Interference Statistics of a Poisson Field of Interferers with Random Puncturing

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***Abstract*—Assume that the transmitters in a random wireless network are distributed according to a Poisson Point Process (PPP) and that the receiver is interested in the signal transmitted by its kth nearest transmitter where $k \in \mathbf{T}$ and $\mathbf{T}$ denotes the set of desired transmitters. Excluding the desired transmitter(s) is equivalent to puncturing the points corresponding to the desired transmitters from the PPP which results in the field of interferers. In this paper, by assuming that the field of interferers keeps its Poisson property, we approximate the cumulants of interference. We show that to find the cumulants, the joint statistics of internodal distances in the PPP are required which we find in closed form. Simulation results show that the obtained analytical derivations provide very good approximation to the interference statistics.**

I. Introduction

Assume that the nodes in a wireless network are distributed according to a Poisson Point Process (PPP). This is a reasonable assumption when the movements of nodes in a mobile network are uncorrelated or when the nodes are deployed randomly (for example, in a wireless sensor network). The field of transmitters in this scenario will remain a PPP when ALOHA MAC protocol is used in the network [1] or can be closely approximated by a PPP when other MAC protocols like CSMA is used [2]. Let us denote this PPP as Φ. Assume that the receiver is interested in the signal transmitted by its kth nearest transmitter where $k \in \mathbf{T}$ and $\mathbf{T}$ shows the set of desired transmitters. By excluding one or few of the nodes as the desired transmitter(s), and since the locations of the desired transmitter(s) are random, the field of interferers will be a randomly punctured version of Φ. Examples for the case of multiple desired transmitters include when these desired nodes are engaged in cooperative transmission and/or relaying to the receiver or when the receiver is equipped with a multi-user detector and wishes to detect the signals transmitted by all of these desired nodes.

A. Related Work and Motivation

The majority of works for interference modeling in random wireless networks assume that the interferers are distributed according to a PPP and isolate the desired transmitter from the point process. The desired transmitter is assumed to have a deterministic distance to the receiver and to be separate from the PPP (e.g., [3]-[5]). The aggregate interference originated from a Poisson field of interferers can be modeled as a random sum over the PPP. Closed form results exist for the characteristic function of such random sums defined over a PPP (see [6] for example). Therefore, by taking advantage of the tractability of Poisson processes, closed form results can be obtained for interference statistics.

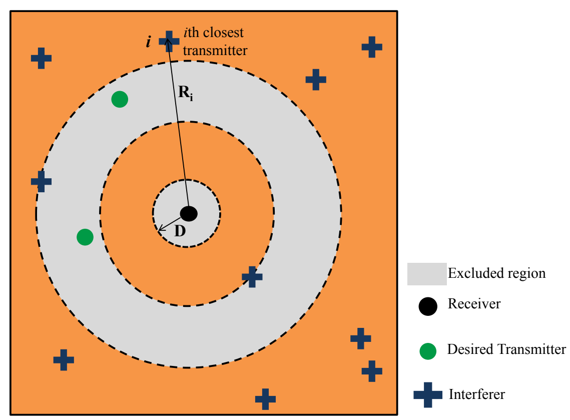

Fig. 1: Random region encompassing the interferers. The guard zone is a disc with radius D around the receiver.

In a more realistic scenario, the desired transmitter (or transmitters) also belongs to the point process that models the field of concurrent transmitters. In fact, with a PPP assumption for the spatial distribution of wireless nodes, the field of concurrent transmitters will remain a PPP with an ALOHA MAC scheme or can be closely approximated as PPP using CSMA (see [1] and [2]). By excluding the desired transmitter(s), the interference is coming from a randomly punctured PPP.

We consider a region with random boundaries which encompasses all of the interferers. By conditioning on the boundaries of this region, we approximate the conditional characteristic function of interference with the simplifying assumption that the nodes' distribution in the region keeps its Poisson property and Campbell's theorem can be applied to find the characteristic function of interference. Subsequently, we find the conditional cumulants and use the law of total cumulance (see [7]) to find the cumulants. We show that this requires the joint statistics of the distances between the transmitters and the receiver which we obtain in closed form and use them to obtain the cumulants.

