Random Power Control in Poisson Networks

Xinchen Zhang, *Student Member, IEEE,* and Martin Haenggi, *Senior Member, IEEE*

***Abstract*—This paper studies power control strategies in interference-limited wireless networks with Poisson distributed nodes. We concentrate on two sets of strategies: single-node optimal power control (SNOPC) strategies and Nash equilibrium power control (NEPC) strategies. SNOPC strategies maximize the expected throughput of the power-controllable link given that all the other transmitters do not use power control. Under NEPC strategies, no individual node of the network can achieve a higher expected throughput by unilaterally deviating from these strategies.**

We show that under mean and peak power constraints at each transmitter, the SNOPC and NEPC strategies are ALOHA-type random on-off power control policies, whose transmit powers and transmit probabilities depend on the knowledge about the network at each transmitter. Moreover, the resulting NEPC strategies achieve a higher spatial average throughput of the network than constant power transmission. These results suggest that ALOHA can be viewed not only as a MAC scheme but also as a stable and efficient power control scheme.

***Index Terms*—Wireless networks, power control, interference, ALOHA, stochastic geometry, game theory.**

I. Introduction

POWER control benefits wireless communication in many different ways (see [1]–[3] and the references therein). In wireless networks, two main approaches have been used to analyze and design sensible power control policies: the optimization approach and the game theory approach. The optimization approach takes a global point of view and aims at finding the assignment of power that maximizes some global metric [4]–[7]. Recent efforts concentrate on finding such assignment by distributed algorithms, and algorithms with different merit have been proposed in both infrastructure (cellular) networks [8] and infrastructureless (ad hoc) networks [4], [7], [9]. Instead of modeling network users as cooperative individuals, the game theory approach views the network as a collection of selfish users with conflicting interests [10]. Many forms of games have been introduced to facilitate the analysis and design (see [11]–[13] and the references therein). Although properly designed the games (*e.g.,* by designing pricing structures) are often used to find solutions that maximize some global utility (*e.g.,* [9], [14]–[16]), the key merit of game theory arguments lies in revealing the robust power control strategies such that malicious users cannot benefit from deviating from them, which can hardly be achieved by other approaches. A detailed survey of existing power control schemes is presented in [17].

Unlike most of the other work listed above where the resulting power control strategy is characterized by a deterministic power level at each transmitter, we allow the transmit power at each node to be a random variable with arbitrary distribution subject to a (unit) mean power and a peak power constraint. The results of this paper show that this additional randomness can potentially be highly beneficial.

We focus on the case where any inter-node coordination is not allowed and the transmitters decide on their own power control strategies to maximize their own expected throughput. Modeling power control as a non-cooperative game among transmitters, we characterize two types of power control strategies: 1. Single-node optimal power control (SNOPC) strategies when only one node in the network uses power control; 2. Nash equilibrium power control (NEPC) strategies when all the nodes in the network use power control. SNOPC strategies maximize the expected throughput of the power-controllable link, whereas NEPC strategies ensure that no individual node of the network can achieve a higher expected throughput by unilaterally deviating from these strategies. In the discussion of each type of strategy, we consider three different levels of information available at the transmitters, which can be interpreted as corresponding to three levels of mobility of the network. It turns out that, in many cases, ALOHA-type random on-off power control policies are single-node optimal and constitute Nash equilibria.

While this paper is not the first one to demonstrate the benefits of randomly varying the transmit power in wireless networks, only a very limited number of papers focus on it [18]–[20]. [20] considers a noise-limited wireless network and shows that an ALOHA-type random on-off power control policy maximizes the expected throughput. This paper extends this result to interference-limited networks, where the choice of power control policy is a function of the power control strategies at all other nodes (the interferers). While [18], [19] demonstrate the benefits of random power control in the presence of interference, both papers fail to justify their choice of random power control distribution (uniform distribution). Here, we show that there is a specific type of simple transmit power distribution (ALOHA-type random on-off) that not only maximizes a single user's throughput but also constitutes the network-wide Nash equilibrium.

Random power control schemes also naturally appear as mixed strategies in game-theoretic frameworks, *e.g.,* [21]. However, in this context, the combination of mean *and* peak power constraints is usually not considered. We show that these constraints have a significant impact on the strategies of interest. In particular, it turns out that both the SNOPC and NEPC strategies are ALOHA-type random on-off policies .

Paper approved by A. MacKenzie, the Editor for Game Theory and Cognitive Networking of the IEEE Communications Society. Manuscript received October 30, 2011; revised February 17, 2012.

The authors are with the Department of Electrical Engineering, University of Notre Dame, 275 Fitzpatrick Hall, Notre Dame, IN, 46556, USA (e-mail: {xzhang7, mhaenggi}@nd.edu). The corresponding author is X. Zhang.

This work was partially supported by the NSF (grant CNS 1016742) and the DARPA IT-MANET program through grant W911NF-07-1-0028.

Digital Object Identifier 10.1109/TCOMM.2012.071312.110730

Another important feature that contrasts this paper with others in the power control literature is the use of a stochastic point process to model the network, and the explicit separation of channel uncertainty due to node location and fading. Although powerful tools from stochastic geometry have been introduced in the analysis of large wireless networks over the last decade [22], only a very small part of the power control literature considers the spatial distribution of wireless networks. Instead, most of the papers do not differentiate the path loss due to fading and path loss due to random location of the nodes. Such simplification does not matter when the (combined) channel state is known at the transmitters, *e.g.,* [4], [9], or stays fixed over time and thus can be learned gradually through channel feedback, *e.g.,* [15], [16], [21]. Yet, it prohibits the discovery of efficient power control schemes when the network has limited capability to acquire perfect channel state information. In this paper, we explicitly account for these two sources of randomness.

The rest of the paper is organized as follows: Section II introduces the system model and specifies three cases of interest (with different levels of information available at the transmitters). Section III, IV and V analyze the SNOPC and NEPC strategies in three cases. The performance of these strategies is evaluated in Section VI. Section VII concludes the paper.

II. System Model

A. Network Model

The network topology is represented as a marked Poisson point process (PPP) $\hat{\Phi} = \{(x_i, y_{x_i})\} \subset \mathbb{R}^2 \times \mathbb{R}^2$, where $\Phi = \{x_i\}$ is a homogeneous PPP with intensity λ and denotes the location of the transmitters, and the mark y_x denotes the location of a dedicated receiver of transmitter x. The link distances $R_x \triangleq \|x - y_x\|$ are iid with distribution f_R.

We consider the following SIR model, where a transmission attempt from z to y_z is considered successful iff

$$\mathrm{SIR}_z \triangleq \frac{S_z}{I_z} > \theta,$$

where $S_z = P_z h_z \|z - y_z\|^{-\alpha}$, $I_z = \sum_{x \in \Phi \setminus \{z\}} P_x h_{xz} \|x - y_z\|^{-\alpha}$, P_x is the transmit power at node $x \in \Phi$, $\alpha > 2$ is the path-loss exponent, θ is the SIR threshold, and h_z and h_{xz} are (power) fading coefficients from the desired transmitter and the interferer x to z respectively. We focus on the iid Rayleigh fading case, thus h_z and the sequence (h_{xz}) are iid exponentially distributed with unit mean. In the following, we use I for I_z for simplicity.

B. Game-Theoretic Formulation

The players in the game are all the transmitters in the network $x \in \Phi$. Each player can select a strategy s_x from a common set of stationary strategies $\mathcal{S}$. Here, $\mathcal{S}$ is the set of distributions with (at most) unit mean and with support (at most) $[0, P_{\max}]$, where $P_{\max} > 1$ (otherwise, the mean power constraint would always be loose).

