Spatial Analysis of Opportunistic Downlink Relaying in a Two-Hop Cellular System

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***Abstract*—We consider a two-hop cellular system in which the mobile nodes help the base station by relaying information to the dead spots. While two-hop cellular schemes have been analyzed previously, the distribution of the node locations has not been explicitly taken into account. In this paper, we model the base station locations deterministically and the mobile stations by a point process on the plane. The node with the best channel to the destination that received information in the first hop acts as a relay to the destination (selection cooperation), and we obtain the success probability of this two-hop scheme, accounting for the interference from all other cells. We use tools from stochastic geometry and point process theory to analyze this two-hop opportunistic relaying scheme. Besides the results obtained, a main contribution of the paper is to introduce a mathematical framework that can be used to analyze arbitrary relaying schemes.**

I. Introduction

Cellular systems are the most widely deployed wireless systems and provide reliable communication services to billions around the world. They consist of base stations that serve a geographical area called cell. In most of the present cellular systems, the base station (BS) communicates directly with the mobile users (MS) in its cell. This single-hop architecture makes it difficult for the BSs to communicate with MSs at the cell boundary because of the distance and the inter-cell interference. So a base station will have to increase its power to maintain the rate of transmission. The dead spots problem can be countered by using more base stations, thereby increasing the spatial reuse. But increasing the number of base stations can be prohibitively expensive or even impossible. The problem can be addressed more effectively by moving away from the paradigm of single-hop communication and permitting the base station to communicate with mobile stations at the boundary by using the other intermediate MSs in its cell in a sequence of hops. Although such multi-hopping requires some significant changes in the present cellular system architecture, it may help to effectively combat the dead spots problem, and hence the cellular multi-hopping problem is worthy to investigate, as argued in [1], [2]. In this paper, we analyze the benefits of two-hop cellular communication by comparing its performance with a traditional single-hop cellular system. A two-hop system

- may provide significant benefits over single-hop communication.
- does not have the implementation complexity of larger number of hops (in terms of routing and scheduling).

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When a BS transmits, multiple MSs will be able to receive the information, and hence these mobile nodes can help the BS transmit information to the cell edge. Since more than one MS can act as a relay, it is not clear how to choose a subset of these relays in a distributed fashion so as to reduce the interference and increase the probability of packet delivery. In this paper, we analyze a simple relay selection scheme and compare its performance with direct transmission. We account for the inter-cell interference and the spatial structure of the transmitting nodes in the analysis.

We use methods from stochastic geometry and point process theory [3], [4] to model and study the two-hop cellular system. In particular, we provide techniques based on the probability generating functional of a point process to analyze the outage probabilities, and we provide asymptotic results for the outage at high signal-to-noise-ratio and low BS density. The techniques presented in this paper can be extended to analyze more complicated relay selection schemes, power control mechanisms and other multi-hop techniques. The main emphasis of the paper is in the methodology and the techniques of the analysis rather than the specifics of the communication system.

A. Previous work

The problem of two-hop extensions of cellular system has been studied extensively, and a provision for a multi-hop technique has been included in the A-GSM standard [1], [2]. There are various techniques [5]–[9] to combat the dead spot problem by using multiple-antenna techniques, fractional frequency reuse, and relaying. In this paper we concentrate on a selection cooperation relaying scheme, and our primary focus is on illustrating the application of analytical tools from stochastic geometry, rather than optimizing the end performance of the system. In [10], a MS is selected to help the BS depending on the large-scale path-loss on the BS-relay link and the relay-destination link. [11] considers a similar problem, but the MSs that can act as relays are assumed to be located on a circle around the BS, and the authors provide various power allocation schemes and verify their performance by simulations. The present problem is also very similar to the problem of opportunistic relay selection. In [12], [13] a detailed analysis of an opportunistic two-hop relaying scheme obtaining full diversity order using distributed space-time codes has been provided. But a distributed space-time code requires very tight coordination and precise signaling between the relays, which increases the overhead and complexity in the system. An alternative approach is to choose the *best* relay, and in opportunistic relaying [14] a relay is chosen so as to maximize the minimum signal-to-noise ratio (SNR) of the source-relay and the relay-destination links. In selection cooperation [15], [16] the relay with maximum relay-destination SNR is chosen and it has been shown that selection cooperation and opportunistic relaying provide a similar diversity order. In [12], [14]–[16], distributed relay selection schemes are analyzed and asymptotes of the outage are provided for high SNR. However these results do not account for the spatial distribution of nodes. Our emphasis is on a low-overhead scheme that can readily be implemented.

The paper is organized as follows: In Section II, the system model is introduced, assumptions stated and the metrics used in the paper defined. In Section III, the outage probability in the direct connection between the BS and its destination is derived. In Section IV, the outage probability of the two-hop scheme, where the relay with the best channel transmits is analyzed. The asymptotic gain of using the two-hop scheme over a direct connection is also studied in these sections. We also provide simulation results to validate the theory.

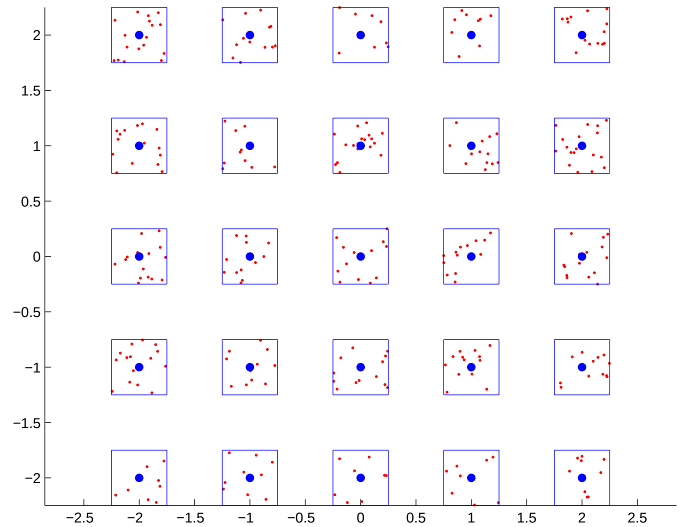

Fig. 1. Illustration of the cellular system with $\lambda_b = 1$ and $\eta(y) = 50 \cdot \mathbf{1}_y([-0.25, 0.25]^2)$. So on a average there are 12.5 MSs per cell. The bold dots represent the BSs and the smaller dots the MSs. The white spaces between the cells may consist of other cells which transmit at a different frequency. We may model the case where the neighboring cells use the same frequency by choosing $\eta(y) = \mathbf{1}_y([-0.5, 0.5]^2)$.

