold T that maximizes the *area spectral efficiency*. A larger T permits higher transmission rates (or spectral efficiencies if normalized by the bandwidth) but results in lower success probabilities (see further discussion in §V-B). Similarly, the *probabilistic progress*, usually defined as the product of distance times success probability can be maximized by finding the optimum link distance R [59]. Choosing a larger R may increase the progress but comes with the disadvantage of a lower success probability [60]. In [58], an expression for progress based on extremal shot noise is derived, and in [61], the distribution and the mean value of the throughput of a typical user (as given by Shannon's formula) are given using Fourier transform techniques.

IV. Percolation and Connectivity

A. Introduction

Percolation theory was originally introduced to model the porosity of materials. It has since then developed into a lively

branch of probability. More recently, percolation models have been used to model the connectivity of wireless multi-hop networks.

The main property of percolation models is that they exhibit a *phase transition* in their connectivity behavior: depending on some (continuous) parameters, the components of the model are either all finite (*sub-critical* case) or one giant component forms (*super-critical* case). In the context of networking, such a transition affects the performance of the system greatly: without a giant component, the network would be completely fragmented and unusable. It is therefore of prime importance to characterize the conditions under which the network is super-critical. In this section, we cover some basics of percolation theory, starting with the simplest models and proof techniques, and then moving on to models that are more appropriate for wireless networks.

Percolation theory deals mostly with models of infinite size. We start with these classical models and briefly address finite networks.

B. Discrete percolation: Bond percolation in infinite lattices

The bond percolation model is defined as follows: consider the infinite square lattice $\mathbb{Z}^2$ and connect each pair of nearest neighbors independently with probability p. Then define the component (or *cluster*) of the origin C as the set of elements of $\mathbb{Z}^2$ that are connected to the origin by a sequence of adjacent edges. We define the *percolation probability* as

$$\theta(p) := \mathbb{P}(\#C = \infty).$$

The central result of percolation theory is the following:

Theorem 1 (Broadbent and Hammersley [33]) *There exists a number $0 < p_c < 1$ such that $\theta(p) = 0$ for $p < p_c$ and $\theta(p) > 0$ for $p > p_c$.*

In the particular case of bond percolation on $\mathbb{Z}^2$, the exact value of p_c is known to be $1/2$ [62]. Other specific cases and properties of $\theta(p)$ can be found in [47]. To prove Theorem 1, we need first to observe that $\theta(p)$ is an increasing function. Although this is a very intuitive property, its proof is not so trivial (see, *e.g.*, [35, Ch. 2.2] for a general method). It is then enough to prove that there exists some probability $p_1 > 0$ such that $\theta(p_1) = 0$ and some probability $p_2 < 1$ such that $\theta(p_2) > 0$. The following two sections are devoted to finding p_1 and p_2.

1) Absence of percolation for small values of p: We start from the observation that if the origin belongs to an infinite cluster, then for any integer n, one can find in the lattice a self-avoiding path of length n starting at the origin. Thus we have

$$\theta(p) \leq \mathbb{P}(\exists \text{ a path of length } n \text{ starting at } o) \quad \forall n\,. \qquad (11)$$

If all direct neighbors of $\mathbb{Z}^2$ were connected by an edge, the number $\kappa(n)$ of such path would be bounded from above by $4 \cdot 3^{n-1}$. Since edges are present with probability p, each of these κ paths exists with probability p^n. Using the union bound, we find that

$$\mathbb{P}(\exists \text{ a path of length } n \text{ starting at } o) \leq 4p(3p)^{n-1}.$$

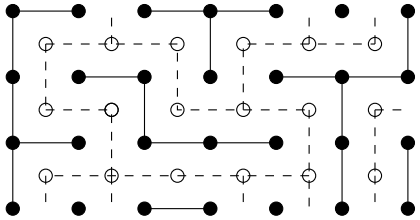

Fig. 1. A realization of the bond percolation model on $\mathbb{Z}^2$ (plain lines) and its dual (dotted lines).

If $p < 1/3$, this quantity tends to zero when n increases, so that (11) implies $\theta(p) = 0$. So we have established that $p_c \geqslant 1/3$.

2) Existence of an infinite cluster for sufficiently large p: Our proof relies on the classical Peierls argument [63]. Consider the dual lattice of $\mathbb{Z}^2$, which consists of vertices that are shifted by half a unit in both directions, as depicted in Figure 1. An edge is placed between two direct neighbors of the dual lattice if it does not intersect an edge of the direct lattice.

The key observation is that if a component is finite in the original lattice, it is necessarily surrounded by a circuit in the dual lattice. To prove that a vertex (*e.g.*, the origin) belongs to an infinite cluster with positive probability, it is thus enough to show that the probability that a circuit surrounds the origin in the dual lattice is less than one. Let us estimate the number $\sigma(n)$ of possible circuits of length $2n$ that surround the origin: it is easy to see that it is bounded by

$$\sigma(n) \leq (n-1) \cdot 3^{2(n-1)}\,.$$

Therefore, the probability that there exists a circuit around the origin with all edges closed upper bounded by

$$\begin{aligned} \mathbb{P}(\text{closed circuit}) &\leq \sum_{n=2}^{\infty} (1-p)^{2n} \sigma(n) \\ &= \frac{9(1-p)^4}{[1-9(1-p)^2]^2}. \end{aligned}$$

One can verify that when $p > 1 - 1/(2\sqrt{3}) \approx 0.71$, the above sum converges to a number smaller than one. As a consequence, the origin belongs to an infinite cluster with positive probability.

Note that since the existence of an infinite cluster does not depend on the state of a finite number of edges, we can use Kolmogorov's zero-one law to conclude that the probability that such a cluster exists is either zero or one (see *e.g.*, [64]). If it was zero, then the origin would belong to an infinite cluster also with probability zero. Therefore, whenever $\theta(p) > 0$, an infinite cluster exists with probability one.