The strategy each node chooses is based on its knowledge about the network. We use $\mathcal{K}_x$ to denote this knowledge available at node x and split our discussion into three cases: case 1: $\mathcal{K}_x = \{f_R\}$, case 2: $\mathcal{K}_x = \{\lambda, R_x, f_R\}$, and case 3: $\mathcal{K}_x = \{\hat{\Phi}\}$. These three cases represent different levels of information in ascending order. Case 1 and case 2 are more suitable models for high-mobility networks, where only very limited network information can be acquired at each node. Case 3 applies to static networks, where the complete network topology information can be either provided off-line or learned gradually by each link.

The pay-off of node $x \in \Phi$ is its own expected throughput (success probability) averaged over all the randomness in the rest of the network, *i.e.,* $\pi_x(s_x) = p_{s|\mathcal{K}_x}(s_x) = \mathbb{P}(\frac{S_x}{I_x} > \theta \mid \mathcal{K}_x, s_x(\mathcal{K}_x))$. The single-node optimal power control (SNOPC) strategy of node x maximizes $\pi_x(\cdot)$ if all the other transmitters in the network transmit with unit power (no power control). If all the transmitters in the network use power control, we say that a strategy set $\{s_x(\mathcal{K}_x), x \in \Phi\}$ is a *Nash equilibrium* and $s_x(\mathcal{K}_x)$ is the Nash equilibrium power control (NEPC) strategy if none of the transmitters is willing to unilaterally deviate from its current strategy as that cannot increase its pay-off (expected throughput).

In addition to the game-theoretic framework above, we study the global impact of SNOPC and NEPC by evaluating the spatially averaged throughput, (or, simply spatial throughput), defined as the throughput (success probability) of a typical node in the network, which can be expressed as

$$p_s = \mathbb{E}^{!x}[\pi_x(s_x)],$$

where $\mathbb{E}^{!x}$ is the expectation with respect to the reduced Palm measure. Loosely speaking, it is the expectation conditioned on the existence of a point at x but not counting it. More details about the Palm measure and its applications in wireless networks can be found in [22] and the references therein. In the case of a PPP, by Slivnyak's theorem, $\mathbb{E}^{!x} = \mathbb{E}$, *i.e.,* having a node at location x does not change the distribution of the point process [23].

III. Case 1: Unknown Link Distances

We first consider the case of $\mathcal{K}_x = \{f_R\}$, *i.e.,* only the distribution of R_x is known at the nodes with power control capability. In particular, we consider the case where the link distances R_x are Rayleigh distributed with mean $1/2\sqrt{\lambda_r}$, *i.e.,* $f_R(x) = 2\lambda_r \pi x \exp(-\lambda_r \pi x^2)$. This distribution is of interest because f_R is the distribution of the link distances when each node of Φ tries to connect to its nearest neighbor in an independent homogeneous PPP of intensity λ_r [24].

Proposition 1. *If only the node $z \in \Phi$ can use power control but all other transmitters transmit with unit power and R is Rayleigh distributed, the SNOPC strategy at z is constant power transmission (no power control).*

Proof: Since Φ is motion-invariant, without loss of generality, place the desired receiver at origin, *i.e.,* $y_z = o$. Then, if we let h_x be the iid fading coefficient from x to o, the

Laplace transform of the interference I can be expressed as

$$\begin{aligned}\mathcal{L}_I(s) &= \mathbb{E}\Big[\prod_{y\in\Phi\setminus\{z\}} e^{sh_y\|x\|^{-\alpha}} \mid z\in\Phi\Big]\\ &\overset{(a)}{=} \mathbb{E}^{!z}\Big[\prod_{y\in\Phi\setminus\{z\}} e^{sh_y\|y\|^{-\alpha}}\Big] \overset{(b)}{=} \mathbb{E}\Big[\prod_{y\in\Phi} e^{sh_y\|x\|^{-\alpha}}\Big]\\ &\overset{(c)}{=} \exp\left(-\lambda\pi s^{\delta}\frac{\pi\delta}{\sin(\pi\delta)}\right),\end{aligned}$$

where (a) is due to the definition of Palm distribution, (b) is due to Slivnyak's theorem, (c) is shown in [25] and $\delta = 2/\alpha$. It is assumed that $\alpha > 2$ (otherwise the interference is infinite almost surely), so $\delta < 1$.

Thus, for an arbitrary power control policy characterized by the random variable P, the success probability can be written as

$$\begin{aligned}&\mathbb{P}(PhR^{-\alpha} > \theta I)\\ &= \mathbb{E}_{P,R}[\mathbb{P}(PhR^{-\alpha} > \theta I) \mid P, R] = \mathbb{E}_P\mathbb{E}_R[\mathcal{L}_I(s)|_{s=\frac{\theta R^\alpha}{P}}]\\ &= \mathbb{E}_P\left[\int_0^\infty 2br\exp\left(-a\left(\frac{\theta r^\alpha}{P}\right)^{\delta}\right)\exp(-br^2)\mathrm{d}r\right]\\ &= \mathbb{E}_P\left[\frac{b}{a\left(\frac{\theta}{P}\right)^{\delta}+b}\right],\end{aligned}$$

where $a = \lambda\pi\frac{\pi\delta}{\sin(\pi\delta)}$ and $b = \lambda_r\pi$. Since $\frac{b}{a(\theta/x)^\delta+b}$ is concave for $a, b, \theta > 0$ and $0 < \delta < 1$, by Jensen's inequality, the throughput is maximized when choosing $P \equiv \mathbb{E}P = 1$. ■

Proposition 1 shows that if R is Rayleigh distributed and the rest of the network uses constant power transmission, the best strategy at node z is constant power transmission, regardless of the values of λ and λ_r. Since z is arbitrarily chosen, this immediately implies that constant power transmission at all nodes is a Nash equilibrium. However, in order to find out whether there are other Nash equilibria, we need to study the interference distribution when the rest of the network uses power control.

Lemma 1. *If the interferers are distributed as a homogeneous Poisson point process Φ with intensity λ and the transmit power at each transmitter is drawn iid from the same distribution f_P, the interference observed at any receiver y_z with $z \in \Phi$ has the Laplace transform*

$$\mathcal{L}_I(s) = \exp(-\lambda c_d\mathbb{E}[P^\delta]\mathbb{E}[h^\delta]\Gamma(1-\delta)s^\delta).$$

Proof: First, by Slivnyak's theorem, $\mathcal{L}_I(s) = \mathbb{E}[\prod_{x\in\Phi} e^{-sP_xh_x\|x\|^{-\alpha}}]$, where P_x is the transmit power at x. Second, since (P_x), $x \in \Phi$, is iid, P_xh_x can be considered as a new fading coefficient $\tilde{h}_y$. The proof is then completed by the Laplace transform of the interference distribution for arbitrary iid fading with finite δ-th moment [25, Sec. 3.2]. ■

Proposition 2. *If all nodes are capable of power control, constant power transmission is the unique NEPC policy in Poisson networks with Rayleigh distributed unknown link distances.*