II. System Model

A. Locations of base stations and mobile stations

A 2-D network of BSs is usually modeled by a hexagonal lattice or other regular geometries. Following this tradition, we assume that the BSs (cell towers) are arranged on a square lattice, and their spatial density is λ_b. The locations of the BSs is denoted by Φ_b, and

$$\Phi_b = \left\{ \frac{a}{\sqrt{\lambda_b}}, \quad a \in \mathbb{Z}^2 \right\}.$$

While in this paper we assume a square grid, the analysis in this paper can be easily generalized to any deterministic arrangement of BSs.

We assume that the mobile users associated with each BS form independent Poisson point processes (PPP). The locations of the MSs associated with BS a are denoted by Φ_a, and we assume that Φ_a is a non-homogeneous PPP with density $\lambda_a(y) = \eta(y - a)$. For example choosing $\eta(y) = \mathbf{1}_y([-1/2, 1/2]^2)$ and $\lambda_b = 1$ would lead to the mobile stations distributed as a Poisson point process in a unit square area centered at each base station. See Figure 1 for an illustration. The cell corresponding to the BS a is the support of the function $\lambda_a(y)$ where the mobile users are located. So $\eta(y) = \mathbf{1}_y([-1/2, 1/2]^2)$ would correspond to unit square cells centered around the BSs. The number of users in the cell of a BS a is denoted by $n_a = |\Phi_a|$; it is a Poisson random variable with mean $\int_{\mathbb{R}^2} \eta(x) \mathrm{d}x$. It follows that the probability that a cell contains at least one MS is

$$\mu \triangleq 1 - \exp\left(- \int_{\mathbb{R}^2} \eta(x) \mathrm{d}x \right). \tag{1}$$

B. Path loss and fading

Independent Rayleigh fading is assumed between any pair of nodes and also across time, and the power fading coefficient between a node x and node y is denoted by h_{xy}. Hence h_{xy} is an exponential random variable with unit mean. The path loss model is denoted by $\ell(x) : \mathbb{R}^2 \setminus \{o\} \to \mathbb{R}^+$, where o denotes the origin $(0, 0)$. We assume that $\ell(x)$ is a continuous, positive, non-increasing function of $\|x\|$, and finite everywhere except possibly at o. The path loss $\ell(x)$ is usually taken to be a power law in one of the forms:

1) Singular path loss model: $\|x\|^{-\alpha}$, $\alpha > 2$.
2) Non-singular path loss model: $(1 + \|x\|^{\alpha})^{-1}$ or $\min\{1, \|x\|^{-\alpha}\}$, $\alpha > 2$.

Assuming simple linear receivers and treating interference as noise, the communication between x and y is successful if

$$\mathsf{SINR}(x, y, \Phi) \triangleq \frac{\mathsf{h}_{xy}\ell(x - y)}{\frac{\sigma^2}{P} + \sum_{z \in \Phi} \mathsf{h}_{zy}\ell(z - y)} \geq \theta, \tag{2}$$

for a threshold θ that depends on the transmission rate and coding. We also assume $\theta > 1$ which implies at most one transmitter can connect to a receiver[1]. Here Φ is the set of interfering transmitters, σ^2 is the the additive white Gaussian noise power at the receiver, and P is the common transmit power. For each cell $x \in \Phi_b$ we add an additional mobile station, the *destination* at $r(x)$ with $\|r(x) - x\| = R$, to which the BS wants to transmit information. This additional node just receives and never transmits. We also assume that the BSs transmit in the even time slots and the MSs transmit in the odd time slots, synchronized across all cells.

C. Metrics

Let P_d denote the probability that a BS can connect to its destination directly in the first hop. Since all the BSs are identical, we focus on the BS at the origin and define

$$\mathsf{P}_d = \mathbb{P}\left(\mathsf{SINR}(o, r(o), \Phi_b \setminus \{o\}) > \theta\right), \tag{3}$$

[1]This follows from the fact that $\frac{a_1}{a_2 + b} > 1$ and $\frac{a_2}{a_1 + b} > 1$ cannot be simultaneously true for any $a_1, a_2, b > 0$.

i.e., the probability that the received SINR from the BS is greater than the threshold θ.

When a BS transmits, it can potentially connect (transmit information) to any MS at which the received SINR is greater than the threshold θ. These MSs are the potential transmitters in the second hop, and we denote this set by $\hat{\Phi}_a$, where a denotes the BS. Mathematically

$$\hat{\Phi}_a = \{y \in \Phi_a, \mathsf{SINR}(a, y, \Phi_b \setminus \{a\}) > \theta\}.$$

In our relaying strategy, the node from the set $\hat{\Phi}_a$ that has the best channel to the destination $r(a)$ transmits in the second hop. For a BS $a \in \Phi_b$, we denote the location of the best relay by ξ_a, *i.e.*,

$$\xi_a = \arg\max_{y \in \hat{\Phi}_a} \{\mathsf{h}_{yr(a)} \ell(y - r(a))\}.$$

Again concentrating on the cell at the origin, denote the probability that the best relay can connect to the destination $r(o)$ in the second hop by

$$\mathsf{P}_r = \mathbb{P}\left(\max_{x \in \hat{\Phi}_o} \{\mathsf{h}_{xr(o)} \ell(x - r(o))\} > \theta\left(\mathtt{I} + \frac{\sigma^2}{P}\right)\right), \quad (4)$$

where $\mathtt{I}$ is the interference at $r(o)$ caused by concurrent transmissions in other cells, *i.e.*,

$$\mathtt{I} = \sum_{a \in \mathbb{Z}^2 \setminus \{o\}} \mathsf{h}_{\xi_a r(o)} \ell(\xi_a - r(o)).$$