C. Continuum percolation: The random geometric graph

The basic random geometric graph or disk graph $G_{\lambda,r}$, as defined in §II-B2, relies on two assumptions: First, the nodes' location follows a two-dimensional Poisson point process. Second, each node can communicate directly to any other node within a radius r around it. The latter assumption comes

from the following model: assume that nodes emit with a certain power P and that this signal is attenuated over distance according to a deterministic decreasing function $\ell(d)$. Assume also that receivers can successfully receive data if the signal is at least β times stronger than the ambient noise, which has power W. Then the transmission radius is defined by

$$r \triangleq \max\{d : \frac{P\ell(d)}{W} \geq \beta\}. \quad (12)$$

Similarly to the discrete model, we denote by $\theta(\lambda, r)$ the probability that a node located at the origin belongs to an infinite cluster. Due to its simplicity, the graph $G_{\lambda,r}$ can be rescaled while keeping its connectivity properties. Indeed, if all distances are divided by γ, the underlying PPP is transformed into another Poisson process with intensity $\gamma^2\lambda$. Thus, the graph $G_{\gamma^2\lambda,r/\gamma}$ has the same connectivity properties as $G_{\lambda,r}$ and we have

$$\theta(\gamma^2\lambda, r/\gamma) = \theta(\lambda, r).$$

As in the bond percolation model, we briefly explain a way to show that a phase transition occurs in $G_{\lambda,r}$.

1) Absence of an infinite cluster for small values of λ: Consider a node o placed at the origin. We populate the set C of the nodes connected to o step by step: at step zero, we set $C = \{o\}$. Then at each step, we add to C all nodes that share an edge with an element of C. As each node has on average $\lambda\pi r^2$ edges, this process can be compared to a Galton-Watson process [34] where each individual gives birth to $\lambda\pi r^2$ children in average. The difference is that in our process, the number of nodes added at each step might be smaller, as some nodes sharing edges with elements of C might be already in C. It is known that if the average number of children per individual in a Galton-Watson process is smaller than one, the process stops after a finite number of steps [34]. As our process grows more slowly, it certainly stops in this case too. Therefore we have that if $\lambda < 1/(\pi r^2)$, the cluster of the origin is finite a.s.

2) Existence of a giant cluster for large λ: We use a mapping onto a bond percolation model: We divide the plane into squares of size $c = r/2\sqrt{2}$ as shown in Figure 2. Each square corresponds to a potential edge of the lattice, which is added if at least one point of the Poisson process falls into this square. Thus, each edge is present with probability $p = 1 - \exp(-\lambda c^2)$, independently of the other edges. As a consequence, if $\lambda \geq \log 2/c^2$, the edge percolation model contains an infinite cluster a.s.

Moreover, if two edges are adjacent, then by construction two points of the Poisson process are located in squares that share at least at one corner. Therefore, the distance between them is less than $2\sqrt{2}c = r$, and they are connected in $G_{\lambda,r}$. Accordingly, an infinite collection of connected edges corresponds to an infinite set of connected points in $G_{\lambda,r}$. We can thus conclude that if $\lambda \geq \log 2/c^2$, $G_{\lambda,r}$ contains an infinite cluster a.s.

The exact value of the critical density $\lambda_c(r)$ or critical radius $r_c(\lambda)$ is unknown. For $\lambda = 1$, the bounds $1.1979 < r_c < 1.1988$ were established with 99.99% confidence in [65].

3) Generalization to the Boolean model: Gilbert's random geometric graph is a particular case of the Boolean model (see §II-B). Let us consider a Boolean model of arbitrary dimension d where grains are balls whose radius is now random. It turns out that for a suitable radius distribution, a phase transition is observed at some critical germ density. Denoting by R the (random) radius of a ball, one can show [35] that whenever the dimension of the model d is greater than one and $\mathbb{E}(R^4) < \infty$, there exists a critical value of λ below which the union of the balls Ξ has only finite components and above which a giant component forms.

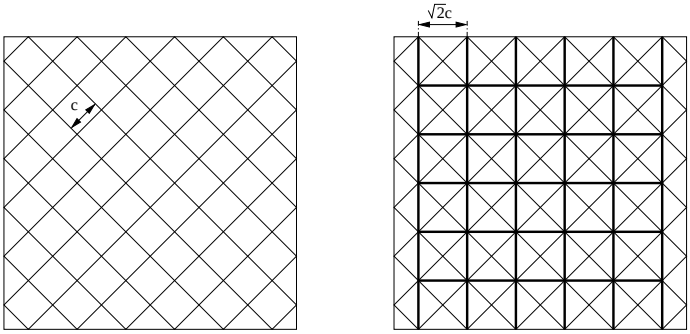

Fig. 2. On the left hand side, the division of the plane into squares. On the right hand side, each square is assigned to an edge of a bond percolation model (bold lines).

D. Other models

In this section, we briefly describe other percolation models that are relevant to communication networks. In all of them, the location of the nodes is modeled by a Poisson point process of density λ over $\mathbb{R}^2$. They differ only by the criterion used for adding edges between nodes.

1) Nearest-neighbors networks: In this model, each node connects to its k nearest neighbors. This model is for example suitable for a dense wireless network where nodes use power control algorithm in order to be connected only to their k first neighbors.

An important property of this model is that the value of λ does not affect the connectivity of the model (it is called a *scale-free* model). Therefore, its only relevant parameter is k. One can show that there exists a critical value of k for which a giant component forms, and below which only finite clusters are observed. In two dimensions, the critical value is conjectured to be $k = 3$ [39].

2) Random connection model: This model is another generalization of Gilbert's model. For each pair of nodes, we consider the distance x between them. Then we add an edge between them with probability $g(x)$, where g is a function from $\mathbb{R}^+$ to $[0, 1]$ such that $\int_0^\infty xg(x)dx < \infty$.

This model takes some randomness of the wireless channel into account: nodes connect to each other probabilistically, depending on their distances. The probability of failed connection can for example model a shadow fading effect. Gilbert's original model can be retrieved by setting $g(x) = u(r - x)$. A similar phase transition as in Gilbert's model can be observed at some critical density of nodes λ [38]. One relevant question is how this critical density changes with the shape of the connection function $g(x)$. The work in [66] has shown that under some spreading transformations on g, the critical

density cannot increase. The spread-out limiting case has also been worked out in [67], showing that in the case of very long, unreliable connections, the critical density has a limit corresponding to that of an independent branching process, i.e. the average degree of the corresponding random graph at the percolation threshold tends to 1. Alternative proofs of these spreading results also appear in [68], [69].

