Statistical Performance Analysis of a Generalized Processor Sharing System by Using Large Deviations

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Abstract

This paper reviews the statistical performance analysis of the tail of the steady-state queue length distribution in a *Generalized Processor Sharing* (GPS) system. In particular, we focus on the *Large Deviations Principle* (LDP) which can be used to estimate a wide range of traffic sources, including short-range dependent sources, like processes with Markovian structure, and long-range dependent sources, like self-similar traffic. Generally, performance bounds, either upper or lower, are derived to characterize the asymptotic behavior of the queue length for each session which shares the GPS server with multiple sessions. Other analysis methods, such as service-curve based method, and bufferless fluid flow approximations are introduced briefly. Finally, a comparison of different methods and the range where they can be applied are given.

1 Introduction

High-speed packet-switched networks carry traffic for a diversity of different applications. Each of these applications has its own traffic characteristics and requires specific Quality of Service (QoS) guarantees. Statistical multiplexing is employed to integrate those services. A salient advantage of such a network is its high resource utilization. The efficient call admission control, which determines the resource utilization, is mainly supported by (i) traffic characterization used to describe the traffic of connections, (ii) the packet scheduling disciplines at each server or switch in the network, and (iii) the accuracy of the performance analysis used for call admission control tests.

A well-defined traffic class is characterized as a Markovian process, which is inherently short-range dependent and can be analyzed through classical queueing models. However, statistical data analysis has in fact shown that traffic patterns may look similar when observed on various time scales. This behavior is a kind of *long-range dependence* (LRD), which means that correlations in the traffic activity decay slowly over time. Since quite a number of traffic types, ranging from local to wide area network traffic, belong to LRD, the system fed by LRD traffic has been paid much more attention recently [6, 9].

Scheduling mechanisms play an important role in achieving differentiated QoS. One of the most important scheduling algorithm is the *Generalized Processor Sharing* (GPS). GPS is characterized by two attractive properties: (i) each backlogged flow is guaranteed a minimum service rate, and (ii) the excess service rate is redistributed among the backlogged flows in proportion to their minimum service rates. Its ability to isolate various service classes as well as allow bandwidth sharing among classes, on the one hand, makes it a desirable fair queueing scheme to satisfy diverse QoS requirements; on the other hand, it leads to increased complexity of the bandwidth allocation and thus makes its performance analysis intractable. As a result, in a GPS system, we are more interested in the performance bounds, rather than the accurate characterization.

Even though only performance bounds are desired in the GPS system, the accuracy of these bounds is still of high importance. An *effective* and *efficient* method to derive the bounds is needed. One of the most relevant QoS parameters is the required Cell Loss Probability (CLP), which can be expressed through the asymptotic behavior of each flow. In this paper, we focus on the derivation of the tail distribution of each session's queue length. By *effective* [2], we mean that the method is able to produce bounds on the tail distribution that are relatively tight. A loose bound would reduce the utilization of the network. By *efficient*, we mean that the method is simple and fast in order to be used as a part of on-line call admission control.

Generally, there are several methods to analyze the cell loss in a network: service-curve based method [15], large deviation approximations [3, 6, 9–13], importance sampling [5, 7, 8] and bufferless fluid flow approximation [4]. The service-curve based method is based on Markovian processes, which possess an interesting property that after traversing through a router or a switch, the output is also Markovian. This property simplifies the derivation of stochastic bounds for a network of queues. An example is the Exponentially Bounded Burstiness (EBB) process. However, not every traffic source satisfies the Markovian condition, e.g., subexponential and superexponential processes. Thus, a more general method is desired. Large Deviations (LD) techniques have been developed on general mathematical settings and are used to investigate the asymptotic behavior of different traffic classes. They can not only estimate the probability of the queue size, but also tell us how the queue reaches that size. An essential step for preventing congestion through a variety of control mechanisms (buffer dimensioning, admission control, resource allocations) is to determine how it occurs, hence, in this sense, the LD approximation method is more attractive than others. Importance Sampling (IS) is usually used in simulation study. When combined with *extreme value theory*, an accurate estimation of the tail distribution could be obtained for Gaussian input sources. In many CAC schemes, the bufferless fluid flow approximation is employed due to its simplicity. However, to find an efficient and effective method to study cell loss is still an open problem.