The direct transmission by the BS fails with a probability $1 - \mathsf{P}_d$, and the selected relay may fail with probability $1 - \mathsf{P}_r$, and hence, the total probability of success for the two-hop scheme is

$$\mathsf{P}_s = 1 - (1 - \mathsf{P}_d)(1 - \mathsf{P}_r).$$

In defining P_s we have assumed that the information received in the two time slots is decoded independently. The BS can potentially transmit in the second hop instead of using a MS as an intermediate relay. We compare the performance of the relaying scheme with the retransmission strategy, and for a fair comparison, we assume that the selected relay also transmits with power P in the second hop. The gain in using the two-hop scheme over the retransmission scheme can be characterized as

$$\mathsf{G}(\mathsf{SNR}, \lambda_b) = \frac{(1 - \mathsf{P}_d)^2}{(1 - \mathsf{P}_d)(1 - \mathsf{P}_r)} = \frac{1 - \mathsf{P}_d}{1 - \mathsf{P}_r}, \quad (5)$$

where $\mathsf{SNR} = \frac{P\ell(R)}{\sigma^2}$, is the received SNR for the direct transmission. It is well known that opportunistic relaying increases the diversity order, and in this paper we analyze the diversity of the relaying scheme in the presence of intercell interference. The diversity gain is defined as

$$\mathrm{d}_2(\lambda_b) = -\lim_{\mathsf{SNR} \to \infty} \frac{\log(1 - \mathsf{P}_s)}{\log(\mathsf{SNR})}.$$

From the definition of the diversity and the gain, the following relation follows:

$$\mathrm{d}_2(\lambda_b) - \mathrm{d}_d(\lambda_b) = \lim_{\mathsf{SNR} \to \infty} \frac{\log(\mathsf{G}(\mathsf{SNR}, \lambda_b))}{\log(\mathsf{SNR})},$$

where d_d is the diversity gain for the single-hop retransmission scheme. In our case we can increase the SNR by increasing the transmit power P or by reducing the noise variance σ^2. While increasing the SNR would combat noise, the interference would not be affected since all nodes transmit with the same power. In cellular systems, to reduce intercell interference and increase coverage, spatial reuse is decreased by frequency planning so that adjacent cells transmit in different frequency bands. Without such frequency planning, it is easy to observe that the probability P_r of any relay selection scheme does not tend to one by increasing the SNR because of the intercell interference. We can easily introduce frequency planning in our model, by decreasing the density of BS, as this will increase the distance between transmitting BSs. We decrease the BS density as

$$\lambda_b = \mathsf{SNR}^{-\beta}, \quad \beta \geq 0. \quad (6)$$

Compared to studying both asymptotics, $\lambda_b \to 0$ and $\mathsf{SNR} \to \infty$, choosing $\lambda_b = \mathsf{SNR}^{-\beta}$ allows us to study only one asymptotic, *i.e.*, $\mathsf{SNR} \to \infty$, while the generality is not compromised thanks to the parameter β. Define the signal-to-interference-ratio as

$$\mathsf{SIR} \triangleq \frac{\ell(R)}{\sum_{a \in \Phi_b \setminus \{o\}} \ell(a - r(o))}. \quad (7)$$

In the next section we show that $\sum_{a \in \Phi_b \setminus \{o\}} \ell(a - r(o)) = \Theta(\lambda_b^{\alpha/2})$, $\lambda_b \to 0$, and hence from (6) we obtain $\mathsf{SIR} = \Theta(\mathsf{SNR}^{\frac{\alpha\beta}{2}})$. We can observe that $\mathsf{SIR} = o(\mathsf{SNR})$ when $\alpha\beta < 2$, *i.e.*, the system is interference-limited, and noise-limited ($\mathsf{SNR} = o(\mathsf{SIR})$) otherwise[2]. Hence the scaling in (6) helps us evaluate the performance of the system by varying β. We now begin with the analysis of the direct transmission scheme.

III. First Hop: Base Station Transmits

A. Direct Connection

When the BSs transmit, the inter-cell interference, fading and the noise may cause the transmission to fail. In the next lemma we compute this probability.

Lemma 1: The probability of direct connection between a BS at the origin and its destination $r(o)$ is given by

$$\mathsf{P}_d = \exp\left(-\frac{\theta}{\mathsf{SNR}}\right)\Delta(r(o)), \quad (8)$$

where

$$\Delta(x) = \prod_{a \in \Phi_b \setminus \{o\}} \frac{1}{1 + \frac{\theta}{\ell(x)}\ell(a - x)}.$$

Proof: From (3), the probability of direct connection is

$$\mathsf{P}_d = \mathbb{P}\left(\frac{\mathsf{h}_{or(o)}\ell(R)}{\frac{\sigma^2}{P} + \sum_{a \in \Phi_b \setminus \{o\}} \mathsf{h}_{ar(o)}\ell(a - r(o))} > \theta\right).$$

[2] $\Theta(.)$ and $o(.)$ follow the standard Landau notation. We denote $F(x) = \Theta(G(x))$, $x \to \infty$, if there exists two constants $C_1 > 0$ and $C_2 > 0$ such that $C_1 G(x) < F(x) < C_2 G(x)$, when $x \to \infty$. The notation $F(x) = o(G(x))$ implies $\lim_{x \to \infty} F(x)/G(x) = 0$. Analogous notation is used when $x \to 0$

Since $\mathsf{h}_{or(o)}$ is exponentially distributed with unit mean, its CCDF is $\exp(-x)$ and

$$\mathsf{P}_d = \mathbb{E}\exp\left(-\frac{\theta}{\ell(R)}\left(\frac{\sigma^2}{P} + \sum_{a\in\Phi_b\setminus\{o\}} \mathsf{h}_{ar(o)}\ell(a-r(o))\right)\right).$$

The fading coefficients $\mathsf{h}_{ar(o)}$ are independent, so P_d equals

$$\exp\left(-\frac{\theta}{\ell(R)}\frac{\sigma^2}{P}\right)\prod_{a\in\Phi_b\setminus\{o\}}\mathbb{E}\exp\left(-\mathsf{h}_{ar(o)}\frac{\theta\ell(a-r(o))}{\ell(R)}\right).$$

Since $\mathsf{h}_{yr(o)}$ is exponentially distributed the result follows from the Laplace transform of the exponential distribution. ■

From (8), it is not clear how the success probability scales with the BS density. In the next lemma, we analyze the asymptotics of the success probability when the BS density is small.