II. System Model

We assume that a PPP with density μ models the locations of the nodes in a wireless network, and we focus on the interference at the receiver. With an ALOHA MAC protocol, the field of concurrent transmitters will be a PPP with smaller density $\lambda = p\mu$ where p is the the transmission attempt probability of ALOHA [1]. Let us index the receiver as node 0 and the ith closest transmitter to the receiver as node i. Without loss of generality, we can assume that the receiver is located at the origin. Also, assume that there is a guard zone around the receiver which is a disc with radius D and the receiver at its center (See Fig. 1). No node can transmit if it is located inside the guard zone. The guard zone is necessary to avoid excessively high interference power, generated by very close neighbors, at the receiver. In practice, nodes can identify whether their distance to the receiver is smaller than D by measuring the power level of the control packets sent by the receiver [8]. We denote the distance of node i to the receiver as R_i and the set of desired transmitters for the receiver as $\mathbf{T}$.

We exclude the guard zone as well as the annular regions with inner and outer radii of R_{k-1} and R_{k+1} for any $k \in \mathbf{T}$ from the 2-D Euclidean space. The resulting region, denoted by $\mathcal{S}$, contains all of the interferers (See Fig. 1). Assuming that all of the interferers transmit with the same power level and using a power-law path loss model, the normalized aggregate interference power can be written as

$$I = \sum_{i \in \mathcal{S}} h_i R_i^{-\alpha}. \tag{1}$$

where α is the path loss exponent, h_i denotes the fading on the channel from node i to the receiver and we assume that $\{h_i\}$ are i.i.d.

As the inner and outer radii of annular regions are random variables, $\mathcal{S}$ has random boundaries. Depending on the indices of the desired transmitters, the annular regions may overlap. For example, for $\mathbf{T} = \{4, 5\}$, the regions: $\{R_3 \le r \le R_5\}$ and $\{R_4 \le r \le R_6\}$ overlap and the excluded region, which is the union of these annuli, is $\{R_3 \le r \le R_6\}$. Considering this possible overlap, the excluded region consists of M disjoint annuli where M is less than or equal to the number of desired transmitters. We denote the inner and outer radii of kth annulus as $R_{k,l}$ and $R_{k,u}$. The boundaries of region $\mathcal{S}$ are therefore $\mathbf{B} = \bigcup_{k=1}^{M} \{R_{k,l}, R_{k,u}\}$. For example, for $\mathbf{T} = \{4, 5\}$, we have $M = 1$ and $\mathbf{B} = \{R_3, R_6\}$ or for $\mathbf{T} = \{4, 7\}$, $\{R_3 \le r \le R_5\}$ and $\{R_6 \le r \le R_8\}$ are excluded, $M = 2$ and $\mathbf{B} = \{R_3, R_5, R_6, R_8\}$.

III. Interference Statistics

The interference defined in (1) is a random sum over the punctured PPP which is distributed in the region $\mathcal{S}$. The number of points in $\mathcal{S}$ is therefore Poisson minus a deterministic number (i.e., the number of desired transmitters)[1]. Since there is no known result for finding statistics of random sums over non-Poisson point processes, we define a random variable $\hat{I}$ as an approximation for the interference assuming that the distribution of nodes inside $\mathcal{S}$ preserves its Poisson property and Campbell's theorem can be applied to obtain the characteristic function. Conditioning on $\mathbf{B}$ and using Campbell's Theorem [6], the conditional characteristic function of $\hat{I}$ can be written as

$$\Psi_{\hat{I}|\mathbf{B}}(\omega) = E\{e^{j\omega\hat{I}}|\mathbf{B}\} = \exp\left(2\pi\lambda \int_h \int_\Upsilon \left[\exp(j\omega h r^{-\alpha}) - 1\right] r dr f_h(h) dh\right) \tag{2}$$