3) Signal-to-interference ratio graph (STIRG) model: The connectivity criterion in (12) compares the received signal to the ambient noise only. However, if several nodes are using the same channel, interference degrades the received signals. In the so-called STIRG model [36], [37], the SNR threshold is replaced by an SINR threshold as in (9) so that the nodes $X_i, X_j \in \Phi$ are connected by an edge if

$$\frac{P\ell(\|X_i - X_j\|)}{W + \gamma\left(\max\{I(X_i); I(X_j)\} - P\ell(\|X_i - X_j\|)\right)} \geq T,$$

where $I(X_i) = \sum_{X_k \in \Phi \setminus \{X_i\}} P\ell(\|X_i - X_k\|)$. This condition ensures that the two nodes have a sufficiently high SINR to exchange data in both directions despite the interference of all the other nodes. The factor $\gamma \leq 1$ serves as a weight for the interference term and models the gain of the spread spectrum scheme (if any).

This model differs from the others by the fact that if has more degrees of freedom. Clearly, when $\gamma = 0$, the model is equivalent to Gilbert's model, and it percolates above the critical node density for Gilbert's graph λ_c. In the case of attenuation function of bounded support, it has been shown in [36] that for large enough λ one can choose γ small enough so that the model percolates. This result has been strengthened in [37] to show that this is the case whenever $\lambda > \lambda_c$ and also for attenuation functions of unbounded support. In other words, whenever the node density is super-critical (in Gilbert's sense), the model can tolerate a certain amount of interference before the giant component disappears. Figure 3 gives a pictorial representation of this mathematical result obtained through computer simulation, illustrating the parameter domain where percolation occurs for a power attenuation function $\ell(\cdot)$ of bounded support.

E. Connectivity in finite networks

In finite networks, there is of course no infinite cluster, and therefore no percolation *stricto sensu*. However, if one considers a sufficiently large network, one expects to observe a similar phenomenon: if the density of nodes is large enough, a component that contains a large fraction of the nodes should emerge. The following theorem follows from [70] and confirms this intuition in the case of Gilbert's disk graph.

Theorem 2 *Consider the restriction of a Boolean model to a square of size $\sqrt{n} \times \sqrt{n}$, whose connectivity graph is denoted by $G_{\lambda,r}(n)$. Denote by $C_{\lambda,r}(\eta, n)$ the event that there exists in $G_{\lambda,r}(n)$ a component that contains at least ηn vertices. Then we have*

$$\lim_{n\to\infty} \mathbb{P}(C_{\lambda,r}(\eta, n)) = \begin{cases} 1 & \text{if} \quad \eta < \theta(\lambda, r) \\ 0 & \text{if} \quad \eta > \theta(\lambda, r) \end{cases}$$

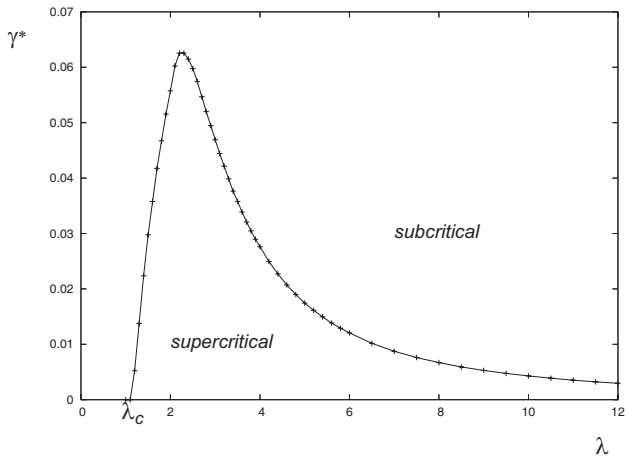

Fig. 3. Percolation threshold of the STIRG model: in this figure, we evaluated by simulation the region of the parameter space (λ, γ) where percolation occurs (supercritical region).

As a consequence of the above theorem, since $\theta(\lambda, r)$ is never equal to one, there is always a non-vanishing fraction of disconnected nodes in the network. However, if one lets the connectivity range r increase with n, the fraction of connected nodes can be made to converge to one. If the connectivity range of the nodes is a function $r(n)$ of the number of nodes, the condition for *asymptotic connectivity* (i.e. the condition under which the probability that all nodes are connected tends to one when the n increases) is given by

$$r(n) = \sqrt{\frac{\log n}{\pi}} + c(n),$$

where $c(n)$ is any function such that $\lim_{n\to\infty} c(n) = \infty$. This result can be deduced from [70] and has been published in its explicit form in [71]. A similar condition on the rate at which $p \to 1$ to observe full connectivity in a bond percolation model on an $n \times n$ grid has been derived in [72].

Nearest-neighbor model: A similar result on asymptotic connectivity has been derived for the nearest-neighbors model. The rate at which the number of neighbors k must increase with n is [73]:

$$0.3043 \log n \leq k(n) \leq 0.5139 \log n\,.$$

In summary, tools from percolation theory and random geometric graphs have enabled analytical studies of the connectivity properties of large ad hoc networks. While connectivity is a fundamental prerequisite for network operation, it does not guarantee any network throughput or capacity. The study of the capacity of wireless networks is the topic of the next section.

V. Capacity and Scaling Laws

The capacity of a communication system is the maximum data-rate in bits per second that can be reliably transferred from transmitter to receiver. In the strict information-theoretic sense, this is an unsurpassable upper bound that, in practice, can only be approached. In a single Tx-Rx link of unit

bandwidth subject to AWGN, the capacity in bits per channel use (*i.e.*, bps/Hz) is given by the Shannon-Hartley formula:

$$\log_2\left(1+\frac{S}{W}\right), \tag{13}$$

where, as before, $S = P\ell(r)$ and W are the received power and Gaussian thermal noise at the receiver, respectively.

The situation becomes more complex in an ad hoc network with n Tx-Rx pairs, where the capacity is much more difficult to define and compute. The most general (and natural) description is given by the so-called *capacity region*, which is an $n\times n$ matrix C where the c_{ij} entry corresponds to the capacity from node i to node j, which is also dependent on the signals being generated by the remaining $n-1$ transmitting nodes in the network. These can either interfere with the communication between i and j, or be cooperative and aid communication between i and j. Clearly, due to the many possible ways of interaction, the capacity region of an ad hoc network is difficult to characterize. Even for very small values of n, such as $n=3$, the capacity region has yet to be determined. Therefore, intermediate descriptive theories that fall short of the strict information-theoretic standard are needed, as pointed out in more detail in [74].