Lemma 2: When $\ell(x) = \|x\|^{-\alpha}$ or $\ell(x) = 1/(1+\|x\|^\alpha)$,

$$\lim_{\lambda_b\to 0}\frac{1-\Delta(x)}{\lambda_b^{\alpha/2}} = \frac{\theta C(\alpha)}{\ell(x)},$$

where

$$C(\alpha) = \frac{\xi(\alpha/2,0)\left[\xi(\alpha/2,1/4)-\xi(\alpha/2,3/4)\right]}{2^{\alpha-2}}. \quad (9)$$

$\xi(s,b) = \sum_{k=0,k\neq -b}^{\infty}(k+b)^{-s}$ is the generalized Riemann zeta function.

Proof: We consider the case $\ell(x) = \|x\|^{-\alpha}$; the other case follows similarly. From the definition of $\Delta(x)$ it follows that

$$\exp\Big(-\theta\ell(x)^{-1}\sum_{a\in\Phi_b\setminus\{o\}}\ell(a-x)\Big) \le \Delta(x)$$
$$\le \Big(1+\theta\ell(x)^{-1}\sum_{a\in\Phi_b\setminus\{o\}}\ell(a-x)\Big)^{-1}. \quad (10)$$

The lower bound follows from $\exp(-x)\le(1+x)^{-1}$ and the upper bound from $\prod_{i=1}^n(1+x_i)\ge 1+\sum_{i=1}^n x_i$, $x_i>0$, $1\le i\le n$. We have

$$\sum_{a\in\Phi_b\setminus\{o\}}\ell(a-x) = \sum_{a\in\mathbb{Z}^2\setminus\{o\}}\ell\left(\frac{a}{\sqrt{\lambda_b}}-x\right) = \lambda_b^{\alpha/2}\sum_{a\in\mathbb{Z}^2\setminus\{o\}}\ell(a-x\sqrt{\lambda_b}).$$

Substituting in (10) we obtain

$$\exp\Big(-\theta\ell(x)^{-1}\lambda_b^{\alpha/2}\sum_{a\in\mathbb{Z}^2\setminus\{o\}}\ell(a-x\sqrt{\lambda_b}))\Big)\le\Delta(x)$$
$$\le\Big(1+\theta\ell(x)^{-1}\lambda_b^{\alpha/2}\sum_{a\in\mathbb{Z}^2\setminus\{o\}}\ell(a-x\sqrt{\lambda_b})\Big)^{-1}.$$

Since $\exp(-y)\sim 1-y$ and $(1+y)^{-1}\sim 1-y$ for small y, as $\lambda_b\to 0$, we obtain

$$\lim_{\lambda_b\to 0}\frac{1-\Delta(x)}{\lambda_b^{\alpha/2}} = \lim_{\lambda_b\to 0}\frac{\theta}{\ell(x)}\sum_{a\in\mathbb{Z}^2\setminus\{o\}}\ell(a-x\sqrt{\lambda_b}) = \frac{\theta}{\ell(x)}\sum_{a\in\mathbb{Z}^2\setminus\{o\}}\ell(a).$$

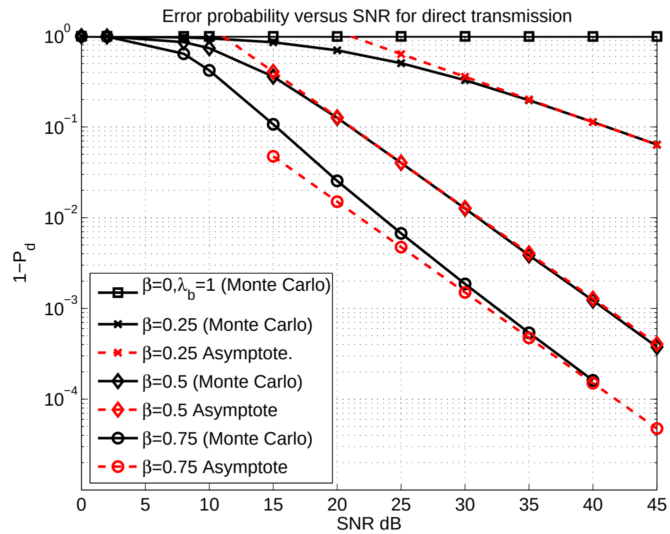

Fig. 2. Outage probability $1-\mathsf{P}_d$ versus SNR for $\lambda_b = \mathsf{SNR}^{-\beta}$ with different β. The system parameters are $\alpha=4$, $\theta=1.5$, $r(o)=(0.5,0.5)$ and $\ell(x)=(1+\|x\|^4)^{-1}$. The dashed lines are the asymptotes derived in (11). Observe the difference in the slopes of the error curve for $\beta<0.5$ and $\beta\ge 0.5$.

The summation in the RHS in the above equation $\sum_{a\in\mathbb{Z}^2\setminus\{o\}}\|a\|^{-\alpha}$ is the Epstein zeta function of order 2, which is equal to $C(\alpha)$ [17]. ■

We have $C(3)\approx 9.03362$ and $C(4)\approx 6.02681$. From the derivation of the above lemma we observe that $\sum_{a\in\Phi_b\setminus\{o\}}\ell(a-x)\sim\lambda_b^{\alpha/2}C(\alpha)$, $\lambda_b\to 0$. Hence defining SIR as in (7), and setting $\lambda_b=\mathsf{SNR}^{-\beta}$, we obtain $\mathsf{SIR}\sim\mathsf{SNR}^{\alpha\beta/2}\ell(R)C(\alpha)^{-1}$. The following result characterizes the asymptotic success probability of a direct connection between the BS and its destination.

Corollary 1: When $\lambda_b=\mathsf{SNR}^{-\beta}$, the success probability of a direct connection at high SNR is

$$\mathsf{P}_d\sim\begin{cases}1-\theta\mathsf{SNR}^{-1} & \alpha\beta>2\\ 1-\theta\left(1+C(\alpha)\ell(R)^{-1}\right)\mathsf{SNR}^{-1} & \alpha\beta=2\\ 1-\theta C(\alpha)\ell(R)^{-1}\mathsf{SNR}^{-\alpha\beta/2} & 0<\alpha\beta<2,\end{cases} \quad (11)$$

and the diversity gain of the direct transmission is $\mathrm{d}_d(\mathsf{SNR}^{-\beta})=\min\left\{1,\frac{\alpha\beta}{2}\right\}$.