where $\Upsilon = [D, \infty) - \bigcup_{k=1}^{M} (R_{k,l}, R_{k,u})$ and $f_h(h)$ is the pdf of the fading. The conditional cumulants of $\hat{I}$ can be obtained from (2) as[2]

$$\kappa_n(\hat{I}|\mathbf{B}) = \frac{1}{j^n} \left[\frac{\partial^n \ln \Psi_{\hat{I}|\mathbf{B}}(\omega)}{\partial \omega^n}\right]_{\omega=0} = 2\pi\lambda E\{h^n\} \int_\Upsilon r^{1-n\alpha}\, dr = \frac{2\pi\lambda E\{h^n\}}{n\alpha - 2} Y_n \tag{3}$$

where $Y_n = D^{2-n\alpha} + \sum_{i=1}^{M} (R_{i,u}^{2-n\alpha} - R_{i,l}^{2-n\alpha})$. Cumulants of $\hat{I}$ can be obtained from law of total cumulance. Using the corollary in [7], $\kappa_n(\hat{I})$ is found as

$$\kappa_n(\hat{I}) = \sum \frac{n!}{\mu_1!\mu_2!\cdots} \frac{1}{(p_1!)^{\mu_1}(p_2!)^{\mu_2}\cdots} \times \kappa\{\underbrace{\kappa_{p_1}(\hat{I}|\mathbf{B}),}_{\mu_1 \text{ times}} \underbrace{\kappa_{p_2}(\hat{I}|\mathbf{B}),}_{\mu_2 \text{ times}} \cdots\} \tag{4}$$

where the summation extends over all partitions of n such that $p_1\mu_1 + p_2\mu_2 + \cdots = n$.

The expected value of $\hat{I}$ (i.e., $\kappa_1(\hat{I})$) can be found as

$$E\{\hat{I}\} = E\{E\{\hat{I}|B\}\} = \frac{2\pi\lambda E\{h\}}{\alpha - 2} E\{Y_1\} \tag{5}$$

where $E\{Y_1\} = \left(D^{2-\alpha} + \sum_{i=1}^{M} E\{R_{i,u}^{2-\alpha}\} - E\{R_{i,l}^{2-\alpha}\}\right)$. In [9], the pdf of R_i is found to be generalized gamma, and $E\{R_i^\beta\}$ is found in [9, 10] in closed form:

$$E\{R_i^\beta\} = \frac{1}{(\lambda\pi)^{\beta/2}} \frac{\Gamma(i + \beta/2)}{\Gamma(i)} \tag{6}$$

where β can be any real number such that $i + \beta/2 > 0$.

For the variance of $\hat{I}$, the law of total cumulance in (4)

[1]This is unlike the case that the boundaries are deterministic and the number of points in both excluded and non-excluded regions are Poisson [6].

[2]The integral in (3) converges for $2 - n\alpha < 0$ which holds if $\alpha > 2$. This condition is always hold except for free space propagation model for which $\alpha = 2$.

degenerates to the law of total variance:

$$\begin{aligned}\text{var}\{\hat{I}\} &= E\left\{\text{var}\{\hat{I}|B\}\right\} + \text{var}\left\{E\{\hat{I}|B\}\right\} \\ &= \frac{2\pi\lambda E\{h^2\}}{2\alpha-2}E\{Y_2\} + \left(\frac{2\pi\lambda E\{h\}}{\alpha-2}\right)^2 \text{var}\{Y_1\} \end{aligned} \quad (7)$$

where

$$\begin{aligned}E\{Y_2\} &= \frac{2\pi\lambda E\{h^2\}}{2\alpha-2} \\ &\times \left(D^{2-2\alpha} + \sum_{i=1}^{M}(E\{R_{i,u}^{2-2\alpha}\} - E\{R_{i,l}^{2-2\alpha}\})\right) \end{aligned} \quad (8)$$

and

$$\text{var}\{Y_1\} = \left(\frac{2\pi\lambda E\{h\}}{\alpha-2}\right)^2 \text{var}\left\{\sum_{i=1}^{M}(R_{i,l}^{2-\alpha} - R_{i,u}^{2-\alpha})\right\} \quad (9)$$

As can be concluded from (7) and (9), to find the variance of $\hat{I}$, the joint statistics of R_i and R_j, $i \neq j$, will be required. The Joint statistics will also be required to find higher cumulants of $\hat{I}$.