In this paper, we mainly focus on deriving the bounds of cell loss probability in a GPS system, by using LD principles. We can show that this method is applied not only for traditional Markovian processes (short-range dependent), but also for long-tailed traffic found empirically.

2 Performance analysis methods in the network

2.1 Service-curve based method

Cruz [17] first presented how to use the service-curve based method to effectively evaluate the end-to-end delay induced by a connection with linearly constrained traffic. Parekh and Gallager [18,19] extended his work to a GPS system and derived the worst-case upper bounds on both the delay and backlog of each session. The basic idea is to find a deterministic, piecewise-linear function that provides lower bounds for the service offered by a GPS server to a session, and then calculate the delay and backlog bounds according to the deterministic arrival curve and service curve. This method is readily extended to the EBB traffic.

Recently, [15] studied the LRD traffic and used, instead of Leaky Bucket mechanism, the Fractal Leaky Bucket (FLB) mechanism to monitor LRD sources, which is more effective than the classic Leaky Bucket (LB) policy. Since a LB has only two parameters, the leaky rate ρ and a token buffer size σ which are not sufficient to completely characterize the target traffic, it is a difficult task for it to police the bursty source. The problem is that the regulated traffic is implicitly bounded by a linear function of time, while it is not true for bursty sources, particularly for LRD. The FLB mechanism has been proposed to overcome the inefficiency of the LB mechanism for policing LRD traffic. This mechanism constrains traffic to the Fractional Brownian Motion (FBM) envelope process, as follows:

$$\hat{A}_{FLB} = \rho t + \psi t^H \tag{2.1}$$

where $\hat{A}$ represents the output traffic of the FLB, ρ corresponds to the mean arrival rate of the source, and H is the Hurst parameter. The parameter ψ is given by $k\sigma$, where σ and k are constants associated to the standard deviation of the arrival process and to the probability of the violation of $\hat{A}_{FLB}$ by the arrival process, respectively. This mechanism, if appropriately choosing the value of k, could well approximate the LRD traffic. By using similar deduction technique to that in [18, 19], a deterministic bound on both end-to-end delay and backlog of each session is obtained for LRD traffic.

2.2 Bufferless fluid flow approximation method

The advantage of the bufferless fluid flow approximation over other methods is its simplicity [4]. Under the bufferless assumption, cell loss due to overflow occurs if and only if the sum of the traffic rates of all active connections denoted by R exceeds the link capacity C. So, a cell loss rate function (CLRF) $F(m)$ is defined as:

$$F(m) \triangleq E[(R-m)^+] \triangleq \sum_x (x-m)^+ f(x) \tag{2.2}$$

where $f(x)$ is the probability density distribution of the traffic. There are several attractive features which facilitate the analysis of cell loss in the bufferless fluid flow model:

- F(C) denotes the cell loss rate of traffic with traffic density distribution $f(x)$;
- Traffic sources with similar CLRF can be regarded as equivalent in the cell loss analysis;
- If $f(x)$ and $g(x)$ are the traffic density distribution of independent traffic sources X_1 and X_2, respectively, then the cell loss rate function of $f(x) \cdot g(x)$ equals to $F(x) \cdot G(x)$;
- Let $f(x)$, $g(x)$ and $q(x)$ be the traffic density distribution of independent traffic sources X_1, X_2, and X_3, respectively. Denote the CLRF of $f(x)$, $g(x)$, $f(x) \cdot q(x)$ and $g(x) \cdot q(x)$ by $F(m)$, $G(m)$, $FQ(m)$ and $GQ(m)$ respectively. If for $F(m)$ and $G(m)$ the condition $F(m) \geq G(m)$ holds for any m, then $FQ(m) \geq GQ(m)$ for any m.