Proof: The success probability from Lemma 1 is $\exp\left(-\frac{\theta}{\mathsf{SNR}}\right)\Delta(r(o))$. The result follows from $\exp\left(-\frac{\theta}{\mathsf{SNR}}\right)\sim 1-\frac{\theta}{\mathsf{SNR}}$, and using the asymptotic result for $\Delta(r(0))$ from Lemma 2. ■

So for the direct transmission, $\beta<2/\alpha$ corresponds to the interference-limited regime, and $\beta>2/\alpha$ corresponds to the noise-limited regime. Also the maximum diversity possible is 1 as expected. From Figure 2, we observe that the asymptotes in (11) are close to the true P_d even at moderate SNR.

B. Properties of the potential relay sets $\hat{\Phi}_o$.

In this subsection, the properties of the node set that the BS at the origin is able to connect to are analyzed. When the BSs transmit, the interference seen by two MSs is independent, since the fading is independent. So the set of MSs to which the BS at the origin can connect to is an independent thinning of Φ_o. Hence $\hat{\Phi}_o$ is also a PPP with intensity $\delta(x) = \eta(x)\mathbb{P}(\mathsf{SINR}(o, x, \Phi_b \setminus \{o\}) > \theta)$ [18]. Using Lemma 1, with the receiver located at x instead of at a distance R, we have

$$\delta(x) = \eta(x) \exp\left(-\frac{\theta}{\mathsf{SNR}}\frac{\ell(R)}{\ell(x)}\right)\Delta(x). \quad (12)$$

Also from the proof technique of Corollary 2, when $\lambda_b = \mathsf{SNR}^{-\beta}$ it follows that

$$\delta(x) \sim \eta(x)\left(1 - \mathsf{SNR}^{-1}\frac{\ell(R)\theta}{\ell(x)}\right)\left(1 - \mathsf{SNR}^{-\alpha\beta/2}\frac{\theta C(\alpha)}{\ell(x)}\right), \quad (13)$$

as $\mathsf{SNR} \to \infty$ The average number of MSs which the BS is able to connect to is

$$\mathbb{E}\sum_{x\in\Phi_o} \mathbf{1}(\mathsf{SINR}(o, x, \mid \Phi_b \setminus \{o\}) \geq \theta) = \int_{\mathbb{R}^2} \delta(x)\mathrm{d}x \quad (14)$$

which follows from the Campbell-Mecke theorem [19].

IV. Second Hop: Relay With Best Channel to Destination Transmits

In this selection procedure, the relay with the best channel to the destination is selected. Hence the fading between a potential relay and the destination is also incorporated in the criterion for the relay selection. This method of relay selection, called selection cooperation, is known to increase the diversity order that depends on the cardinality of the set of plausible relays.

In the second hop, each relay of the set $\hat{\Phi}_o$ can send a channel estimation packet to the destination in an orthogonal fashion, and the destination can choose the relay with the best channel. Alternatively, if channel reciprocity is assumed, the relays can estimate the channel between themselves and the destination when receiving the NACK and use this information to elect the best relay in a distributed fashion. We begin with the computation of Laplace transform of the intercell interference, which we then utilize to obtain asymptotics of the success probability.

Lemma 3: When $\lambda_b \sim \mathsf{SNR}^{-\beta}$, the Laplace transform of the interference $\mathtt{I}$ seen by the receiver $r(o)$ in the second hop is asymptotically

$$\mathcal{L}_{\mathtt{I}}(s) \sim 1 - \mu s C(\alpha)\mathsf{SNR}^{-\alpha\beta/2}, \quad \mathsf{SNR} \to \infty,$$

where $C(\alpha)$ is given in (9), and μ is the probability that a cell is non-empty as defined in (1).

Proof: The Laplace transform of the interference in the second hop at the receiver $r(o)$ is $\mathcal{L}_{\mathtt{I}}(s) = \mathbb{E}\left[\exp(-s\mathtt{I})\right]$, where

$$\mathtt{I} = \sum_{a\in\Phi_b\setminus\{o\}} \mathsf{h}_{\xi_a r(o)}\ell(\xi_a - r(o))\mathbf{1}(|\hat{\Phi}_a| > 0).$$

Recall that ξ_a denotes the location of the selected relay in cell a. We require $\mathbf{1}(|\hat{\Phi}(a)| > 0)$ multiplying the path loss, since the cell corresponding to BS a might have been empty to start with, and hence there is no relay to transmit at the second hop. Hence we obtain

$$\mathcal{L}_{\mathtt{I}}(s) = \mathbb{E}\prod_{a\in\Phi_b\setminus\{o\}} e^{-s\mathsf{h}_{\xi_a r(o)}\ell(\xi_a - r(o))\mathbf{1}(|\hat{\Phi}_a|>0)},$$

$$= \mathbb{E}\prod_{a\in\Phi_b\setminus\{o\}} 1 - (1 - e^{-s\mathsf{h}_{\xi_a r(o)}\ell(\xi_a - r(o))})\mathbf{1}(|\hat{\Phi}_a| > 0).$$

Averaging with respect to the fading random variables $\mathsf{h}_{\xi_a r(o)}$, we obtain

$$\mathcal{L}_{\mathtt{I}}(s) = \mathbb{E}\prod_{a\in\Phi_b\setminus\{o\}} 1 - \frac{\mathbf{1}(|\hat{\Phi}_a| > 0)}{1 + s^{-1}\ell(\xi_a - r(o))^{-1}}.$$

Since for any $b_i > 0$, $1 - \sum_i b_i \leq \prod_i 1 - b_i \leq \exp(-\sum_i b_i)$, we have

$$1 - \mathbb{E}\sum_{a\in\Phi_b\setminus\{o\}} \frac{\mathbf{1}(|\hat{\Phi}_a| > 0)}{1 + s^{-1}\ell(\xi_a - r(o))^{-1}} \leq \mathcal{L}_{\mathtt{I}}(s)$$

$$\leq \mathbb{E}\exp\left(-\sum_{a\in\Phi_b\setminus\{o\}} \frac{\mathbf{1}(|\hat{\Phi}_a| > 0)}{1 + s^{-1}\ell(\xi_a - r(o))^{-1}}\right).$$