A. Joint Statistics of distances in PPP

It is known that if nodes are distributed according to a two-dimensional Poisson point process with density λ, the squared ordered distances from the receiver have the same distribution as the arrival times of a one-dimensional PPP with density $\lambda\pi$ (see [11] and [12]). Consequently, for any set of indices $\{l_1, l_2, \cdots, l_n\}$ where $l_1 < l_2 \cdots < l_n$, the joint pdf $f_{R_{l_1}^2, R_{l_2}^2, \cdots R_{l_n}^2}(x_1, x_2, \cdots, x_n)$ can be found as follows:

$$\begin{aligned}&f_{R_{l_1}^2, R_{l_2}^2, \cdots R_{l_n}^2}(x_1, x_2, \cdots, x_n) = \\ &f_{R_{l_1}^2, R_{l_2}^2 - R_{l_1}^2, \cdots, R_{l_n}^2 - R_{l_{n-1}}^2}(x_1, x_2 - x_1, \cdots, x_n - x_{n-1}) = \\ &\frac{(\lambda\pi)^{l_n}}{\Gamma(l_1)\Gamma(l_2 - l_1)\cdots\Gamma(l_n - l_{n-1})} \times \\ &x_1^{l_1 - 1}(x_2 - x_1)^{l_2 - l_1 - 1} \cdots (x_n - x_{n-1})^{l_n - l_{n-1} - 1} \times \\ &e^{-\lambda\pi x_n}\end{aligned} \quad (10)$$

which is found using $R_{l_1}^2$ and $R_{l_i}^2 - R_{l_{i-1}}^2$, $i > 2$ are independent Erlang random variables with rate parameter $\lambda\pi$ and shape parameters l_1 and $l_i - l_{i-1}$ respectively[3]. Using this joint pdf, we find $E\left\{R_{l_1}^\beta R_{l_2}^\beta \cdots R_{l_n}^\beta\right\}$.

Proposition 1. *We have*

$$\begin{aligned}&E\left\{R_{l_1}^\beta R_{l_2}^\beta \cdots R_{l_n}^\beta\right\} = \int_0^\infty \int_0^{x_n} \cdots \int_0^{x_2} x_1^{\beta/2} x_2^{\beta/2} \cdots x_n^{\beta/2} \\ &\times f_{R_{l_1}^2, R_{l_2}^2, \cdots R_{l_n}^2}(x_1, x_2, \cdots, x_n) dx_1 \cdots dx_n = \\ &\frac{1}{(\lambda\pi)^{n\beta/2}} \prod_{k=1}^{n-1} \frac{\Gamma(l_k + k\beta/2)}{\Gamma(l_k + (k-1)\beta/2)}\end{aligned} \quad (11)$$

[3]In a one-dimensional Poisson process with density $\lambda\pi$, the inter-arrival times are i.i.d exponential random variables with mean $\frac{1}{\lambda\pi}$ and sums of inter-arrival times are Erlang distributed.

Proof: We let $A = \frac{(\lambda\pi)^{l_n}}{\Gamma(l_1)\Gamma(l_2 - l_1)\cdots\Gamma(l_n - l_{n-1})}$. We then have

$$\begin{aligned}&E\left\{R_{l_1}^\beta R_{l_2}^\beta \cdots R_{l_n}^\beta\right\} = \\ &A \int_0^\infty \int_0^{x_n} \cdots \int_0^{x_2} x_1^{\beta/2} x_1^{l_1 - 1}(x_2 - x_1)^{l_2 - l_1 - 1} \cdots \\ &\times x_{n-1}^{\beta/2}(x_n - x_{n-1})^{l_n - l_{n-1} - 1} x_n^{\beta/2} e^{-\lambda\pi x_n} dx_1 \cdots dx_{n-1} dx_n\end{aligned}$$