These properties of the CLRF enable us to decompose the complex analysis of the aggregation of several traffic sources into the analysis of individual traffic sources, hence the analysis is greatly simplified. Furthermore, for heterogeneous sources, as long as they have a similar CLRF, they can be treated as homogeneous sources. Therefore, as an application, the cell loss due to the multiplexing of the N heterogeneous on-off sources are upper bounded by the cell losses due to the multiplexing of N independent homogeneous on-off sources with peak cell rate (pcr) as the maximum among all N sources' *pcr*, and mean cell rate (mcr) as the average of the N mcrs. From this perspective, the bufferless fluid flow approximation method is applicable in on-line CAC for the heterogeneous sources.

2.3 Importance Sampling (IS)

The basic idea underlying the IS technique is the *biasing* of the examined system [5,7,8], so that the target rare event becomes more likely to occur. This is mainly used in simulation study. In the case of a stationary Gaussian inut process, the IS technique can be used to derive a tight bound on the tail distribution of the queue length. The Gaussian process model is useful for two reasons. First, Gaussian processes have several appealing properties, e.g., any stationary Gaussian process can be completely specified by its mean and covariance, and the superposition of independent Gaussian processes is still Gaussian. Thus, unlike the case of Markovian processes, analyzing a queue with a large number of Gaussian input processes is no more difficult than for a single Gaussian input. Second, and more importantly, in a high-speed network, the input flow is always an aggregation of multiple applications. According to the Central Limit Theorem, even though the individual application cannot be characterized by a Gaussian process, the aggregated traffic to the multiplexer can be effectively modeled as a Gaussian process.

Under some mild assumptions (such as the stationarity and ergodicity of the net amount of fluid input, denoted by $\gamma_n = \lambda_n - \mu$ where λ_n and μ represent the fluid input rate and the service rate, respectively), the supremum distribution of $X_n := \sum_{m=1}^{n} \gamma_{-m}$ is equal to the steady-state queue length distribution, i.e.,

$$\mathbb{P}(Q > x) = \mathbb{P}(\sup_{n \geq 0} X_n > x) \tag{2.3}$$

This relation converts the estimation of the queue length distribution to that of the accumulated net fluid input. For Gaussian processes, after scaling and shifting X_n, this conversion simplifies the analysis procedure. Based on extreme value theorem for Gaussian processes, [7] derived two bounds. The first is of a single exponential form

$$\mathbb{P}(Q > x) \sim Ce^{-\eta x} \tag{2.4}$$

and results in an accurate *upper bound* to the asymptotic constant C. However, Eqn.(2.4) captures only the leading (fastest decaying) term in $\log \mathbb{P}(Q > x)$ and thus suffers from the limitation inherent in all single-exponential based approximation for $\mathbb{P}(Q > x)$, i.e., when the tail probability converges slowly, a single exponential approximation may fail to accurately approximate $\mathbb{P}(Q > x)$ even for fairly large values of x. Motivated by this problem, other asymptotic bounds were developed which exploit the variance of the Gaussian process.

$$\Psi\left(\sqrt{\frac{x}{\langle\sigma_x^2\rangle}}\right) \leq \mathbb{P}(Q > x) \leq \exp\left[-\left(\frac{x}{2\langle\sigma_x^2\rangle}\right)\right] \sim \exp\left\{-\left[\left(2\frac{\kappa}{S}\right)\left(x + \frac{\kappa D}{S}\right)\right]\right\}. \tag{2.5}$$

This method, unlike LDP which is for general traffic and thus would suffer from a loss of accuracy, is specified for Gaussian random processes, hence a tight bound is obtained.

3 Introduction to the Large Deviations Principle (LDP)

Large Deviation theory [1, 14] is a modern branch of probability, concerned with estimating the probabilities of rare events. This makes it well-suited to study high-performance communications networks, in which dropping a packet (cell loss) should be a rare event. More precisely, large deviations theory is concerned with limiting regimes. In queueing problems, it is very rare to solve all equations exactly to obtain the performance measures of interest, so instead we seek limiting results. Normally, large deviations estimates are governed by the *principle of the largest term*, which means that if a rare event occurs, it is overwhelmingly likely that it occurs in just one way. If we can calculate which is the most likely way, we know the typical behavior of the system. That is one of the two important factors we are interested when analyzing the system.