A. Capacity scaling laws

One attempt to simplify the problem came in 2000 by Gupta and Kumar [75]. Their approach was to introduce two main simplifications: on the one hand, they proposed to study the case in which all the nodes in the network are required to transmit at the same bit-rate. This implies that the whole capacity region reduces to a single scalar quantity. On the other hand, they proposed to compute only the *scaling limit*, *i.e.*, the order-of growth of such a scalar quantity as the number of nodes in the network increases. In addition, Gupta and Kumar's scaling law was also derived under some assumptions on the physics of propagation (*i.e.*, channel gains that decay as a power law of the distance between transmitters and receivers), and on some restrictions on the cooperation strategy employed by the nodes (*i.e.*, multi-hop operation and pairwise coding and decoding at each hop). Their main result was the so-called square-root law, namely, as n increases the per-node bit-rate decreases as $1/\sqrt{n}$.

Due to their restrictive model, the result of Gupta and Kumar cannot be considered an unsurpassable bound in the strict information-theoretic sense. It does not allow sophisticated network coding strategies but only point-to-point transmission across multiple hops, and it relies on a specific physical propagation model in which the signal power received at distance $\|x\|$ from the transmitter is given by $\ell(x) = \|x\|^{-\alpha}$. Under this model, at each hop the collective interference can be approximated as Gaussian and its power added to the noise term W in (13). In this way, any point-to-point link in the network from an arbitrary chosen origin to point $x \in \mathbb{R}^2$ can support a rate of

$$\log_2\left(1+\frac{Ph_x\ell(\|x\|)}{W+I(x)}\right).$$

Since there is no a priori reason to operate the network in a multi-hop fashion and treat the interference term $I(x)$ as pure noise, the square root law of Gupta and Kumar can in principle be surpassed. One can, for example, envisage a network strategy in which groups of nodes help each other, rather than interfere, by coherently summing their signals at the receiver.

Perhaps the main contribution of Gupta and Kumar has been to show that such a simple geometric interference-based model can lead to meaningful insights on the capacity limitations of wireless networks operated with current multi-hop technologies. Furthermore, their paper showed that stochastic geometry tools such as random Voronoi tessellations and random geometric graphs can be used to analyze the performance of network protocols.

Gupta and Kumar's work sparked an enormous interest in the field. On one side, under the same restrictive model, simpler strategies achieving the same square root law have been proposed [41], [76]. The work in [41] is particularly relevant in the context of this paper, as it showed a connection between protocol design and percolation theory. In that paper, the flow of information through the network is compared to the number of disjoint paths crossing the network from side to side in an underlying percolation model. It is shown that in a network of area proportional to the number of nodes n, the number of disjoint paths crossing the network area from side to side in the underlying percolation model grows with $\sqrt{n}$. Hence, roughly speaking, the amount of information that can flow across the network is only of the order of $\sqrt{n}$, and since n nodes must share this flow, the $1/\sqrt{n}$ bound follows.

On the other side, researchers were interested in discovering whether the square root law holds in a more general context. Hence, they started to seek bounds on the capacity scaling that were independent on the network operation strategy. This more general approach led to considerable success. Starting with the work of Xie and Kumar [77], *information-theoretic* scaling laws, independent of any strategy used for communication, have been established by many authors [78]–[82]. These arise from the application of the information-theoretic cut-set bound [83, Ch. 15] which allows one to bound the total information flow across any network cut, allowing arbitrary cooperation among the nodes. Among these works, a short information-theoretic derivation of the square root law relying on geometric elements of spatial point processes is given in [80]. It is important to notice, that while essentially confirming the square root law in a more general context, all the results above still depend on the assumptions made on the electromagnetic propagation process.

Indeed, more striking results appear in [84], and [81]. These papers show that a much higher per-node bit-rate than $\Theta(1/\sqrt{n})$ can be achieved in wireless networks by changing the assumptions on the physical propagation process. These authors introduce the presence of an additional source of randomness by adding fading. Under some assumptions on the fading process and when the path loss exponent α is sufficiently small, these authors describe node cooperation strategies based on space-time codes which can achieve an almost constant per-node bit rate: a great improvement com-

pared to Gupta and Kumar's original bound! Hence, the main message of the above papers is that there is a gain to be expected when adopting more complex node cooperation strategies than simple multi-hop operation.

A recent additional effort has been made in [85], which recognizes that the strong dependence of information-theoretic results on heuristic physical propagation models is somehow undesirable for a theory that seeks the fundamental limits of communication. They showed that the square root law also arises from physical limitations dictated by Maxwell's physics of wave propagation, in conjunction to the information-theoretic cut-set bound. This result shows that the original prediction of Gupta and Kumar is also due to a degrees-of-freedom limitation that is independent of empirical path-loss models and stochastic fading models. In other words, stochastic fading assumptions such as in [84], and [81] are open to debate, as they can lead to non-physical results.

But does this lead to the conclusion that the sophisticated cooperative strategies described in [84] and [81] do not lead to any improvement over multi-hop operation? The general answer is no. Recall that scaling results are only up to order and pre-constants can make a huge difference in practice. Sophisticated cooperative communication schemes could certainly improve upon nearest-neighbor routing. The precise amount of this improvement, if any, remains unknown; it is only known that this improvement vanishes as n grows [85].

B. Transmission capacity and area spectral efficiency

Although scaling laws provide significant insight on the large-scale performance of ad hoc networks, a finer view of throughput limits is needed to understand how different technologies and protocols affect the baseline performance of distributed wireless networks. Many, even most, communication design choices will have a significant effect on the achievable SINR and hence throughput, while not affecting the scaling law. In this section, we show how stochastic geometry and, in particular, the techniques for characterizing interference and outage of §III, can be used to determine the area spectral efficiency (ASE) in a specific ad hoc network design; in other words, the number of bits per second per unit of bandwidth that can be transmitted in a given area.