Proof: By Proposition 1, the fact that constant power transmission is a NEPC strategy is evident. To show its uniqueness, we start by noting that the information available at each individual node $\mathcal{K}_x = \{f_R\}$ is the same. Thus, given that all the nodes are completely selfish and sufficiently and equally smart, at any Nash equilibrium, their choice of power control strategy must be the same. Assume the common choice of power control strategy (of the rest of the network) is characterized by the random variable $\tilde{P}$ with pdf $f_{\tilde{P}} \in \mathcal{S}$, and let the interference observed at arbitrary receiver y_z under this power control policy be $\tilde{I}$. Then, Lemma 1 gives the Laplace transform of the $\tilde{I}$. Straightforward manipulation shows

$$\mathbb{P}(PhR^{-\alpha} > \theta\tilde{I}) = \mathbb{E}_P\left[\frac{b}{\tilde{a}\left(\frac{\theta}{P}\right)^{\delta}+b}\right], \quad (1)$$

where $\tilde{a} = \lambda\pi\frac{\pi\delta}{\sin(\pi\delta)}\mathbb{E}[\tilde{P}^\delta]$, $b = \lambda_r\pi$, and P is the transmit power at z. As in the proof of Proposition 1, we can show that $P \equiv 1$ maximizes (1) under the unit mean power constraint. Since z is arbitrarily chosen, we have $P = \tilde{P} \equiv 1$. ■

Corollary 1. *If the transmitters and receivers are distributed as two independent homogeneous Poisson point processes $\Phi, \Phi_r \subset \mathbb{R}^2$ and all the transmitters in Φ try to connect to their nearest neighbor in Φ_r, constant power transmission is the NEPC strategy.*

This corollary is a straightforward extension of Proposition 2. However, this result hinges critically on the special form of Nash equilibrium (constant power transmission), in Proposition 2. A similar result cannot be obtained in the two cases discussed in the next two sections.

IV. Case 2: Known Link Distance

In this section, we consider the case where $\mathcal{K}_x = \{\lambda, R_x, f_R\}$, *i.e.,* the nodes with power control capability know the network density, the distances to their own dedicated receivers and the distribution of the link distance of the whole network. We first derive the form of the SNOPC and NEPC strategies for general f_R. Then, we take the Poisson bipolar network, *i.e.,* $f_R(x) = \delta(x - r)$, as an example to further illustrate the NEPC strategy.

A. General f_R

In this subsection, we start with the SNOPC strategy and then study the Nash equilibrium. First, we present a lemma:

Lemma 2. *Given a link of length $R = r$, if there exists $x_0 > 0$ such that $x\mathcal{L}_I(\theta r^\alpha x)$ is monotonically increasing for $x < x_0$ and monotonically decreasing for $x > x_0$, the power control strategy that maximizes the throughput at node x is random on-off power control with transmit power γ and transmit probability γ^{-1} where $\gamma = \max\{1, \min\{P_{\max}, x_0^{-1}\}\}$.*

Proof: For interference-limited Rayleigh fading networks, the success probability of a transmission at power P is $\mathcal{L}_I(s)|_{s=\frac{\theta r^\alpha}{P}}$. Thus, the success probability of any power control strategy characterized by the pdf f_P of the random variable P is

$$p_s = \mathbb{E}_P\left[\mathcal{L}_I(s)|_{s=\frac{\theta r^\alpha}{P}}\right] = \int_0^\infty \mathcal{L}_I\left(\frac{\theta r^\alpha}{x}\right)f_P(x)\mathrm{d}x. \quad (2)$$

It is easy to show that $\mathcal{L}_I(x)$ is a valid ccdf, *i.e.,* $\mathcal{L}_I(0) = 1$, $\lim_{x\to\infty}\mathcal{L}_I(x) = 0$, and $\mathcal{L}_I(x)$ is monotonically decreasing on $[0, \infty)$. So, instead, we can consider an interferenceless link of distance r with another fading random variable $\tilde{h}$ whose ccdf is $\bar{F}_{\tilde{h}}(x) = \mathcal{L}_I(x)$. The success probability is

$$\begin{aligned}\tilde{p}_s &= \mathbb{P}(P\tilde{h}r^{-\alpha} > \theta) = \mathbb{E}_P\left[\bar{F}_{\tilde{h}}\left(\frac{\theta r^\alpha}{P}\right)\right] \\ &= \int_0^\infty \mathcal{L}_I\left(\frac{\theta r^\alpha}{x}\right) f_P(x)\mathrm{d}x.\end{aligned} \tag{3}$$

Comparing (2) and (3), we find that finding the SNOPC strategy that maximizes p_s and finding the one for $\tilde{p}_s$ are two identical problems. The latter problem has already been solved in [20]. In particular, Theorem 2 in [20] shows that if there exists a x_0 as in the statement of the lemma, subject to the constraints $\mathbb{E}P \leq 1$ and $P \leq P_{\max}$, $\tilde{p}_s$ is maximized when $f_P(x) = (1-\gamma^{-1})\delta(x) + \gamma^{-1}\delta(x-\gamma)$, where $\gamma = \max\{1, \min\{P_{\max}, x_0^{-1}\}\}$. ■

Corollary 2. *If the Laplace transform of the interference I has the form $\mathcal{L}_I(s) = \exp(-as^\delta)$, where $\delta = 2/\alpha$ and $a > 0$, the throughput-maximizing power control strategy at any transmitter $z \in \Phi$ with $R_z = r$ is a random on-off power control strategy with transmit power γ and transmit probability γ^{-1}, where $\gamma = \max\{1, \min\{P_{\max}, (a\delta)^{1/\delta}\theta r^\alpha\}\}$.*

Corollary 2 is proved by simply verifying that the form of the Laplace transform of the interference satisfies the conditions in Lemma 2.

Proposition 3. *If only one node $z \in \Phi$ with $R_z = r$ uses power control and all other nodes $\Phi\backslash\{z\}$ transmit at unit power, the SNOPC strategy of z is an ALOHA-type random on-off power control strategy with transmit power γ and transmit probability γ^{-1}, where $\gamma = \max\{1, \min\{P_{\max}, \left(\lambda\frac{\pi^2\delta^2}{\sin(\pi\delta)}\right)^{1/\delta}\theta r^\alpha\}\}$.*

Proof: The proposition follows directly from Lemma 1 ($P \equiv 1$) and Corollary 2. ■

Moreover, since the transmit power at each node $x \in \Phi$ is a (stochastic) function of the link distances $R_x = r$, where the R_x are spatially iid, Lemma 1 shows that the interference always has a Laplace transform in the form $\exp(-as^\delta)$, regardless of what kind of power control strategy is applied at each node. Then, the proposition below follows.

Proposition 4. *ALOHA-type random on-off power control is the unique NEPC strategy in a wireless network where the transmitters are distributed as a homogeneous Poisson point process Φ and $\mathcal{K}_x = \{\lambda, R_x, f_R\}$, for all $x \in \Phi$.*

Proof: The fact that ALOHA-type random on-off power control at each node is a Nash equilibrium can be deduced directly from Lemma 1 and Corollary 2. In particular, we can write $\mathbb{E}[P^\delta]$ in terms of the throughput-maximizing random on-off strategy at each link, which yields

$$\begin{aligned}\mathbb{E}[P^\delta] &= \mathbb{E}_R[P_R^\delta] = \mathbb{E}_R[\gamma_R^{-1}\gamma_R^\delta] \\ &= \mathbb{E}_R\left[\min\left\{1, \max\left\{P_{\max}^{\delta-1}, \frac{(a\delta\mathbb{E}[P^\delta])^{1-1/\delta}}{(\theta R^\alpha)^{1-\delta}}\right\}\right\}\right],\end{aligned} \tag{4}$$

where $a = \lambda\pi\frac{\pi\delta}{\sin(\pi\delta)}$. Note that the RHS of (4) is a monotonically decreasing function of $\mathbb{E}[P^\delta]$ (since $1 - 1/\delta < 0$), and when $\mathbb{E}[P^\delta] = 0$, its value is $P_{\max}^{\delta-1} > 0$. Thus, there is a unique $\mathbb{E}[P^\delta] > 0$ satisfying (4). Once this value is found, the optimal power control strategy at x is simply an ALOHA policy with transmit power γ and transmit probability γ^{-1}, where $\gamma = \max\{1, \min\{P_{\max}, \left(\lambda\mathbb{E}[P^\delta]\frac{\pi^2\delta^2}{\sin(\pi\delta)}\right)^{1/\delta}\theta R_x^\alpha\}\}$.