Large deviations theory has been widely studied, and much work has been done on LD in queueing theory. These approaches differ by the limiting regime we look at. There are three possibilities [1], *many sources*, *large buffer*, and *moderate deviations.*

1. **many sources**: here the total input flow is the aggregate of many independent input flows. This sort of scaling is well-suited to modern telecommunication networks, e.g., the Internet, where a router may have thousands of inputs. So the L^{th} queue can be thought of as multiplexing together L different flows, with its resources growing in proportion, i.e., it has service rate LC and buffer size LB. When there are many input flows or when the sources exhibit long-range dependence, the observation has prompted some of the work on the *many sources* asymptotic.

2. **large buffer**: in which buffer size of a router increases but the number of flows stays fixed. So the L^{th} queue can be regarded as having a single input and fixed service C, but increasing buffer size LB. For Markov modulated fluid sources and for many others, the probability of loss decays exponentially in buffer size, so a good way to reduce loss is to make the buffers larger. And it is natural to study the *large buffer* asymptotic.

3. **moderate deviations**: in which the impacts of the mean arrival rate and burstiness are treated differently. Usually, the central limit theorem looks at the limiting behavior of $L^{\frac{1}{2}}(\mathbf{X}^L - \mu)$ and produces estimates based on the normal distribution and involving only the mean and covariance; while large deviations, on the other hand, look at the limiting behavior of $(\mathbf{X}^L - \mu)$ and produces estimates involving the entire distribution, but they depend only on the most likely path. Moderate deviations lies between large deviations theory and the central limit theorem. It looks at the limiting behavior of $L^{\frac{\gamma}{2}}(\mathbf{X}^L - \mu)$ for $0 < \gamma < 1$, and produces estimates involving only the mean and covariance and depending only on the most likely path.

In order to estimate the quantities of interest, we first need to find a sample path LDP in a space appropriate for queueing applications. This will be done in four steps.

1. The first step is to find an LDP for the finite truncation of the process. Think of a sequence of processes $\mathbf{X}_L$, define the logarithmic moment generating function $\mathbf{\Lambda}_t^L(\theta)$ for $\theta \in \mathbb{R}^t$ by

$$\mathbf{\Lambda}_t^L(\theta) = \frac{1}{L} \log \mathbb{E} \exp(L\theta \cdot \mathbf{X}^L(0, t]) \tag{3.6}$$

   Assume that for each t and θ, the limiting moment generating function $\mathbf{\Lambda}_t(\theta) = \lim_{L\to\infty} \mathbf{\Lambda}_t^L(\theta)$ exists and is an essentially smooth, lower semicontinuous function. Then for any fixed t, the sequence $\mathbf{X}^L(0, t]$ satisfies an LDP with good rate function

$$\mathbf{\Lambda}_t^*(\mathbf{x}(0, t]) = \sup_{\theta\in\mathbb{R}^t} \theta \cdot \mathbf{x}(0, t] - \mathbf{\Lambda}_t(\theta) \tag{3.7}$$

2. The next step is to extend the LDP to the entire process, i.e., from $\mathbf{X}(0, t]$ to $\mathbf{X}(0, \infty]$. This can be done by taking projective limits.

3. The third step is to strengthen the LDP to a more appropriate topology such that the queueing functions of interest are continuous with respect to the projective limit topology.

4. Finally, restrict the LDP by incorporating a notion of stability, i.e., the limiting moment generating function $\mathbf{\Lambda}_t$ corresponds to a stationary process.

Then, the process $\mathbf{X}^L$ is bounded from both above and below by:

$$\liminf_{L\to\infty} \frac{1}{L} \log \mathbb{P}(\mathbf{X}^L \in B) \quad \geq \quad -\inf_{x\in B} \tag{3.8}$$

$$\limsup_{L\to\infty} \frac{1}{L} \log \mathbb{P}(\mathbf{X}^L \in B) \quad \geq \quad -\inf_{x\in \bar{B}} \tag{3.9}$$

where B and $\bar{B}$ represent a closed and an open set, respectively.