The ASE is formalized by a metric termed the *transmission capacity*, first proposed in [86], which is the maximum number of bits per second sent by all users in the network per unit area, subject to a constraint on outage probability relative to a SINR threshold. Formally, the transmission capacity is defined as

$$c(\lambda) \triangleq \lambda_\epsilon(1-\epsilon) \tag{14}$$

measured in transmissions/area, where λ_ϵ is the maximum density of transmissions supported such that $\mathbb{P}[\mathsf{SINR} < T] \leq \epsilon$, for an SINR target T. Adding the per-user data rate (which would be on the order of $\log_2(1+T)$ bps/Hz) results in the area spectral efficiency. Transmission capacity is used as the capacity metric in a few papers in this issue [87], [88], and a more extensive tutorial on transmission capacity is available online [89].

There are some shortcomings of this metric, namely that it is usually a single-hop metric rather than end-to-end, presumes a common SINR target and outage probability (conceptually like the packet error rate), and is more the description of a given technology through its achieved SINR than of technology-independent fundamental limits. Nevertheless, it does capture key aspects of capacity – "good" communication techniques should provide higher transmission capacity – and combined with a homogeneous Poisson distribution for the interfering nodes, yields superior analytical tractability to other network throughput metrics. We now provide the simplest baseline model for transmission capacity. The key aspects of the model are as follows, with generalizations noted.

- Fixed transmit distance R. Variable transmit distances can be used but reduce the tractability: in general a loss factor of $\mathbb{E}[R^2]/(\mathbb{E}[R])^2$ is experienced if the transmit distance R is a random variable [90].
- Single transmit and receive antennas. Multiple antennas can obviously increase the transmission capacity [60], [91]–[93].
- Homogeneous PPP for interferers, which implies an ALOHA-type transmission scheme. Generalizations are nontrivial, but one to clustered PPPs has been undertaken [94], and exclusion regions are considered in [95].
- Interference is treated as noise, although it can in principle be cancelled or suppressed by an appropriate receiver to get higher capacity [96]–[98].

The key to transmission capacity is outage probability, which for the case of Rayleigh fading can be exactly derived. Setting this equal to the outage probability target ϵ and solving (10) for λ_ϵ (see (14)), the transmission capacity in this simple case is (two-dimensional networks, thermal noise neglected)

$$c_o(\epsilon) = \frac{(1-\epsilon)\ln(1-\epsilon)}{C(\alpha)R^2T^{2/\alpha}} = \frac{\epsilon}{C(\alpha)R^2T^{2/\alpha}} + \Theta(\epsilon^2), \tag{15}$$

where $C(\alpha) = \pi^{1+2/\alpha}/\sin(2\pi/\alpha)$. This simple expression shows precisely how the number of supportable users in the network depends on outage probability (about linearly, for low outage), path loss exponent α, and target SIR T (noise can be included at the expense of more bulky expressions).

If there is no fading – just large-scale path loss – it is possible to get tight bounds but not an exact solution. In this case, bounds on the transmission capacity are as follows, where we have included the noise power η and transmit power ρ for $\mathsf{SNR} = \rho/\eta$

$$\frac{(\alpha-1)\epsilon}{\alpha\pi}\left(\frac{1}{R^2T^{2/\alpha}} + \mathsf{SNR}^{2/\alpha}\right) + \Theta(\epsilon^2) \;\leq\; c_o(\epsilon) \;\leq\; \frac{\epsilon}{\pi}\left(\frac{1}{R^2T^{2/\alpha}} + \mathsf{SNR}^{-2/\alpha}\right) + \Theta(\epsilon^2). \tag{16}$$

Note the similarity between (15) and (16): they are within a small constant of each other, all the parameters are of the exact same order. Arbitrary fading distributions can be accommodated at further loss of tractability [90], but again there is no change in the first-order effects of the system parameters.

What is useful about the transmission capacity is that it allows candidate technologies and design choices to be compared objectively, analytically, and fairly simply. For example, one might ask how adding spread spectrum modulation (CDMA) to the system would change the number of supportable users. The answer with the transmission capacity metric is fairly immediate. With asynchronous binary direct-sequence spread spectrum (DSSS), the target SINR is effectively decreased to $2T/3M$ at the cost of a bandwidth penalty of M[2]. If frequency-hopping (FH) was used instead, M independent channels are created with an effective interference density of λ/M on each of them. With some straightforward manipulations it can be seen that the transmission capacities become

$$c^{\mathrm{DS}}(\epsilon) = \left(\frac{3M}{2}\right)^{\frac{2}{\alpha}} c_o(\epsilon), \quad c^{\mathrm{FH}}(\epsilon) = M c_o(\epsilon)\,. \tag{17}$$

The ratio

$$\frac{c^{\mathrm{FH}}}{c^{\mathrm{DS}}} = k_1 M^{1-\frac{2}{\alpha}} \tag{18}$$

implies that frequency-hopping is a superior form of spread spectrum in an ad hoc network, for example by a factor of $\sqrt{M}$ when $\alpha = 4$. In principle, any modulation technique, multiple access or even scheduling protocol can be analyzed using the transmission capacity metric to predict relative gains.

C. The road forward

Scaling laws on transport capacity and exact results on transmission capacity both provide important views into the network capabilities, but both presently fall short of providing a complete metric for the achievable network throughput. Future research should attempt to bridge this gap, by utilizing stochastic geometry to quantify end-to-end achievable rates. For example, each hop in the network can be considered to have some outage probability, and an aggregation of such stochastic links comprises an end-to-end connection in the network, with some aggregate outage probability, achievable data rate, and queueing delays at relay nodes. A recent example in this vein can be found in [99], where a delay-minimizing routing strategy for ad hoc networks is proposed. The analysis is complicated by the spatial and temporal correlations that exist in the interference due to the common randomness in the nodes' positions [100].

Another promising related approach is the increasing popularity of erasure channels and erasure networks to model the performance of links that are occasionally in outage [101]. Although this has never been done, one can envision combining emerging results on the capacity of wireless erasure networks with outage (erasure) probabilities computed with the help of stochastic geometry tools. Two important new techniques from information theory for characterizing interference in wireless networks include the deterministic capacity [102] and the degrees-of-freedom region [103]. These approaches both require relative values for the channel gains of each link, which again may require stochastic geometry to characterize in a statistical sense for a typical node placement. In short, stochastic geometry can be viewed as a supplement and tool for many approaches to determining network capacity.