We first find

$$\int_0^{x_2} x_1^{\beta/2} x_1^{l_1 - 1}(x_2 - x_1)^{l_2 - l_1 - 1} dx_1 = x_2^{l_2 + \beta/2 - 1} B(l_1 + \beta/2, l_2 - l_1)$$

where $B(.)$ is the beta function. Using induction,

$$\begin{aligned}&\int_0^{x_n} \cdots \int_0^{x_2} x_1^{\beta/2} x_1^{l_1 - 1}(x_2 - x_1)^{l_2 - l_1 - 1} \cdots \\ &\times x_{n-1}^{\beta/2}(x_n - x_{n-1})^{l_n - l_{n-1} - 1} dx_1 \cdots dx_{n-1} = \\ &B(l_1 + \beta/2, l_2 - l_1) \cdots \times B(l_{n-1} + (n-1)\beta/2, l_n - l_{n-1}) \\ &\times x_n^{l_n + (n-1)\beta/2 - 1}\end{aligned}$$

and

$$\begin{aligned}&E\left\{R_{l_1}^\beta R_{l_2}^\beta \cdots R_{l_n}^\beta\right\} = AB(l_1 + \beta/2, l_2 - l_1)B(l_2 + \beta, l_3 - l_2) \cdots \\ &\times B(l_{n-1} + (n-1)\beta/2, l_n - l_{n-1}) \int_0^\infty x_n^{l_n + n\beta/2 - 1} e^{-\lambda\pi x_n} dx_n\end{aligned}$$

Noting that

$$\int_0^\infty x_n^{l_n + n\beta/2 - 1} e^{-\lambda\pi x_n} dx_n = \frac{\Gamma(l_n + n\beta/2)}{(\lambda\pi)^{l_n + n\beta/2}},$$

and using the equality $B(x, y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$, the result in (11) is found after simplification. ■

B. Single Desired Transmitter Scenario

Let us consider the case of a single desired transmitter, i.e., $T = \{i\}$ for some i. In this case: $Y_n = D^{2-n\alpha} + R_{i+1}^{2-n\alpha} - R_{i-1}^{2-n\alpha}$. The mean interference can be found from (5) as

$$E\{\hat{I}\} = \frac{2\pi\lambda E\{h\}}{\alpha - 2}\left(D^{2-\alpha} + E\{R_{i+1}^{2-\alpha}\} - E\{R_{i-1}^{2-\alpha}\}\right) \quad (12)$$

and $E\{R_{i+1}^{2-\alpha}\}$ and $E\{R_{i-1}^{2-\alpha}\}$ are known from (6). The variance of the interference can be found as:

$$\text{var}\{\hat{I}\} = \frac{2\pi\lambda E\{h^2\}}{2\alpha - 2}E\{Y_2\} + \left(\frac{2\pi\lambda E\{h\}}{\alpha - 2}\right)^2 \text{var}\{Y_1\} \quad (13)$$

where

$$E\{Y_2\} = D^{2-2\alpha} + E\{R_{i+1}^{2-2\alpha}\} - E\{R_{i-1}^{2-2\alpha}\}$$

and

$$\begin{aligned}\text{var}\{Y_1\} &= \text{var}\{D^{2-\alpha} + R_{i+1}^{2-\alpha} - R_{i-1}^{2-\alpha}\} \\ &= \text{var}\{R_{i+1}^{2-\alpha}\} + \text{var}\{R_{i-1}^{2-\alpha}\} - 2\text{cov}\{R_{i-1}^{2-\alpha}, R_{i+1}^{2-\alpha}\}.\end{aligned}$$

We can find $E\{Y_2\}$, $\text{var}\{R_{i-1}^{2-\alpha}\}$ and $\text{var}\{R_{i+1}^{2-\alpha}\}$ from (6) and $\text{cov}\{R_{i-1}^{2-\alpha}, R_{i+1}^{2-\alpha}\}$ can be found from (11) and (6).