Moreover, Lemma 1 also says that no matter what kind of power control policy is applied in the rest of the network, the interference distribution observed at an arbitrary receiver has a Laplace transform of the form $\mathcal{L}_I(s) = \exp(-as^\delta)$. Thus, Corollary 2 also indicates the uniqueness. ■

B. Bipolar Networks

In general, analytically solving for the Nash equilibrium in (4) is difficult. As a special case, when all the link distances are known and constant, the network model becomes a Poisson bipolar model [26], for which we have the following result:

Corollary 3. *If all the link distances are r, the NEPC strategy is an ALOHA-type random on-off policy with transmit power γ and transmit probability γ^{-1} where $\gamma = \max\{1, \min\{P_{\max}, \lambda\frac{\pi^2\delta^2}{\sin(\pi\delta)}\theta^\delta r^2\}\}$, λ is the density of the transmitters and $\delta = 2/\alpha$.*

Proof: For Rayleigh fading, h is exponentially distributed with mean 1, and thus $\mathbb{E}[h^\delta] = \Gamma(1+\delta)$. Then, when the link distances are the same, (4) becomes

$$\mathbb{E}[P^\delta] = \left(\max\left\{1, \min\left\{P_{\max}, \left(a\delta\mathbb{E}[P^\delta]\right)^{1/\delta}\theta r^\alpha\right\}\right\}\right)^{\delta-1}, \tag{5}$$

where $a = \lambda\pi\frac{\pi\delta}{\sin(\pi\delta)}$. Solving this equation for $\mathbb{E}[P^\delta]$ and applying to $\max\{1, \min\{P_{\max}, (\lambda\mathbb{E}[P^\delta]\frac{\pi^2\delta^2}{\sin \pi\delta})^{1/\delta}\theta r^\alpha\}\}$ yields the desired result. ■

Corollary 3 says that in any case, an ALOHA-type random on-off policy is the NEPC policy in a Poisson bipolar network. For ALOHA-type random on-off strategies with transmit power γ and transmit probability γ^{-1}, we define the following regimes to facilitate our illustration.

Definition 1. *A random on-off power control strategy is said to be in its peak-power-limited regime if $\gamma = P_{\max}$.*

Definition 2. *A random on-off power control strategy is said to be in its bandwidth-limited regime if $\gamma = 1$.*

As a MAC scheme, ALOHA in Poisson bipolar networks is well analyzed in the literature. In particular, [26] derived the ALOHA scheme that maximizes the spatial throughput in Poisson bipolar networks, which we call the globally optimal (GOPT) ALOHA scheme. GOPT ALOHA maximizes the spatial throughput by properly choosing transmit probability, and when only the SIR is considered, the absolute transmit power does not affects its optimality. However, in order to make a fair comparison with the NEPC strategy, we interpret GOPT as a power control scheme and always choose the maximum transmit power for GOPT under the mean and peak power constraints. Then, it can be shown that the transmit probability of GOPT is $p_{\text{GOPT}} = \min\{1, \left(\lambda\frac{\pi^2\delta}{\sin(\pi\delta)}\theta^\delta r^2\right)^{-1}\}$, and the transmit power is $P_{\text{GOPT}} = \min\{p_{\text{GOPT}}^{-1}, P_{\max}\}$. For the same set of parameters, we always have $P_{\text{GOPT}} \geq \gamma$ and

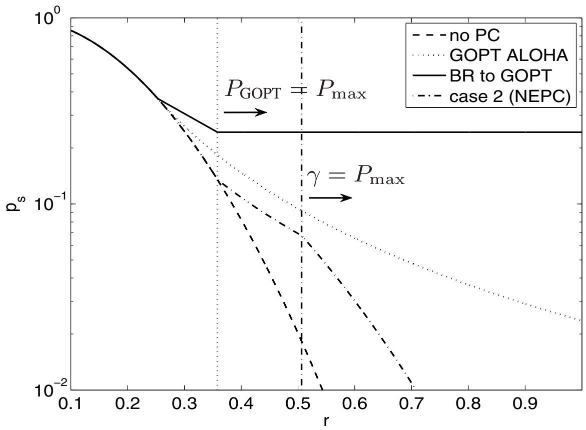


Fig. 1: Comparison of throughput using 1) constant power transmission (no power control) 2) GOPT ALOHA 3) the best response to the GOPT ALOHA and 4) NEPC strategy. Here, $\lambda = 1$, $P_{\max} = 2$, $\alpha = 2$, $\theta = 10$. To the right of the two vertical lines, the transmit power of GOPT and NEPC strategy hits their corresponding peak power limits.

$p_{\rm GOPT} \leq \gamma^{-1}$, where γ is the transmit power of the Nash equilibrium power control strategy. In other words, GOPT achieves higher spatial throughput by forcing each transmitter to back off on their transmit probability.

However, GOPT is unstable in the sense that any selfish link can apply another power control strategy and thus obtain a performance far better than anyone else. It is not difficult to see (by slight variation to Proposition 3) that the best response of any individual link in a Poisson bipolar network applying GOPT is an ALOHA policy with transmit power $\gamma_{\rm BR}$ and transmit probability $\gamma_{\rm BR}^{-1}$, where the subscript BR stands for best response and

$$\begin{aligned}\gamma_{\rm BR} &= \max\left\{1, \min\left\{P_{\max}, \left(a\delta p_{\rm GOPT} P_{\rm GOPT}^{\delta}\right)^{1/\delta}\theta r^{\alpha}\right\}\right\}\\ &= \max\{1, P_{\rm GOPT}\delta^{\delta}\}.\end{aligned}$$

Here, $\gamma_{\rm BR}^{-1} \geq p_{\rm GOPT}$, and the equality holds only when $\gamma_{\rm BR}^{-1} = p_{\rm GOPT} = 1$, *i.e.,* both strategies operate in the bandwidth-limited regime.

Fig. 1 compares the spatial throughput of 4 strategies: constant-power transmission (no power control), NEPC strategy, the globally optimal (GOPT) ALOHA, and the best response to GOPT in Poisson bipolar networks. We can see from the figure that the Nash equilibrium power control policy has a better performance than constant power transmission. As expected, outside the bandwidth-limited regime of both GOPT and NEPC, NEPC has a spatial throughput strictly smaller than GOPT. However, the performance gain of GOPT over NEPC mostly comes from forcing each transmitter in the network to reduce its mean transmit power and thus manage the interference, *i.e.,* for large r, $p_{\rm GOPT}P_{\rm GOPT} < 1$. Fig. 1 shows that in such cases, if any node cheats by using another power control strategy, in particular, the best response to GOPT, its expected throughput gain is significant. Such gain can be a strong incentive for individual links to cheat.

V. Case 3: Static Network

This section considers the case where $\mathcal{K}_x = \{\hat{\Phi}\}$. This assumption is particularly interesting in a static network, where the topology of the network can be acquired either directly off-line or gradually by observing the interference from the rest of the network. In fact, in this context, the assumption on the spatial distribution of the network becomes unimportant. We are in fact considering an arbitrary realization of the random network. Results in this section thus also apply to deterministic networks.