When discussing the capacity in its information-theoretic sense, that is as an upper bound to the best possible network throughput, it is important to note that the capacity is not likely to be achieved by a purely random choice of transmitters. Rather, capacity-approaching techniques will almost surely require some degree of cooperation or at least coordination among the contending transmitters, which will degrade the relevance of the 2-D Poisson distribution upon which most results in this tutorial are based. As again highlighted in the conclusions, particularly from the standpoint of understanding network capacity, new stochastic geometric tools that go beyond a homogeneous PPP are urgently needed to better characterize networks with cognition and intelligent transmission scheduling. Some recent results in this direction can be found in [94], [104].

VI. Other Applications: Routing, Information Propagation, Point Processes with Fading, and Secrecy

While the connectivity and capacity have been the main applications of stochastic geometry and random geometric graphs to wireless networks, there have recently been other problems areas where these techniques have led to interesting results. Some of them are briefly described in this section.

A. Routing

While many of the analytical results discussed so far focus on single-hop metrics (outage, single-hop throughput and progress), recently progress has been made toward analyzing routing protocols on a PPP using stochastic geometry tools to evaluate the mean cost of the route and its fluctuations [105]. A typical example is that of greedy forward routing where a transmitter sends a packet to the nearest node which is closer to the packet's destination than the transmitter. In the Poisson case, the geometry of the associated routes can be analyzed thanks to the locality of the definition of the next hop [105]. Another approach is taken in [106] where nearest-neighbor routing in a sector pointed at the destination is compared with routing schemes that use longer hops. In these papers, interference is not taken into account to determine the feasibility of a link. A first attempt to combine routing with an SINR-based link model can be found in [107].

B. Epidemic models; first-passage percolation

Random geometric graphs are useful to model the propagation of information (or disease, fire, or anything else) in a network of randomly placed nodes. Here we briefly cover some broadcasting strategies and elements of first-passage percolation.

We denote by $G^*_{\lambda,r}$ the graph obtained by adding a node at the origin in the standard random geometric graph $G_{\lambda,r}$. As explained in §II-A1, adding this point is equivalent to conditioning $G_{\lambda,r}$ on the presence of a node at the origin.

[2]Asynchronous binary sequences of M ± 1 bits have a cross-correlation variance of $\frac{2}{3M}$, perfectly synchronized sequences have $\frac{1}{M}$ which is actually not as desirable.

1) Broadcasting in multi-hop networks: Consider the scenario where a message is to be broadcast in a static network whose connectivity is represented by the graph $G^*_{\lambda,r}$. Let us assume that the MAC layer prevents collisions perfectly, and that each nodes forwards the message when it receives it for the first time (flooding). Under this algorithm, the message propagates to the entire component to which the source belongs. Therefore, the probability that the message reaches an infinite number of nodes is equal to the probability that the source belongs to an infinite cluster $\theta(\lambda, r)$.

a) Probabilistic broadcast (gossiping): The number of transmission occurring in the above algorithm is exactly equal to the number of nodes who received the message. This number is unnecessarily large, since each transmission reaches all the neighbors of the sender. Thus, each node receives the message from each neighbor while once would be enough. A strategy to reduce the number of transmission is to let the nodes forward the message only with a probability $p < 1$.

The decision whether to forward or not can be made by the nodes before the broadcast starts. Thus, in order to analyze the propagation of the message, one can thin the point process and retain only the nodes who are willing to forward the message (called hereafter *active* nodes). We obtain a thinned PPP of intensity $p\lambda$ on which we can construct restricted graph $G_{p\lambda,r}$. Thus the message originating at the origin reaches an infinite number of nodes if the origin is active and belongs to an infinite cluster of the thinned graph. This happens with probability $\theta_p(\lambda, r) := p\theta(p\lambda, r)$, as the two conditions are independent. As a consequence, if the original graph $G_{\lambda,r}$ is super-critical, there is a critical value for p above which the probability $\theta_p(\lambda, r)$ of a *successful* broadcast (*i.e.*, a broadcast where an infinite number of nodes are reached) is strictly positive.

The next value of interest is the fraction of nodes reached by the message in case of a successful broadcast. Nodes reached by the message include the active nodes that belong to the infinite cluster in $G_{p\lambda,r}$ and the inactive nodes that are within distance r from them. The fraction of nodes belonging to the latter category can be computed as follows: Consider a location x of $\mathbb{R}^2$. If an active node were located at x, it would belong to the infinite cluster of $G_{p\lambda,r}$ with probability $\theta(p\lambda, r)$. This means that the point x is within a distance less than r from a node of the infinite cluster with that probability. Thus, the total fraction of nodes reached by the message is precisely $\theta(p\lambda, r)$, which is a surprisingly simple result.

b) Other models: Probabilistic broadcasting has been studied in [108], and an extension to a model with collisions can be found in [109].

Another variation of the gossiping algorithm, where nodes forward the message only if their node degree is less than a certain threshold, can be addressed using the STIRG model presented in §IV-D3 by using the step function $\ell(x) = 1 - u(x-r)$ as an attenuation function. A sophisticated algorithm to realize degree-dependent activation in a sensor network can be found in [110], where the authors show the existence of a phase transition for the propagation of messages under their algorithm.

2) First-passage percolation: First-passage percolation is a branch of percolation theory that addresses the actual length of the shortest path in percolation models (see *e.g.*, [111] for an introduction). It is useful to compute the propagation speed of messages in a multi-hop network.

a) Asymptotic shape: Consider the graph $G^*_{\lambda,r}$, and define the *hop distance* (also called *chemical distance*) between two nodes as the number of hops on the shortest path between them (or infinity if no such path exists). Let S_k be the set of nodes that are at distance k from the origin. We expect that the shape of S_k is relatively circular around the origin. The following theorem confirms this intuition:

Theorem 3 (see, *e.g.*, [112]) *There is a $\mu > 0$ such that for any $0 < \varepsilon < 1$ almost surely*

$$S_k \subset B(o; (1+\varepsilon)k\mu) \backslash B(o; (1-\varepsilon)k\mu)$$

for all sufficiently large k.

b) Blinking model: First-passage percolation can also be used to assess the speed of propagation of a message in a dynamic model. An example is given in [113], where nodes alternate between active and sleep mode in a random and independent fashion: At any instant, only a fraction $f < 1$ of the nodes are active, so that the connectivity graph is $G_{f\lambda,r}$. As the message is emitted by the source, it instantaneously propagates to all active nodes that are connected to the source. If $f\lambda$ is below the critical density, the initial propagation is a.s. limited to a finite number of nodes. Then, the propagation continues as further nodes switch to active mode. First-passage percolation allows to show that in this case, the asymptotic shape the the area where the message has propagated after a time t is still circular for large t despite the sub-criticality of the graph at each instant.