The location of the interferer in cell a, *i.e.*, ξ_a can be represented as $\xi_a = a + f(a)$, where $f(a)$ is the relative location of the selected relay in cell a from the BS a. It is easy to observe that $|f(a)| < \infty$, as $f(a)$ is bounded by the diameter of the cell, which is finite. Hence it follows that $\|\xi_a\| \sim \|a\|\lambda_b^{-1/2}$ almost surely. Since $\exp(-x) \sim 1 - x$, $x \to 0$, for small λ_b we obtain

$$\mathcal{L}_{\mathtt{I}}(s) \sim 1 - \mathbb{E}\sum_{a\in\Phi_b\setminus\{o\}} \frac{\mathbf{1}(|\hat{\Phi}_a| > 0)}{s^{-1}\|a\|^{\alpha}\lambda_b^{-1/2}}.$$

Denoting by $\hat{\mu}$ the probability that $\hat{\Phi}_o$ is non empty, *i.e.*, $\hat{\mu} = \mathbb{P}(|\hat{\Phi}_o| > 0)$,

$$\mathcal{L}_{\mathtt{I}}(s) \sim 1 - \hat{\mu}s\lambda_b^{\alpha/2}\sum_{a\in\mathbb{Z}^2\setminus\{o\}} \|a\|^{-\alpha}. \quad (15)$$

As in Lemma 2, using the fact that $\sum_{a\in\mathbb{Z}^2\setminus\{o\}} \|a\|^{-\alpha}$ is the Epstein zeta function, and substituting $\lambda_b = \mathsf{SNR}^{-\beta}$, we obtain

$$\mathcal{L}_{\mathtt{I}}(s) \sim 1 - \hat{\mu}sC(\alpha)\mathsf{SNR}^{-\alpha\beta/2}. \quad (16)$$

We have $\hat{\mu} = 1 - \exp\left(-\int_{\mathbb{R}^2}\delta(x)\mathrm{d}x\right)$. Using (13),

$$\hat{\mu} \sim 1 - \exp\left(-\int_{\mathbb{R}^2}\eta(x)\left(1 - \mathsf{SNR}^{-1}\frac{\ell(R)\theta}{\ell(x)}\right)\cdot \left(1 - \mathsf{SNR}^{-\alpha\beta/2}\frac{\theta C(\alpha)}{\ell(x)}\right)\mathrm{d}x\right),$$

Using $\exp(-x) \sim 1 - x$ as $x \to 0$ and simplifying , we obtain

$$\hat{\mu} \sim \mu - (1-\mu)\theta\left(\mathsf{SNR}^{-1}\ell(R) + \mathsf{SNR}^{-\alpha\beta/2}C(\alpha)\right)\int_{\mathbb{R}^2}\frac{\eta(x)}{\ell(x)}\mathrm{d}x, \quad (17)$$

where (a) follows from (13). Using (17) in (16), we obtain the result.

$$\mathsf{P}_r \mid (n_o > 0) \sim \begin{cases} 1 - \mathsf{SNR}^{-1}\left(\frac{1-\mu}{\mu}\right)\theta\ell(R)\int_{\mathbb{R}^2}\left[\frac{1}{\ell(x-r(o))}+\frac{1}{\ell(x)}\right]\eta(x)\mathrm{d}x & \alpha\beta > 2, \\ 1 - \mathsf{SNR}^{-1}\left(\frac{1-\mu}{\mu}\right)\theta\int_{\mathbb{R}^2}\left[\frac{\ell(R)+\mu C(\alpha)}{\ell(x-r(o))}+\frac{\ell(R)+C(\alpha)}{\ell(x)}\right]\eta(x)\mathrm{d}x & \alpha\beta = 2, \\ 1 - \mathsf{SNR}^{-\alpha\beta/2}\left(\frac{1-\mu}{\mu}\right)\theta C(\alpha)\int_{\mathbb{R}^2}\left[\frac{\mu}{\ell(x-r(o))}+\frac{1}{\ell(x)}\right]\eta(x)\mathrm{d}x & \alpha\beta < 2. \end{cases} \tag{19}$$

$$\begin{aligned}\mathsf{P}_r \mid (n_o > 0) &\leq \mu^{-1}\left(1-\exp\left[-\int_{\mathbb{R}^2}\mathbb{E}\exp\left(-\frac{\theta(\sigma^2P^{-1}+\mathtt{I})}{\ell(x-r(o))}\right)\delta(x)\mathrm{d}x\right]\right), \\ &= \mu^{-1}\left(1-\exp\left[-\int_{\mathbb{R}^2}\exp\left(-\frac{\theta\sigma^2}{P\ell(x-r(o))}\right)\mathbb{E}\exp\left(-\frac{\theta\mathtt{I}}{\ell(x-r(o))}\right)\delta(x)\mathrm{d}x\right]\right). \end{aligned} \tag{21}$$

$$\mathsf{G}(\mathsf{SNR},\mathsf{SNR}^{-\beta}) \sim \begin{cases} \frac{\mu}{1-\mu}\ell(R)^{-1}\left[\int_{\mathbb{R}^2}\left[\frac{1}{\ell(x-r(o))}+\frac{1}{\ell(x)}\right]\eta(x)\mathrm{d}x\right]^{-1} & \alpha\beta > 2, \\ \frac{\mu}{1-\mu}(1+C(\alpha)\ell(R)^{-1})\left[\int_{\mathbb{R}^2}\left[\frac{\ell(R)+\mu C(\alpha)}{\ell(x-r(o))}+\frac{\ell(R)+C(\alpha)}{\ell(x)}\right]\eta(x)\mathrm{d}x\right]^{-1} & \alpha\beta = 2, \\ \frac{\mu}{1-\mu}\ell(R)^{-1}\left[\int_{\mathbb{R}^2}\left[\frac{\mu}{\ell(x-r(o))}+\frac{1}{\ell(x)}\right]\eta(x)\mathrm{d}x\right]^{-1} & \alpha\beta < 2. \end{cases} \tag{22}$$