Higher order cumulants of $\hat{I}$ can similarly be found by application of law of total cumulance and using (4). To find these cumulants, the joint statistics of R_{i-1} and R_{i+1} will be required which can be found from (11).

C. Multiple Desired Transmitters Scenario

For multiple desired transmitters, the excluded region consists of M disjoint annuli where M is less than or equal to the number of transmitters. For the case of a single annulus, say $(R_{1,l} \leq r \leq R_{1,u})$, the mean and variance of the interference can be found using the results in previous subsection with $R_{1,l}$ and $R_{1,u}$ replaced by R_{i-1} and R_{i+1}. For multiple annuli, we consider an example and find the mean and variance of the interference.

Let us consider $T = \{4, 7\}$. In this case, the excluded region is $(R_3, R_5) \cup (R_6, R_8)$. The mean and variance of the interference can be found as

$$E\{\hat{I}\} = \frac{2\pi\lambda E\{h\}}{\alpha - 2} \times \left(D^{2-\alpha} + E\{R_5^{2-\alpha}\} - E\{R_3^{2-\alpha}\} + E\{R_8^{2-\alpha}\} - E\{R_6^{2-\alpha}\}\right) \tag{14}$$

and

$$\text{var}\{\hat{I}\} = \frac{2\pi\lambda E\{h^2\}}{2\alpha - 2} E\{Y_2\} + \left(\frac{2\pi\lambda E\{h\}}{\alpha - 2}\right)^2 \text{var}\{Y_1\} \tag{15}$$

where

$$E\{Y_2\} = E\{D^{2-2\alpha} + R_5^{2-2\alpha} - R_3^{2-2\alpha} + R_8^{2-2\alpha} - R_6^{2-2\alpha}\}$$

and

$$\text{var}\{Y_1\} = \text{var}\{R_5^{2-\alpha} - R_3^{2-\alpha} + R_8^{2-\alpha} - R_6^{2-\alpha}\}$$

which can be obtained using (6) and (11).

IV. Numerical Results

We consider a receiver located at the origin and a PPP with density $\lambda = 0.1$ which models the locations of the desired transmitter(s) and the interferers. A guard zone with radius $D = 1$ is considered around the receiver, and no node inside the zone is allowed to transmit. We consider Rayleigh fading and model h as an exponential random variable with parameter 1[4]. Using this model, we have $E\{h^n\} = n!$. We generate a new realization of PPP at each step of simulation, exclude the desired transmitter(s) along with the nodes with distances closer than D to the receiver from the point process and measure the interference at the receiver. Interference statistics are then found empirically and compared with the analytical derivations.

In Fig. 2, we show analytical and simulation results for $E\{R_3^\beta R_6^\beta\}$ for different values of β. We have considered other combinations of distances as well and the result found in (11) match with the simulation.

In Figures 3 and 4, assuming that there is a single desired transmitter ($T = \{5\}$, i.e. the 5th closest neighbor to the receiver), we show the analytical results for $E\{\hat{I}\}$ and var$\{\hat{I}\}$ (using (12) and (13)) along with the simulation results for mean and variance of interference. The results indicate that obtained approximation for interference statistics match very well with the simulation results.

[4]h (the power gain) is the square of a Rayleigh random variable and exponentially distributed.