In the ALOHA case, initial results on the propagation speed in interference-limited ad hoc networks can be found in [114], [115]. It is shown that a similar shape theorem holds as in the interference-free case (Th. 3).

C. Point processes with fading

The path loss over a wireless link is well modeled by the product of a distance component (often called large-scale path loss) and a fading component (called small-scale fading or shadowing). It is usually assumed that the distance part is deterministic while the fading part is modeled as a random process. This distinction, however, does not apply to many types of wireless networks, where the distance itself is subject to uncertainty. In this case it may be beneficial to consider the distance and fading uncertainty jointly, *i.e.*, to define a PP that incorporates both.

We introduce a framework that offers such a geometrical interpretation of fading and some new insight into its effect on the network.

Let $\{Y_i\}$, $i \in \mathbb{N}$, be a stationary Poisson point process in $\mathbb{R}^d$ of intensity 1, and define the *path loss point process* (before fading) as $\Phi = \{X_i \triangleq \|Y_i\|^\alpha\}$ for a path loss exponent α. Let $\{h, h_1, h_2, \ldots\}$ be an iid stochastic process with h drawn

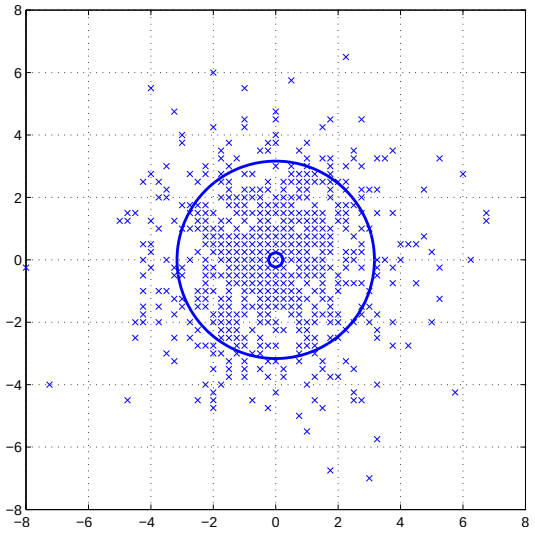

Fig. 4. A Poisson point process of intensity 1 in a 16×16 square. The reachable nodes by the center node are indicated by a bold $\times$ for a path gain threshold of $s = 0.1$, a path loss exponent of $\alpha = 2$, and Rayleigh fading (standard network). The circle indicates the range of successful transmission in the non-fading case. Its radius is $1/\sqrt{s} \approx 3.16$, and there are about $\pi/s \approx 31$ nodes inside.

from a distribution $F \triangleq F_h$ with unit mean and let $\Xi = \{\xi_i \triangleq X_i/h_i\}$ be the *path loss process with fading*.

Assume that there is a transmitter at the origin, and all other nodes are receivers. So there is no interference, and the network is purely noise-limited. Nodes can receive the transmission if their path loss ξ_i is smaller than $1/s$. These nodes are said to be connected to the origin. The processes of connected points are denoted by $\hat{\Phi}$ and $\hat{\Xi}$, respectively. So

$$\hat{\Phi} = \{X_i \in \Phi \colon \xi_i < 1/s\}\,; \quad \hat{\Xi} = \Xi \cap [0, 1/s)\,.$$

Fig. 4 shows a PPP of intensity 1 in a 16×16 square, with the nodes marked that can be reached from the center, assuming a path gain threshold of $s = 0.1$. The disk shows the maximum transmission distance in the non-fading case.

Since Φ and Ξ are constructed from a uniform PPP using mapping and independent random scaling (fading), the processes Φ, $\hat{\Phi}$, Ξ, and $\hat{\Xi}$ are Poisson. We would like to find the number of connected nodes and their expected sum-distance $\mathbb{E}(\sum_{X\in\hat{\Phi}} X^{1/\alpha})$ which may also be also termed the *broadcast transport sum-distance*.

1) Connectivity: Using the Nakagami-m fading model, we have [116]

- $\hat{\Phi}$ is Poisson with $\hat{\lambda}(x) = \lambda(x)(1 - F(sx))$ (independent thinning).
- With Nakagami-m fading, the number $\hat{N} = \hat{\Phi}(\mathbb{R}^+)$ of connected nodes is Poisson with mean

$$\mathbb{E}\hat{N}_m = \frac{c_d}{(ms)^\delta} \frac{\Gamma(\delta + m)}{\Gamma(m)}\,, \tag{19}$$

where $\delta = d/\alpha$ as before, and the *connectivity fading gain*, defined as the ratio of the expected numbers of connected nodes with and without fading, is

$$\frac{\mathbb{E}\hat{N}_m}{\mathbb{E}\hat{N}_\infty} = \frac{1}{m^\delta} \frac{\Gamma(\delta + m)}{\Gamma(m)} = \mathbb{E}(h^\delta)\,. \tag{20}$$

2) Broadcast transport sum-distance and capacity: The *broadcast transport sum-distance* D, *i.e.*, the expected sum over the all the distances $\hat{X}_i^{1/\alpha}$ from the origin is defined as

$$D_m \triangleq \mathbb{E}\left(\sum_{X\in\hat{\Phi}} X^{1/\alpha}\right)\,. \tag{21}$$

Using Campbell's theorem (5), it can be shown that the broadcast transport sum-distance for Nakagami-m fading is

$$D_m = c_d \frac{\delta}{\Delta} \frac{1}{(ms)^\Delta} \frac{\Gamma(m + \Delta)}{\Gamma(m)}\,, \tag{22}$$

where $\Delta = (d+1)/\alpha$ and the (broadcast) *fading gain* D_m/D_∞ is

$$\frac{D_m}{D_\infty} = \frac{1}{m^\Delta} \frac{\Gamma(m + \Delta)}{\Gamma(m)} = \mathbb{E}(h^\Delta)\,. \tag{23}$$

So the fading gain D_m/D_∞ is the Δ-th moment of h.