4 Asymptotic behavior of GPS by using LDP

GPS is a work-conserving scheduling discipline, defined in terms of fluid sources where source traffic is treated as infinitely divisible fluid (hence an ideal model). One of its important features is its ability to provide isolation among different classes, while, at the same time, allowing bandwidth sharing among classes. Consider n ($n \geq 2$) sessions sharing a GPS server with rate c, each session with its own queue. Associated with each session are share parameters $\{\phi_i\}_{i=1}^n$ which determine the guaranteed rate of each session

$$g_i = \frac{\phi_i}{\sum_{j=1}^n \phi_j} c$$

The actually received service by each session is given by

$$r_i = \frac{\phi_i}{\sum_{j\in\mathcal{B}(t)} \phi_j} c \tag{4.10}$$

where $\mathcal{B}(t) \subseteq \{1, 2, \ldots, n\}$ represents the set of all backlogged sessions at time t. Eqn. (4.10) shows that the unused bandwidth by sessions which are not backlogged will be shared by all backlogged sessions in proportion to their share weights. This is an ideal fairness. On the other hand, however, the received service of one session is related to not only its own input traffic pattern and queue length distribution, but also the inputs and queues of other sessions. This correlation makes the performance analysis, particularly stochastic analysis of the GPS system intractable. In [20], we have shown that by the queueing analysis method, even for the simple two on-off sources with exponentially distributed on and off periods, the performance bounds are not tight enough, and the computation is quite complex. If extended to multiple-queue and non-Markov input traffic, the analysis would be prohibitively complicated. As a result, LDP, which can be applied to many input patterns, is considered. In [1], under the *large buffer* asymptotic, the large deviations approximation of the queue length amounts to

$$\log \mathbb{P}(Q = \beta) \approx -\beta^{2(1-H)} I(1), \qquad \text{for large } \beta \tag{4.11}$$

where $I(\cdot)$ is a good rate function associated with $Q(\cdot)$. When $H = \frac{1}{2}$, the decay is exponential in β: many other sources including MMFSs share this exponential decay. While when $H > \frac{1}{2}$, the source has long-range dependence and the decay is less than exponential.

In the LRD case, increasing the buffer size does not give as much of a reduction in loss probability, and thus the *many sources* asymptotic is taken into account to approximate the tail distribution. In other words, using LDP in different limiting regimes, we can obtain an estimate for most classes of traffic.

4.1 Traditional Markovian Processes

Zhang applied LDP to the study of GPS systems in [10–12]. In order to decouple an individual session from the effect of other sessions, a decomposition method is employed. The basic idea is to decompose a one-queue GPS system shared by n sessions into a n-queue system, each of which is associated with a service rate s_i in such a manner that

$$\sum_{i=1}^{n} s_i \le c \tag{4.12}$$

By appropriately selecting the values of $\{s_i\}_{i=1}^{n}$ and then analyzing the single queue system, either an upper or lower bound on $\mathbb{P}(Q > x)$ is available to obtain. A noticeable concept applied in the approximation is the *effective bandwidth*. Assume an input process has a limiting moment generating function $\Lambda_A(\theta)$, its effective bandwidth $\alpha_A(\theta) = \frac{\Lambda_A(\theta)}{\theta}$ reflects its impact at the queue. The range lies between the mean and peak rate of the source. It has been proved that a resource can deliver a performance guarantee expressed in terms of loss or delay by limiting the sources served so that their effective bandwidths sum to less than a threshold. Similarly, we can define the *effective bandwidth* $\alpha_D(\theta)$ for the departure/output process in the same way. The queue length distribution is determined by both the input and output processes.

In a two-queue system, an explicit relation exists between the two sessions' service rate $s_i, i = 1, 2$:

$$\begin{aligned} s_1(t) + s_2(t) &= c \\ \alpha_{A_i}(\theta) + \alpha_{D_j}(\theta) &= c \qquad (i,j) \in \{(1,2),(2,1)\} \end{aligned} \tag{4.13}$$

Let session 1 be the one considered. Its service rate $s_1(t)$ can be bounded by restricting $s_2(t)$ directly. [11] discusses this problem in two cases: $\mathbb{E}\, a_2(0) < g_2$, and $\mathbb{E}\, a_2(0) \ge g_2$, where $\mathbb{E}\, a_i(0), i = 1, 2$ represents the mean arrival rate.