First, we provide a lemma to explicitly address the case where $x\mathcal{L}_I(\theta r^{\alpha} x)$ is monotonically increasing for all $x > 0$. This lemma complements Lemma 2 as it is essentially considering the case where $x_0 = \infty$.

Lemma 3. *If the Laplace transform of the interference I is $\mathcal{L}_I(s)$ and $x\mathcal{L}_I(\theta r^{\alpha} x)$ is monotonically increasing for all $x > 0$, the optimal power control strategy is constant power transmission,* i.e., $P \equiv 1$.

Proof: Construct another random variable $\tilde{I}$ with Laplace transform

$$\mathcal{L}_{\tilde{I}}(s) = \mathcal{L}_I(s)1_{[0,\theta r^{\alpha}]}(s) + \frac{1}{s^2}\mathcal{L}_I(\theta r^{\alpha})1_{(\theta r^{\alpha},\infty)}(s),$$

for all $s > 0$. Since $x\mathcal{L}_I(\theta r^{\alpha} x)$ is monotonically increasing, it can be verified that $x\mathcal{L}_{\tilde{I}}(\theta r^{\alpha} x)$ is monotonically increasing on $[0, 1]$ and strictly decreasing on $(1, \infty)$. Applying Lemma 2, we know that

$$\max_{f_P \in \mathcal{S}} \int_0^{\infty} \mathcal{L}_{\tilde{I}}\left(\frac{\theta r^{\alpha}}{x}\right) f_P(x)\mathrm{d}x = \mathcal{L}_{\tilde{I}}(\theta r^{\alpha}) = \mathcal{L}_I(\theta r^{\alpha}),$$

where the maximum is achieved when $P \equiv 1$.

Let f_P^* be the distribution of the optimal power control policy for interference I. With the help of Lemma 5 in [20], it is straightforward to show that $f_P^*(x) = 0$, for $x < 1$, and

$$\begin{aligned}\max_{f_P \in \mathcal{S}} \int_1^{\infty} \mathcal{L}_I\left(\frac{\theta r^{\alpha}}{x}\right) f_P(x)\mathrm{d}x &\overset{(a)}{=} \max_{f_P \in \mathcal{S}} \int_1^{\infty} \mathcal{L}_{\tilde{I}}\left(\frac{\theta r^{\alpha}}{x}\right) f_P(x)\mathrm{d}x\\ &\overset{(b)}{=} \max_{f_P \in \mathcal{S}} \int_0^{\infty} \mathcal{L}_{\tilde{I}}\left(\frac{\theta r^{\alpha}}{x}\right) f_P(x)\mathrm{d}x = \mathcal{L}_I(\theta r^{\alpha}),\end{aligned}$$

where (a) is due to the definition of $\tilde{I}$ and (b) is due to the fact that SNOPC policy for $\tilde{I}$ is constant power transmission. The lemma then follows the fact that when $P \equiv 1$, $\int_0^{\infty} \mathcal{L}_I(\frac{\theta r^{\alpha}}{x}) f_P(x)\mathrm{d}x = \mathcal{L}_I(\theta r^{\alpha})$, *i.e.,* the maximum success probability is achieved. ■

Proposition 5. *If only one transmitter at $z \in \Phi$ uses power control and the transmitter knows the positions of all interferers, the SNOPC strategy at this link is an ALOHA-type power control policy.*

Proof: Assume the desired receiver is located at o and the positions of all interferers are ϕ_z, *i.e.,* ϕ_z is one realization of $\Phi \backslash \{z\}$. Then, the Laplace transform of the interference $I \mid \phi_z = \sum_{x\in\phi_z} h_x \|x\|^{-\alpha}$ is

$$\mathcal{L}_{I|\phi_z}(s) = \prod_{x\in\phi_z} \frac{1}{\|x\|^{-\alpha}s + 1}.$$

In order to simplify the notation and eliminate ambiguity, in the following we use $l = \|x\|$ to be the label of each interferer. $\{l\}$ forms another PPP on $\mathbb{R}^+$ [27]. Thanks to the fact that $\{l\}$ is a simple point process, there will be no ambiguity introduced by this change of labeling[1]. With a slight abuse of notation, we let $l = \|x\| \in \phi$ iff $x \in \phi$.

Then, the variable x can be reserved for examining the condition in Lemma 2. Since $\log(\cdot)$ preserves the monotonicity, once the (monotonicity) conditions are proved for $\log(x\mathcal{L}_I(\theta r^\alpha x))$, they are proved for $x\mathcal{L}_I(\theta r^\alpha x)$. Thanks to the continuity of $\log(x\mathcal{L}_I(\theta r^\alpha x))$, we examine the monotonicity of $\log(x\mathcal{L}_I(\theta r^\alpha x))$ by calculating its derivative

$$\frac{\mathrm{d}}{\mathrm{d}x}\log(x\mathcal{L}_I(\theta r^\alpha x)) = \frac{1}{x} - \sum_{l\in\phi_z}\frac{1}{x + l^\alpha/\theta r^\alpha}. \tag{6}$$

We rearrange the equation $\frac{\mathrm{d}}{\mathrm{d}x}\log(x\mathcal{L}_I(\theta r^\alpha x)) = 0$ as

$$\sum_{l\in\phi_z}\frac{1}{1 + l^\alpha/\theta r^\alpha x} = 1, \tag{7}$$

where the LHS is a (strict) monotonically increasing function of x. Thus, there is at most one positive zero-crossing for $\log(x\mathcal{L}_I(\theta r^\alpha x))$. Moreover, it can be shown that $\lim_{x\downarrow 0}\frac{\mathrm{d}}{\mathrm{d}x}\log(x\mathcal{L}_I(\theta r^\alpha x)) = +\infty$. Therefore, $\log(x\mathcal{L}_I(\theta r^\alpha x))$ is either monotonically increasing on $(0,\infty)$ or has a unique $x_0 \in (0,\infty)$, such that $x\mathcal{L}_I(\theta r^\alpha x)$ is monotonically increasing for $x < x_0$ and monotonically decreasing for $x > x_0$. Applying Lemma 3 in the former case and Lemma 2 in the latter case yields the desired result. ∎

In addition to finding the SNOPC policy, Proposition 5 provides the NEPC policy for a network of two links. Interestingly, no matter what kind of topology these two pairs of transmitter and receiver form, the NEPC strategy for this network is always constant power transmission at both transmitters. To see this, we can first assume one of the two transmitters is transmitting with constant power. For the other transmitter, (7) becomes $l/\theta r^\alpha x = 0$, where l is the distance between the desired the receiver and the interferer. This equation does not have a finite solution and thus by applying Lemma 3, constant power transmission is single-node optimal. The same analysis applies to the other node. Thus, constant power transmission is a NEPC strategy in two-link networks. However, for networks of more than two links, the NEPC strategy is in general not constant power control, as is shown in the following proposition.

Proposition 6. *ALOHA random on-off power control is a NEPC strategy for Rayleigh fading wireless networks, where the positions of the nodes of the whole network (both the transmitters and the receivers) are available at each transmitter.*

Proof: We first focus on the power control strategy of an arbitrary transmitter $z \in \phi$ and assume the rest of the network uses certain ALOHA random on-off power control policies which are known at z. We show that in such scenario the best response power control strategy for z is ALOHA.

[1]Even in a deterministic network where $\|x_1\| = \|x_2\|$ may exist, one can still avoid ambiguity by simply denoting $l_1 = \|x_1\|$ and $l_2 = l_1 + \epsilon = \|x_2\|$ for ϵ sufficiently small.