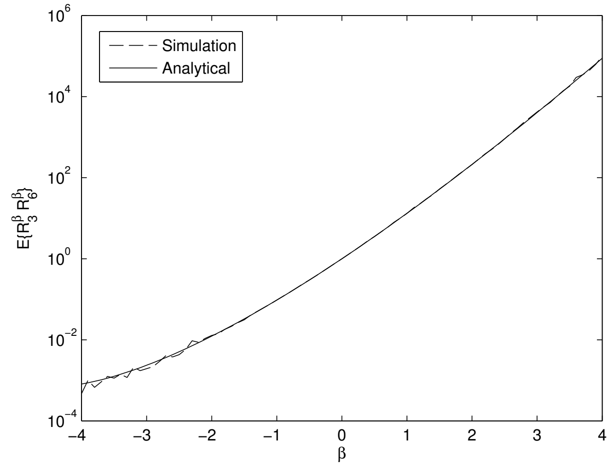

Fig. 2: Analytical and Simulation results for $E\{R_3^\beta R_6^\beta\}$

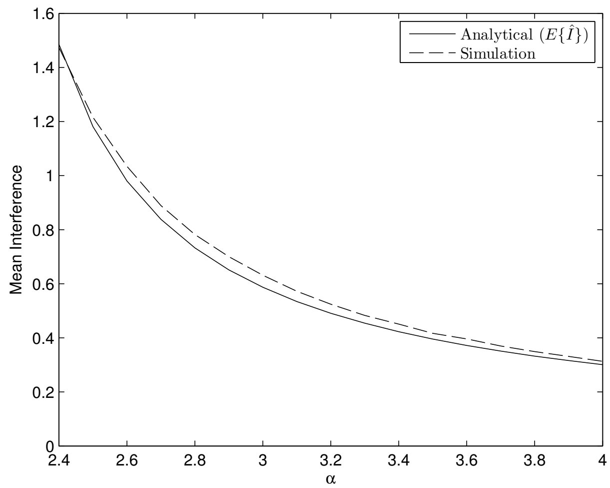

Fig. 3: Analytical and simulation results for mean of interference vs. path loss exponent for $T = \{5\}$

In Figures 5 and 6, simulation results for the mean and the variance of interference are compared with the analytical approximation results assuming $T = \{4, 7\}$, i.e. for two desired transmitters. Results indicate that the interference statistics are very well approximated for both single as well as multiple desired transmitters scenarios.

V. Conclusions

In random wireless network, in which the desired transmitter(s) as well as the interferers all belong to the same Poisson point process (PPP), the field of interferers is a randomly punctured version of the PPP. For example, multiple nodes may cooperatively transmit to the receiver or the receiver may

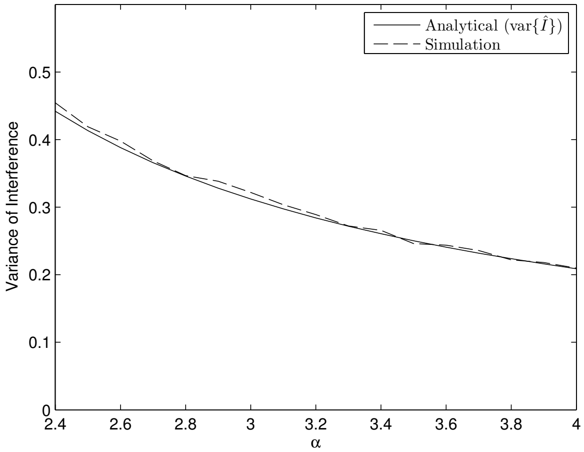

Fig. 4: Analytical and simulation results for variance of interference vs. path loss exponent for $T = \{5\}$

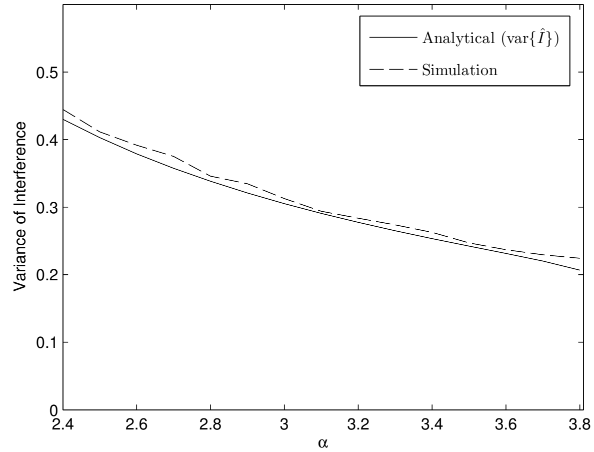

Fig. 6: Analytical and simulation results for variance of interference vs. path loss exponent for $T = \{4, 7\}$