■

To begin with, if the cell at the origin is empty then this two-hop scheme is not possible and hence for a fair comparison we condition on the cell at the origin being non-empty. The probability of success in two hops conditioned on the event that the cell is not empty is obtained as follows:

$$\begin{aligned}\mathsf{P}_r &= (\mathsf{P}_r \mid (n_o>0))\mathbb{P}(n_o>0) + (\mathsf{P}_r \mid (n_o=0))\mathbb{P}(n_o=0) \\ &\overset{(a)}{=} (\mathsf{P}_r \mid (n_o>0))\left(1-\exp\left(-\int_{\mathbb{R}^2}\eta(x)\mathrm{d}x\right)\right). \end{aligned} \tag{18}$$

(a) follows since the probability of decoding correctly by relaying when there are no relays is zero, *i.e.*, $(\mathsf{P}_r \mid (n_o = 0)) = 0$. Hence the success probability when the cell is not empty, *i.e.*, $|\Phi_x| > 0$, is equal to $\mathsf{P}_r \mid (n_o > 0) = \mathsf{P}_r\mu^{-1}$. The next theorem characterizes the conditional success probability for the opportunistic relay selection scheme.

Theorem 1: The success probability in the opportunistic relay selection scheme in the second hop as $\mathsf{SNR} \to \infty$ is given in (19), where $C(\alpha)$ is given in (9).

Proof: We first begin with the analysis of the unconditional outage probability in the second hop of the relaying scheme. From (4), the outage probability is

$$\begin{aligned}1-\mathsf{P}_r &= \mathbb{P}\left(\max_{x\in\hat{\Phi}_o}\{\mathsf{h}_{xr(o)}\ell(x-r(o))\} < \theta\left(\mathtt{I}+\frac{\sigma^2}{P}\right)\right), \\ &\overset{(a)}{=} \mathbb{E}\left[\prod_{x\in\hat{\Phi}_o}1-\exp\left(-\frac{\theta(\mathtt{I}+\sigma^2P^{-1})}{\ell(x-r(o))}\right)\right], \end{aligned}$$

where (a) follows since $\mathsf{h}_{xr(o)}$, $x \in \hat{\Phi}_o$, are independent exponential random variables. Since $\hat{\Phi}_o$ is a PPP with intensity function $\delta(x)$, using the probability generating functional [19] of a PPP,

$$1-\mathsf{P}_r = \mathbb{E}\exp\left[-\int_{\mathbb{R}^2}\exp\left(-\frac{\theta(\sigma^2P^{-1}+\mathtt{I})}{\ell(x-r(o))}\right)\delta(x)\mathrm{d}x\right]. \tag{20}$$

We now obtain bounds on the conditional probability $\mathsf{P}_r \mid (n_o > 0)$ using the relation (18). We first begin with an upper bound on $\mathsf{P}_r \mid (n_o > 0)$. The upper bound follows from Jensen's inequality for the first exponential and is given in (21). From Lemma 3, we have an asymptotic expression of $\mathbb{E}\exp\left(-\frac{\theta\mathtt{I}}{\ell(x-r(o))}\right)$, and for $\delta(x)$ in (13). We obtain (19) with straightforward algebraic manipulations and the fact $\exp(-x) \sim 1-x$, $x \to 0$. A lower bound to $\mathsf{P}_r \mid (n_o > 0)$ can be obtained by using the inequality $\exp(-x) \geq 1-x$, for the inner exponential in (20):

$$\begin{aligned}\mathsf{P}_r \mid (n_o > 0) \geq& \mu^{-1}\left(1-\exp\left(-\int_{\mathbb{R}^2}\delta(x)\mathrm{d}x\right)\cdot\right. \\ &\left.\mathbb{E}\exp\left(\int_{\mathbb{R}^2}\frac{\theta(\sigma^2P^{-1}+\mathtt{I})}{\ell(x-r(o))}\delta(x)\mathrm{d}x\right)\right). \end{aligned}$$

Using the same substitutions as in the upper bound, it can be shown that the lower bound also equals to (19), which completes the proof. ■

The gain of using the two-hop scheme over direct transmission and the diversity follow directly from the above Theorem, and is stated in the following corollary.

Corollary 2: The gain of using a two-hop relay scheme with opportunistic scheduling over direct transmission is given in (22) as $\mathsf{SNR} \to \infty$, and the diversity of this scheme is $\mathrm{d}_2(\mathsf{SNR}^{-\beta}) = \min\left\{1, \frac{\alpha\beta}{2}\right\}$.

Proof: The proof follows from Corollary 2, Theorem 1, and the definitions of the gain and diversity. ■

In the above analysis we assumed that the cell is non-empty and hence obtained a maximum diversity of 1. Conditioned on k nodes in each cell, a maximum diversity of k can be obtained as expected.

Let $\eta(y) = \lambda_m\mathbf{1}_y([-L/2, L/2]^2)$ be the intensity of the mobile nodes in a cell. In this case the gain is proportional to $(1-\exp(-\lambda_mL^2))\exp(\lambda_mL^2)$, and we observe that the gain increases exponentially with λ_m. The gain depends on the link distance R as $\ell(R)^{-1} = R^\alpha$ and hence increases with the distance. This highlights the importance of two-hop relaying schemes over simple direct transmissions specially for the cell edge users. We now check the validity of the theoretical results obtained by Monte-Carlo simulations. For the purpose of simulation we truncate the BS lattice to $\lambda_b^{-1/2}\{-2,-1,0,1,2\}^2$, and $\theta = 1.5$ is used as the decoding threshold. The cells are