The proof is analogous to that of Proposition 5. Again, we let $y_z = o$ without loss of generality and denote $\phi\setminus\{z\}$ by ϕ_z. We assume that the transmit probability of node with label $l = \|x\| \in \mathbb{R}^+$ is γ_l^{-1} and its transmit power is γ_l. Then, the Laplace transform of the interference is

$$\mathcal{L}_{I|\phi}(s) = \prod_{l\in\phi_z}\left(\frac{\gamma_l^{-1}}{\gamma_l l^{-\alpha}s + 1} + 1 - \gamma_l^{-1}\right). \tag{8}$$

Again, we use the logarithm to change the product into a sum and examine the derivative of $\log(x\mathcal{L}_I(\theta r^\alpha x))$, which can be calculated as

$$\frac{1}{x} - \sum_{l\in\phi_z}\left(\frac{1}{x + l^\alpha/\theta r^\alpha\gamma_l} - \frac{1}{x + l^\alpha/\theta r^\alpha(\gamma_l - 1)}\right),$$

where $\gamma_l > 0,\ \forall l$. Then, $\frac{\mathrm{d}}{\mathrm{d}x}\log(x\mathcal{L}_I(\theta r^\alpha x)) = 0$ can be rearranged as

$$\sum_{l\in\phi_z}\left(\frac{1}{1 + l^\alpha/\theta r^\alpha\gamma_l x} - \frac{1}{1 + l^\alpha/\theta r^\alpha(\gamma_l - 1)x}\right) = 1,$$

where the LHS is a (strict) monotonically increasing function of x. Thus, there is at most one positive zero crossing for $\log(x\mathcal{L}_I(\theta r^\alpha x))$. Moreover, it can be shown that $\lim_{x\downarrow 0}\frac{\mathrm{d}}{\mathrm{d}x}\log(x\mathcal{L}_I(\theta r^\alpha x)) = +\infty$. The rest of the proof follows the one of Proposition 5.

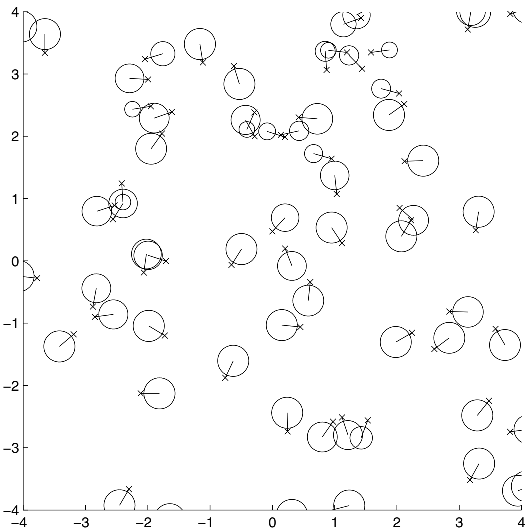
Since z is an arbitrary transmitter in the network, a Nash equilibrium is established. The assumption that each node knows the power control policy of the rest of the network is justified by the fact that each node has the same knowledge when deciding its own power control policy. Therefore, if a particular node can calculate its optimal power control policy, all the other nodes can do that as well. ∎

While Proposition 6 shows that a set of random on-off power control strategies is a Nash equilibrium, the exact equilibrium point, *i.e.* the transmit power $\gamma_l,\ \forall l \in \phi$, is typically difficult to determine. In general, for a finite network of n transmitter-receiver pairs, let l_{ij} be the distance from the transmitter i to the receiver j, and r_k be the distance of the transmitter-receiver pair k, $\forall i,j,k \in [n]$[2]. Then, for each $j \in [n]$, we have the following two equations

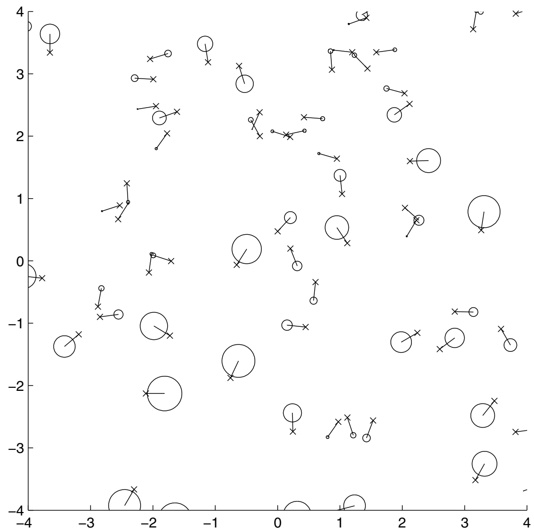
$$\begin{cases} \frac{1}{x_j} = \sum_{i\in[n]\setminus\{j\}}\left(\frac{1}{x_j + l_{ij}^\alpha/\theta r_j^\alpha\gamma_i} - \frac{1}{x_j + l_{ij}^\alpha/\theta r_j^\alpha(\gamma_i - 1)}\right) \\ \gamma_j = 1/\min\{1, \max\{P_{\max}^{-1}, x_j\}\}. \end{cases} \tag{9}$$

Since $\gamma_j, x_j,\ \forall j \in [n]$ are unknowns, altogether, there are $2n$ equations and $2n$ unknowns. Although x_j does not carry a particular physical meaning, it helps solve for the power level γ_j at each node j. Also, Proposition 6 implies that there is at least one solution for the $2n$ unknowns, despite the fact that finding it analytically is almost hopeless. In the following, we use numerically found solutions to evaluate this Nash equilibrium. Fig. 2 shows an example of the NEPC policy as well as its throughput for a realization of a Poisson bipolar network.

[2]We use $[n]$ to denote the set $\{1, 2, 3, \cdots, n\}$.



(a) transmit probaility



(b) throughput

Fig. 2: The NEPC in a Poisson bipolar network in case 3. Each transmitter-receiver pair is linked by a line, where a circle is centered at the transmitter and the receiver is labeled by x. The radius of each circle is proportional to the Nash equilibrium transmit probability (Fig. 2a) or the throughput (Fig. 2b) at the corresponding link. Here, $\lambda = 1$, $r \equiv 0.3$, $P_{\max} = 2$, $\theta = 10$.

VI. Performance Evaluation

A. The Single-node Optimal Power Control (SNOPC) Strategies

When all the interferers are transmitting with unit power, the interference distribution does not dependent on the distribution of the link distance. Thus, if the throughput conditioned on the link distance r is $p_s(r)$, the mean throughput is just $\mathbb{E}_R[p_s(R)]$. For this reason, in this subsection, we only focus on $p_s(r)$.

If no power control is applied, the throughput can be expressed in terms of the Laplace transform of the interference distribution; it is given by [25]

$$p_s(r) = \exp\left(-\frac{\lambda\pi^2\delta}{\sin(\pi\delta)}\theta^\delta r^2\right).$$

The optimal power control policy for the case where only the link distance is available at the transmitter is given by Proposition 3. The throughput conditioned on the link distance r is $p_s(r) =$

$$\begin{cases} \exp\left(-\lambda\pi(\theta r^\alpha)^\delta\frac{\pi\delta}{\sin(\pi\delta)}\right), & r \le R_1 \\ \frac{\exp(-1/\delta)}{\theta r^\alpha}\left(\frac{\sin(\pi\delta)}{\lambda\pi^2\delta^2}\right)^{1/\delta}, & R_1 < r \le R_2 \\ P_{\max}^{-1}\exp\left(-\lambda\pi\left(\frac{\theta r^\alpha}{P_{\max}}\right)^\delta\frac{\pi\delta}{\sin(\pi\delta)}\right), & r > R_2, \end{cases}$$

where $R_1 = \theta^{-1/\alpha}\sqrt{\frac{\sin(\pi\delta)}{\lambda\pi^2\delta^2}}$ and $R_2 = \left(\frac{P_{\max}}{\theta}\right)^{1/\alpha}\sqrt{\frac{\sin(\pi\delta)}{\lambda\pi^2\delta^2}}$.