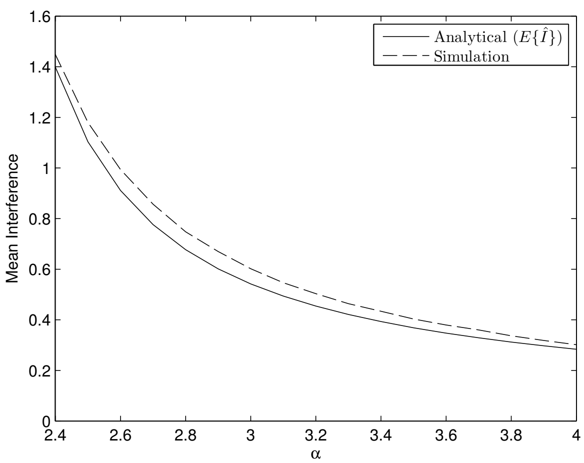

Fig. 5: Analytical and simulation results for mean of interference vs. path loss exponent for $T = \{4, 7\}$

be equipped with multiuser detector and wishes to detect the signals transmitted by multiple desired transmitters. Considering a region with random boundaries which encompasses all of the interferers, we first approximate the characteristic function of the interference assuming that nodes' distribution in this region preserves the Poisson property. We then obtain the conditional characteristic function and cumulants of the approximated interference. The law of total cumulance is then invoked to obtain the cumulants. Joint statistics of internodal distances in the PPP will be required for this purpose which we find in closed form. The approximated interference statistics are shown to match very well with the simulation results.

References

[1] M. Haenggi and R. K. Ganti, "Interference in Large Wireless Networks", *Foundations and Trends in Networking*, vol. 3, no. 2, pp. 127-248, NOW Publishers.

[2] M. Haenggi, "Mean Interference in Hard-Core Wireless Networks," to appear in *IEEE Comm. Letters*.

[3] E. Sousa, "Performance of a Spread Spectrum Packet Radio Network Link in a Poisson Field of Interferers," *IEEE Trans. Inf. Theory,* vol. 38, no.6, pp. 1743-1754, 1992.

[4] A. Babaei and B. Jabbari, "Interference Modeling and Avoidance in Spectrum Underlay Cognitive Wireless Networks," In *Proceedings of ICC 2010*, pp. 1-5, May 2010.

[5] K. Gulati, B. L. Evans, J. G. Andrews and K. R. Tinsley, "Statistics of Co-channel Interference in a Field of Poisson and Poisson-Poisson Clustered Interferers," *IEEE Trans. Signal Process.*, vol. 58, no. 12, Dec. 2010.

[6] J. F. C. Kingman, *Poisson Processes*, Oxford University Press, 1993.

[7] D. Brillinger, "The Calculation of Cumulants via Conditioning", *Annals of the Institute of Statistical Mathematics*, vol. 21, pp. 215-218, 1969.

[8] A. Hassan and J. Andrews, "The Guard Zone in Wireless Ad hoc Networks," *IEEE Trans. Wireless Comm.*, vol. 6, no. 3, pp. 897-906, 2007.

[9] M. Haenggi, "On Distances in Uniformly Random Networks," *IEEE Trans. Inf. Theory*, vol. 51, no. 10, pp. 3584-3586, Oct. 2005

[10] A. Babaei and B. Jabbari, "Distance Distribution of Bivariate Poisson Network Nodes," *IEEE Comm. Letters*, vol. 14, no. 9, pp. 848-850, Sept. 2010.

[11] X. Liu and M. Haenggi, "Throughput Analysis of fading Sensor Networks with Regular and Random Topologies," *EURASIP Journal of Wireless Communications and Networking,*, pp. 554-564, Sept. 2005.

[12] R. Mathar and J. Mattfeldt, "On the Distribution of the Cumulated Interference Power in Rayleigh Fading Channels," *Wireless Networks*, vol. 1, no. 1, pp. 31-36, 1995.