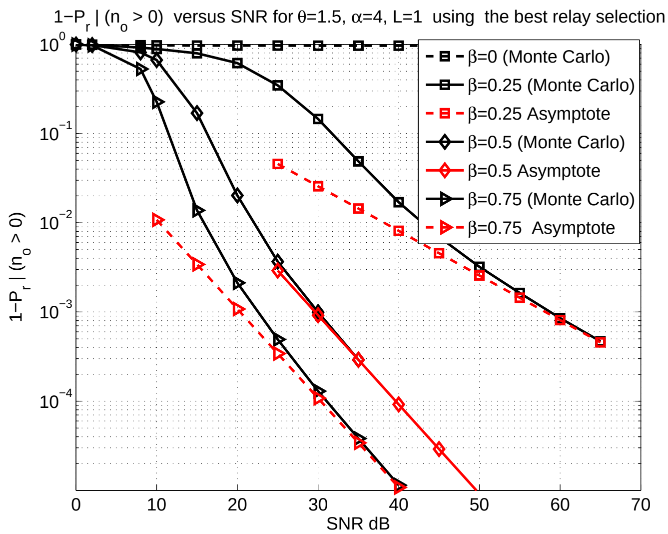

Fig. 3. Outage probability $1-\mathsf{P}_r \mid (n_o>0)$ versus SNR for $\lambda_b = \mathsf{SNR}^{-\beta}$ and various β. The system parameters are $\alpha = 4$, $\theta = 1.5$, $r(o) = (0.5, 0.5)$ and $\ell(x) = (1+\|x\|^4)^{-1}$. The dashed lines are the asymptotes derived in (19) and are approximately equal to $0.812\mathsf{SNR}^{-0.5}$(interference limited) and $0.108\mathsf{SNR}^{-1}$ (noise limited).

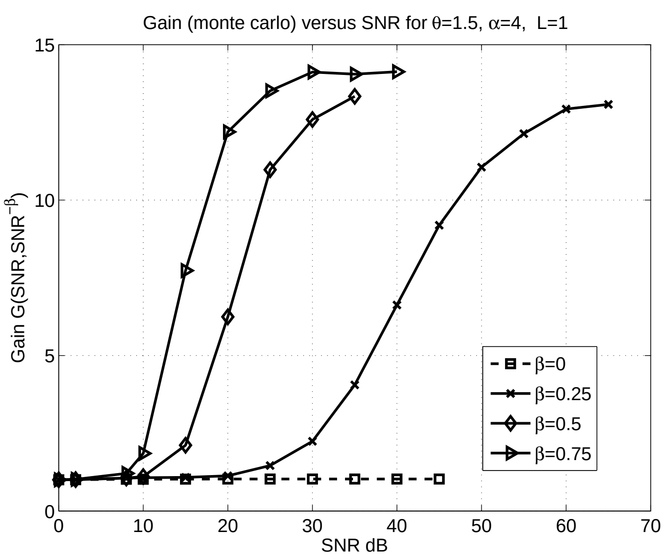

Fig. 4. $G(\mathsf{SNR}, \mathsf{SNR}^{-\beta})$ versus SNR for various β. The system parameters are $\alpha = 4$, $\theta = 1.5$, $r(o) = (0.5, 0.5)$ and $\ell(x) = (1+\|x\|^4)^{-1}$.

modeled as squares, and the destination of each BS is located at a random vertex of the square. The spatial density of mobiles in each cell is taken as $\eta(y) = 5\mathbf{1}_y([-0.5, 0.5]^2)$. In Figure 3, the error probability of the two-hop scheme is plotted. We observe that the asymptotes obtained from theory match perfectly with the simulation results, and the performance becomes better with increasing β. Also, as predicted by theory, the diversity obtained is 1 when $\alpha\beta > 2$, and is equal to $\alpha\beta/2$ otherwise. From Figure 4, we plot the gain $G(\mathsf{SNR}, \mathsf{SNR}^{-\beta})$ with respect to SNR for different β. When $\mathsf{SNR} \to \infty$ we observe that the gain increases to a constant further validating Corollary 2. In Figure 5, we observe that the asymptotic

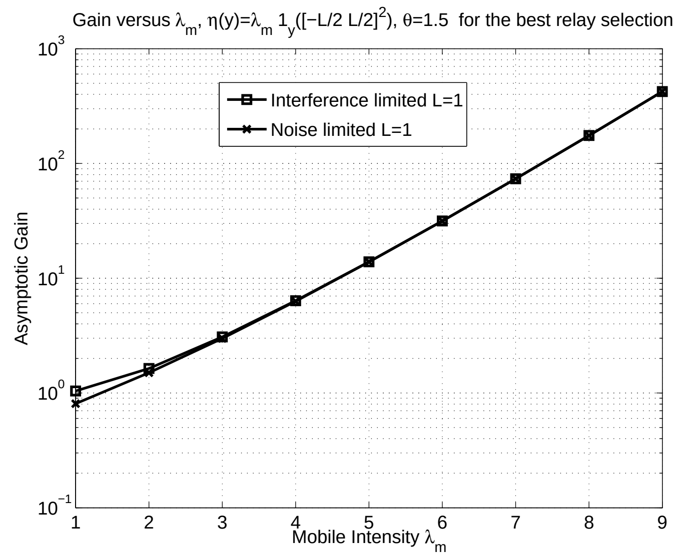

Fig. 5. Asymptotic gain versus λ_m where λ_m is the intensity in $\eta(y) = \lambda_m \mathbf{1}_y([-L/2, L/2]^2)$, $\ell(x) = \|x\|^{-\alpha}$, $\theta = 1.5$ and $r(o) = (-L/2, L/2)$.

gain increases exponentially with λ_m because of the factor $(1-\mu)/\mu$ factor in the expression for the asymptotic gain.

V. Conclusions

In this paper we have analyzed the outage in a two-hop cellular system inclusive of all the node location statistics. Asymptotic outage results were provided for opportunistic relay selection scheme. From these results we observed that the diversity obtained is $\min\{1, \alpha\beta/2\}$ where α is the path loss exponent, when the density of the base stations scales as $\lambda_b = \mathsf{SNR}^{-\beta}$. We show that the interference scales as $\lambda_b^{\alpha/2}$ which implies $\mathsf{SIR} = \Theta(\mathsf{SNR}^{\alpha\beta/2})$. From this result we can infer that the system is noise-limited (even for high SNR) when $\alpha\beta > 2$ and interference-limited otherwise. The asymptotic outage gain of the two-hop system over direct transmission takes only two values as a function of β. This implies that at very high SNR and low SIR, it only matters if the system is interference-limited or noise-limited. The gain in selecting a relay with the best channel over a direct transmission increases exponentially with the density of the *available* relays. The gain also increases with increasing source-destination distance. The techniques introduced in this paper can be easily extended for the spatial analysis of other relay selection schemes.

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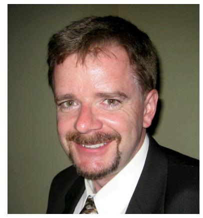

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