When the complete network topology $\Phi = \phi$ is available at the central node (case 3), the optimal performance can be achieved by applying the policy suggested by the proof of Proposition 6. However, analytically characterizing the spatial throughput of such power control strategy requires a closed form expression for $\max_{1\le\gamma\le P_{\max}}\gamma^{-1}\prod_{l\in\phi}\frac{1}{1+\theta r^\alpha/l^\alpha\gamma}$, which seems hopeless. Therefore, we first solve for the single node optimal transmit power $\gamma(\phi)$ and transmit probability $1/\gamma(\phi)$. The expected throughput is given by $\gamma^{-1}\prod_{l\in\phi}\frac{1}{\theta r^\alpha/l^\alpha\gamma+1}$. We can then use simulation to average over a large number of realizations of Φ and compare the result with other two cases.

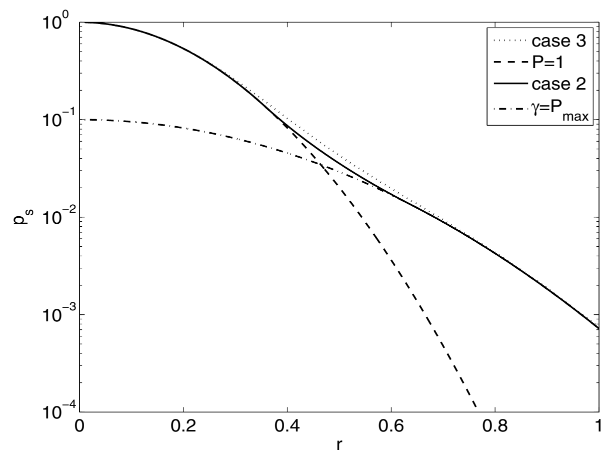


Fig. 3: Comparison of throughput using three different SNOPC strategies. Here, $\lambda = 1$, $P_{\max} = 10$, $\alpha = 4$, $\theta = 10$. The throughput of case 3 is averaged over 10,000 PPP realizations.

Fig. 3 compares the three SNOPC strategies for different levels of information. As expected, the SNOPC strategies with more information does strictly better than the SNOPC strategies based on less information. However, while the gain of knowing the link distance is significant especially when the link distance is large, the gain of knowing the complete network topology is marginal.

B. Nash Equilibrium Power Control (NEPC) Strategies

In contrast to SNOPC, it is not clear whether more information results in a higher throughput for NEPC. In this subsection, we evaluate the spatial throughput for different NEPC strategies.

1) Bipolar Networks: When the link distance is the same and constant throughout the network, the NEPC strategy for case 2 is the one described in Corollary 3, and the expected throughput at each link is

$$p_s(r) = \gamma^{-1} \exp\left(-\lambda\gamma^{-1}\frac{\pi^2\delta}{\sin(\pi\delta)}\theta^\delta r^2\right),$$

where $\gamma = \max\{1, \min\{P_{\max}, \lambda\pi r^2 \frac{\pi\delta^2}{\sin(\pi\delta)}\theta^\delta\}\}$.

Evaluating the performance of NEPC for complete network topology information (case 3) involves solving (9). Once (γ_l) is (numerically) found, the throughput can be determined by making use of the Laplace transform in (8).

Fig. 4 compares the spatial throughput of NEPC strategies in the bipolar network and the throughput of SNOPC. The case where all nodes in the network transmit with power $P_{\max}$ and probability $P_{\max}^{-1}$ is also plotted for reference. We see for case 2 when the link distance is larger than 0.5, the NEPC strategy enters its peak-power-limited regime.

A key observation of Fig. 4 is that by allowing all the transmitters in the network to selfishly use power control, the spatial throughput of the network can be improved (although not necessarily maximized). In particular, the comparison between the performance of SNOPC and NEPC strategy in case 2 shows that the throughput gain of a smart user is larger when all the other users are also smart. This result is somewhat surprising, since it is natural to conjecture that a smart user should be able to take more advantage of others if they are all dumb. The root of this counter-intuitive phenomenon lies in the special form of the Nash-equilibrium, *i.e.,* each node transmits with (the same) power $\gamma \geq 1$ and probability γ^{-1}. At this equilibrium, the interference I_γ observed at any receiver has the Laplace transform $\mathcal{L}_{I_\gamma}(s) = \exp(-\lambda\pi\gamma^{\delta-1}\frac{\pi\delta}{\sin(\pi\delta)}s^\delta)$, which is larger than the Laplace transform of the interference without power control $\mathcal{L}_I(s) = \exp(-\lambda\pi\frac{\pi\delta}{\sin(\pi\delta)}s^\delta)$ for all $s > 0$. Due to the relation between success probability and Laplace transform, this implies that any power control strategy achieves a higher expected throughput when the network operates at a certain the Nash equilibrium than when all other nodes transmit with constant power. Moreover, the NEPC strategy, by definition, maximizes the (individual) throughput at the Nash equilibrium, and thus the spatial throughput of NEPC is always higher than what SNOPC can achieve if all other transmitters do not use power control.

The fact that $\mathcal{L}_{I_\gamma}(s) > \mathcal{L}_I(s),\ \forall s > 0$ in case 2 suggests that by selfishly choosing its power control strategy, each node is essentially *reducing* its interference to other nodes. Therefore, the spatial throughput of NEPC is always larger than without power control (this is also confirmed in Fig. 4).

However, as is shown in Section IV-B, NEPC does *not* maximize the spatial throughput, which interestingly is also due to the special form of the NEPC strategy, which always ensures that $\mathbb{E}[P] = 1$. In contrast, if we replace the mean power constraint $\mathbb{E}[P] \leq 1$ with the constraint $\mathbb{E}[P^\delta] \leq 1$, it can be shown that the resulting NEPC policy coincides with GOPT ALOHA, *i.e.,* the spatial throughput is maximized at the Nash equilibrium. This can be interpreted as follows: A (smart) selfish power control strategy always has $\mathbb{E}[P^\delta] = 1$, since otherwise it is easy to construct another random variable (as a function of P) which statistically dominates P but still

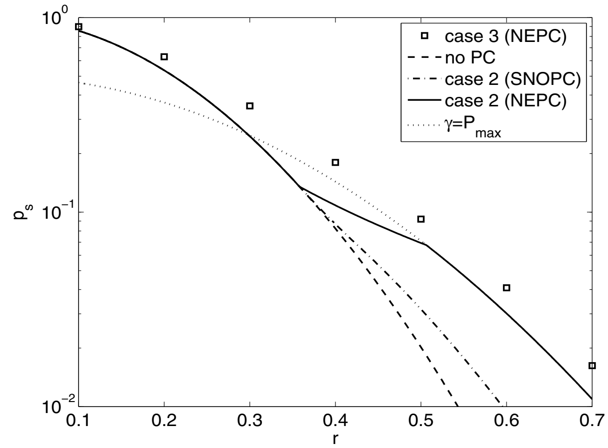


Fig. 4: Comparison of throughput in bipolar network using constant power transmission, NEPC strategies in case 2 and case 3, the case where all nodes transmit with power $P_{\max}$ and probability $P_{\max}^{-1}$ ($\gamma = P_{\max}$), and SNOPC strategy in case 2. Here, $\lambda = 1$, $P_{\max} = 2$, $\alpha = 4$, $\theta = 10$.

satisfies both power constraints. This, by Lemma 1, ensures that the power control strategy does not affect the interference distribution. Then, by adjusting the power control strategy, each node only maximizes its own throughput without inflicting more or less interference on others.