1. **Upper bound** of $\mathbb{P}(Q > x)$ implies that session 1 receives minimum service, which in turn implies the maximum service received by session 2.

   - $\mathbb{E}\, a_2(0) < g_2$: set the service rate of session 2 by $s_2(t) = g_2$ and upper bound the actually received service by the decomposed queue length. This will minimize the received service of session 1 $S_1(0, t)$.
   - $\mathbb{E}\, a_2(0) \ge g_2$: this case is easier to analyze since the fact that session 2 is always backlogged indicates that the available maximum service rate for session 1 is only g_1.

2. **Lower Bound**: during one busy period, the total queue length is the sum of the two individual queue lengths. So, the lower bound of queue 1 means the queue 2 reaches its maximum.

However, when more than two sessions share a multiple GPS server, there does not exist such an explicit relation between the service rates of different queues and the exact analysis of the bandwidth sharing dynamics among the sessions becomes significantly more complicated. In that case, a partial feasible partition [12], which is a generalization of the feasible ordering proposed by Parekh and Gallager in [18, 19], was introduced. This provides an avenue to capture the asymptotic bandwidth sharing dynamics among the sessions in the GPS system. Based on the effective bandwidth of each session, a *partial feasible set* F of session 1 with respect to θ is defined in such a manner that for $F = F_1 \cup F_2 \ldots \cup F_k \subseteq N_1 = N\backslash\{1\} = \{2, 3, \ldots, n\}$, $i \in F_1$ if and only if $\alpha_i(\theta) < \frac{\phi_i}{\sum_{j \in N} \phi_j} c$, and recursively, for $2 \le l \le k$, $i \in F_l$ if and only if

$$\alpha_i(\theta) < \frac{\phi_i}{\sum_{j \in N \backslash F^{l-1}} \phi_j} (c - \sum_{j \in F^{l-1}} \alpha_j(\theta)) \tag{4.14}$$

The sequence of disjoint subsets $F_1, F_2, \ldots, F_k$ is called the *partial feasible partition* of F. If we further define

$$\gamma_l^F(\theta) = \frac{1}{\sum_{j \in N \backslash F^{l-1}} \phi_j} (c - \sum_{j \in F^{l-1}} \alpha_j(\theta)) \tag{4.15}$$

clearly, $\phi_i \gamma_{l-1}^F(\theta) \le \alpha_i(\theta) < \phi_i \gamma_l^F(\theta)$ for $i \in F_l$. The numbers $\{\gamma_l^F(\theta)\}_{l=1}^k$ are referred to as the *associated delimiting numbers* for F. Then, in the decomposition system, assign the service rate of each fictitious queue as follows:

$$\begin{aligned} s_i^F(\theta) &= \alpha_i(\theta), \qquad i \in F \\ s_i^F(\theta) &= \phi_i \gamma_l, \qquad i \in N_1 \backslash F \end{aligned} \tag{4.16}$$

$s_i^F(\theta)$ is called the *feasible rate* of session i with respect to F and θ. Zhang proved that when the stability condition $\sum_{i=1}^n \mathbb{E}\, a_i(0) < c$ holds, the decay rate of the queue length tail distribution for session 1 is bounded by:

$$\limsup_{x \to \infty} \frac{1}{x} \log \mathbb{P}(Q_1 > x) \le -\theta^* \tag{4.17}$$

$$\liminf_{x \to \infty} \frac{1}{x} \log \mathbb{P}(Q_1 > x) \le -\mu^* \tag{4.18}$$

For more details of θ^* and μ^*, refer to [12]. The problem here is that the upper and lower bounds do not match exactly in general, except in a two-queue GPS server. That indicates the bounds are still not tight.

4.2 Long-range dependent traffic

For LRD traffic sources, since their decay is less than exponential, we cannot continue using the single-exponent form as shown in Eqn.(4.17) to bound their decay rate. Bertsimas and Borst [3, 9, 13] have addressed this problem and presented their solutions in different ways.