To obtain the (broadcast) transport capacity, we may multiply D by the rate of transmission R. Assuming at-capacity signaling, $R = \log_2(1 + s)$. When maximizing the product DR over R (or s), we find that an optimum rate only exists if $\Delta \leqslant 1$. If $\Delta > 1$ (or $\alpha < d + 1$), DR can be made arbitrarily large by letting the rate go to zero — irrespective of the transmit power! This follows from $R(s) = \Theta(s)$ and $D(s) = \Theta(s^{-\Delta})$ as $s \to 0$. So $DR = \Theta(s^{1-\Delta})$ which diverges if $\Delta > 1$.

D. Secrecy

There has been growing interest in information-theoretic secrecy in wireless networks. To study the impact of the secrecy constraint on the connectivity of ad hoc networks, we introduce a new type of random geometric graph, the so-called *secrecy graph*, that represents the network or communication graph including only links over which secure communication is possible. We assume that a transmitter can choose the rate very close to the capacity of the channel to the intended receiver, so that any eavesdropper further away than the receiver cannot intercept the message. This translates into a simple geometric constraint for secrecy which is reflected in the secrecy graph. Here we describe some of the properties of the secrecy graph.

Let $\hat{G}_r = (\Phi, \hat{E})$ be a disk graph in $\mathbb{R}^d$ (see §IV-C), where $\Phi = \{X_i\} \subset \mathbb{R}^d$ is a PPP of intensity 1 representing the locations of the nodes, also referred to as the "good guys". We can think of this graph as the unconstrained network graph that includes all possible edges over the good guys could communicate if there were no secrecy constraints.

Take another PPP $\Psi = \{Y_i\} \subset \mathbb{R}^d$ of intensity λ representing the locations of the eavesdroppers or "bad guys". These are assumed to be known to the good guys.

Based on $\hat{G}$, we define the following secrecy graphs:

The directed secrecy graph: $\vec{G} = (\phi, \vec{E})$. Replace all edges in $\hat{E}$ by two directional edges. Then remove all edges $\overrightarrow{X_iX_j}$ for which $\Psi\left(B(X_i; \|X_i - X_j\|)\right) > 0$, *i.e.*, there is at least one eavesdropper in the ball.

From this directed graph, two undirected graphs are derived:

The basic secrecy graph: $G_{\lambda,r} = (\Phi, E)$, where the (undirected) edge set E is

$$E \triangleq \{X_i X_j \,:\, \overrightarrow{X_i X_j} \in \vec{E} \text{ and } \overrightarrow{X_j X_i} \in \vec{E}\}\,.$$

The enhanced secrecy graph: $G'_{\lambda,r} = (\phi, E')$, where

$$E' \triangleq \{X_i X_j \,:\, \overrightarrow{X_i X_j} \in \vec{E} \text{ or } \overrightarrow{X_j X_i} \in \vec{E}\}\,.$$

With $\theta(\lambda, r)$ being the probability that the component in $G_{\lambda,r}$ containing the origin (or any arbitrary fixed node) is infinite, we know from §IV-C that $\theta(0, r) > 0$ for $r > r_c$, where $r_c \approx 1.198$ is the critical radius for percolation of the (unrestricted) disk graph. For radii larger than r_c, we define

$$\lambda_c(r) \triangleq \inf\{\lambda \,:\, \theta(\lambda, r) = 0\}\,, \quad r > r_c, \tag{24}$$

as the critical density for percolation on the secrecy graph.

Some of the properties of these secrecy graphs can be analytically determined, such as the out-degree in directional graph $\vec{G}_{\lambda,\infty}$ which turns out to be geometric with mean $1/\lambda$. This is easily seen if we consider the sequence of nearest neighbors of o in the combined process $\Phi \cup \Psi$. $N^{\text{out}} = n$ if the closest n are in Φ and the $(n+1)$-st is in Ψ. Since these are independent events,

$$\mathbb{P}[N^{\text{out}} = n] = \frac{\lambda}{1+\lambda}\left(\frac{1}{1+\lambda}\right)^n\,. \tag{25}$$

The node degree distribution for $r < \infty$ can also be found analytically; it is a distribution that includes the Poisson and geometric distributions as special cases. As $\lambda \to 0$ it is Poisson (standard disk graph), while for $r \to \infty$ it is geometric (this is the case considered above). Depending on the values of r and λ, we can identify a power-limited and a secrecy-limited operating regime.

Further results and bounds on the percolation threshold for the secrecy graph are given in [117]. It turns out that $\lambda = 0.15$ already makes percolation impossible. Hence a small density of eavesdroppers can have a drastic impact on the connectivity properties of the network.

VII. Concluding Remarks

In this tutorial article we have argued that stochastic geometry and random graph theory are indispensable tools for the analysis of wireless networks that allow analytical results on a number of concrete and important problems. We have shown how to apply these tools to model and quantify interference, connectivity, outage probability, throughput, and capacity of wireless networks. We have also argued that there are many other possible future applications of these techniques, a few of which are mentioned in §VI, and in the accompanying special issue, highly mobile wireless networks [118] and information propagation [119], [120] (also discussed in §VI).

Because the techniques presented in this paper have emerged from the fields of applied probability, point processes, queueing theory, percolation theory, and are now being adapted to the engineering arena, it is important that cross-disciplinary dialogues continue, in part through dedicated journal issues like the present one and workshops like SpaSWiN[3]. We also hope that an increasing number of engineering graduate programs will offer formal classroom training in these methods.

Many important areas for future study remain. In order to better model cooperative wireless networks – including techniques as basic as carrier sensing and as sophisticated as interference alignment and network coding – tractable results or approximations that go beyond the stationary Poisson distribution for the node locations are highly desirable but presently lacking. The capacity of wireless networks is one of the most important open problems in information theory, and stochastic geometry and random graphs appear destined to play a key role in characterizing it, given the primacy of network geometry in determining interference and hence achievable rates. Present attempts at determining achievable end-to-end rates using these tools are still at an early stage.

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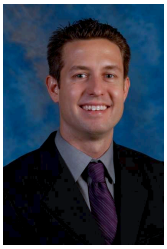

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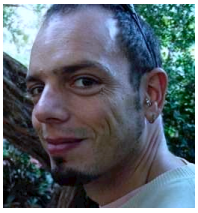

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