In [13], the problem of estimating the queue length tail distribution is converted to an *optimal control* problem, and thus some standard approaches of optimal control can be used to derive the bounds on cell loss probability. Here the ability of large deviations theory to determine in which way the overflow is most likely to occur is utilized. Optimal state trajectories of the control problem correspond to the most likely modes of overflow. Certainly, from the solution of the control problem, we can obtain a detailed characterization of these modes. We have to point out that the authors considered only the case of multiplexing two different traffic streams. (The general case of N streams is supposed to be more complicated by using this method since there is an exponential explosion of the number of overflow modes.)

Under the GPS policy, there are three distinguished regions of system dynamics, depending on which of the two queues is empty. Denote the input rates of the two sessions and the service rate by $r_1(t), r_2(t), s(t)$, respectively. In particular, we have

- Region $\mathcal{R}_1$: $Q^1(t) > 0$ and $Q^2(t) > 0$, where according to the GPS policy

$$\begin{cases} \dot{Q}^1 &= r_1(t) - \phi_1 s(t) \\ \dot{Q}^2 &= r_2(t) - \phi_2 s(t) \end{cases}$$

- Region $\mathcal{R}_2$: $Q^1(t) = 0$ and $Q^2(t) > 0$, where according to the GPS policy

$$\dot{Q}^2 = r_1(t) + r_2(t) - s(t)$$

- Region $\mathcal{R}_3$: $Q^1(t) > 0$ and $Q^2(t) = 0$, where according to the GPS policy

$$\dot{Q}^1 = r_1(t) + r_2(t) - s(t)$$

Above state trajectories $\{Q^i\}$ constitutes a set of *GPS-DYNAMICS*. The overflow modes are two special cases of GPS-DYNAMICS. (see Eqn.(21) of [13]). Then use the available well-developed control tools to explicitly solve that deterministic optimal control problem. It should be noted that in the analysis procedure, the service rate is assumed to be time-varying, which makes it straightforward to extend the result to more complicated scheduling systems, such as a mixture of GPS and priority scheduling, in which the service rate for low priority sessions is a stochastic process.

Differently, in [3], a similarity relation between the desired queue length under GPS policy and that given by a constant server is derived. The main result is

$$\mathbb{P}(Q_i > x) \sim \mathbb{P}(Q_i^{\gamma_i} > x) \tag{4.19}$$

where γ_i is the mean service rate that source i would receive if it continuously claimed capacity, and the condition $\rho_i < \gamma_i < \sigma_i$ holds with traffic intensity ρ_i. The result shows that an individual source with long-tailed traffic characteristics is effectively served at constant rate γ_i. This suggests that the most likely scenario for source i to build a large queue is to generate a large burst, or to experience a long on-period, while the other sources show average behavior.

This method was extended to the feedforward network case by Uitert [9]. After traversing a node, the input process at the node 2 is exactly the output of node 1. Generally, it can be regarded as an on/off process with on periods identical to the busy periods at node 1, and the on rate is equal to the service rate at node 1; while the off periods correspond to the idle periods at node 1, which are exponentially distributed. Furthermore, the on and off periods at node 2 are independent. Applying this result to the network case, an interesting conclusion is obtained: the workload at a particular node in the network depends on two rates, the rate at which traffic is sent into the node and the rate at which traffic is served by the node. In fact, the first rate is the rate at which traffic is served by the bottleneck node on the path to the relevant node, and the other rate is the service rate for flow i. Therefore, the long-tailed flow is only affected by the traffic characteristics of the other flows through their average rates and is not influenced by the excessive behavior of any of the other flows.

5 Conclusion

In this paper we review several methods to estimate the cell loss probability (or called queue length tail distribution) of the individual session under GPS policy. Due to the generalization and mature development, LDP attracted more attention. However, for specific processes, we prefer other methods. For example, service-curve based method for Markov processes, Importance Sampling method for Gaussian processes, and bufferless fluid flow approximation for heterogeneous on-off sources. Because of the complexity of bandwidth sharing mechanism of the GPS system, the tightness of the derived bounds is still an open problem.

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