The comparison between the two NEPC strategies in Fig. 4 also indicates that the more information is available at each transmitter, the higher the throughput. This phenomenon is also observed in the scenario of variable link distances which is discussed in the next subsection.

2) Networks with Variable Link Distances: Here, we focus on the case where the link distance R is Rayleigh distributed with mean $1/2\sqrt{\lambda_r}$. As discussed earlier, in case 1, the NEPC strategy is constant power transmission, and the resulting throughput is

$$\mathbb{E}_R\left[p_s(R)\right] = \frac{\lambda_r}{\lambda_r + \lambda\frac{\pi\delta}{\sin(\pi\delta)}\theta^\delta}.$$

For case 2, the NEPC strategy hinges on solving for $\mathbb{E}[P^\delta]$ in (4). A closed-form solution is not available in this case, but a numerical solution is easy to obtain. Given $\mathbb{E}[P^\delta]$, the spatial throughput can be calculated by averaging over the distribution of R, which yields $\mathbb{E}_R[p_s(R)] =$

$$\frac{\lambda_r}{\lambda_r + a\theta^\delta/\pi}\left(1 - e^{-(\lambda_r\pi + \tilde{a}\theta^\delta)R_1^2}\right) + \frac{\lambda_r/P_{\max}}{\lambda_r + \tilde{a}\left(\theta/P_{\max}\right)^\delta/\pi}\left(1 - e^{-(\lambda_r\pi + \tilde{a}(\theta/P_{\max})^\delta)R_m^2}\right) + (\lambda_r\tilde{a}\pi\delta e)^{-1/\delta}\left(\Gamma(1+\tfrac{1}{\delta}, \lambda_r\pi\tilde{R}_1^2) - \Gamma(1+\tfrac{1}{\delta}, \lambda_r\pi\tilde{R}_m^2)\right),$$

where $\tilde{a} = \frac{\lambda\pi^2\delta}{\sin\pi\delta}\mathbb{E}[P^\delta]$, $\tilde{R}_1 = \theta^{-\delta/2}(\tilde{a}\delta)^{-1/2}$, $\tilde{R}_m = (\theta/P_{\max})^{-\delta/2}(\tilde{a}\delta)^{-1/2}$ and $\Gamma(\cdot,\cdot)$ is the upper incomplete Gamma function.

Fig. 5 compares the Nash equilibrium power control strategies in three cases, where the throughput of case 3 is calculated by simulation, averaged over 10,000 network topologies (realizations of the PPP). Again, we see that the more information

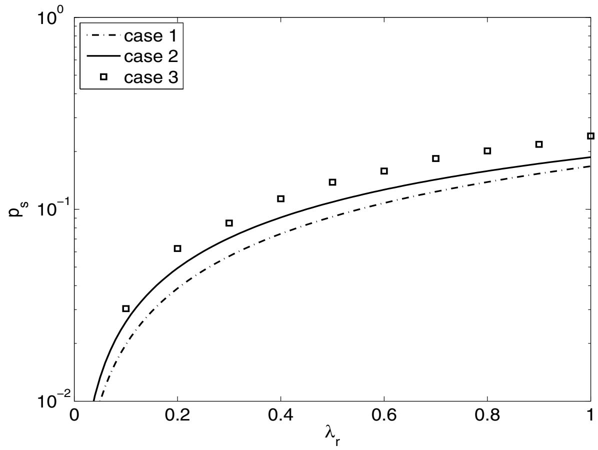


Fig. 5: Comparison of spatial throughput under NEPC when link distance are Rayleigh distributed in three cases. Here, $\lambda = \lambda_r = 1$, $P_{\max} = 2$, $\alpha = 4$, $\theta = 10$. The throughput in case 3 is averaged over 10,000 realizations of PPPs.

available at each node, the higher the spatial throughput will be, even if each node acts selfishly.

Although this paper focuses on Rayleigh distributed links, the results are applicable to general link distributions. More specifically, for case 3, the link distance distribution does not matter since the complete network topology is assumed to be available at the transmitters. For cases 1 and 2, as long as the link distance is iid, the transmit power at each individual transmitter is iid. Therefore, the link distance distribution enters the interference distribution only by the δ-th moment of the transmit power. In case 2, this means the interference distribution property required by Lemma 2 always holds, and thus results for Rayleigh-distributed link distance hold for arbitrary link distance distribution. In case 1, this means all the proofs in Section III can stay the same as long as the concavity of $\mathbb{E}_R[\mathcal{L}_I(s)|_{s=\frac{\theta R^\alpha}{P}}]$ (w.r.t. P) holds.

VII. Conclusions

A. Main Contributions

This paper considers the single-node optimal power control (SNOPC) strategies and the Nash equilibrium power control (NEPC) strategies in wireless networks whose node distribution is governed by a Poisson point process (PPP). The SNOPC strategies maximize one link's expected throughput given that the rest of the network uses constant power transmission. When all the nodes in the network can use power control, the NEPC strategies are the stable strategies in the sense that no individual link would deviate from these strategies as that cannot increase the expected throughput at this link.

With the basic assumption that the channel fading state is unobservable at each node, we analytically characterize SNOPC and NEPC strategies in three cases of different levels of knowledge at each node. We show that in all cases the optimal and Nash equilibrium power control strategies are ALOHA-type random on-off strategies whose transmit power level and transmit probability are functions of the knowledge at each transmitter. While ALOHA is generally considered to be inefficient as a MAC scheme, our results show that, as a power control scheme, it constitutes a natural Nash equilibrium in interference-limited Poisson networks.

In bipolar networks (fixed link distance), the performance comparison between the NEPC strategy and the globally optimal (GOPT) power control strategy reveals the inefficiency of the NEPC strategy. However, it is also shown that in such a scenario the NEPC strategy achieves a spatial throughput larger than without power control. Since nodes in a real network are likely to cooperate to some degree, the performance of the NEPC strategy provides a worst case scenario for wireless networks with random power control capability. In this sense, this paper demonstrates the potential benefits of random power control in wireless networks.

Numerical results suggest, just like SNOPC, under NEPC, the more information is available, the higher the spatial throughput. In other words, spreading network topology information over all nodes can increase the spatial throughput even if all the nodes are selfish. Thus, there exists a trade-off between the gain in spatial throughput and the overhead of acquiring topology information, which can be quantified by the tools provided in this paper.

B. Extensions and Future Work

1) Adding Noise: This paper focuses on the analysis of an interference-limited network and ignores noise. However, in the case of Rayleigh fading, it is straightforward to extend the results to the scenario where both interference and noise exist. To see this, recall that the success probability of a transmission attempt over link distance r with power P is

$$\mathbb{P}(Phr^{-\alpha} > \theta I) = \mathcal{L}_I(s)|_{s=\frac{\theta r^\alpha}{P}}.$$

In the presence of noise of power N_0, the success probability becomes

$$\mathbb{P}\left(\frac{Phr^{-\alpha}}{I+N_0} > \theta\right) = \mathcal{L}_I(s)e^{-N_0 s}|_{s=\frac{\theta r^\alpha}{P}},$$

which can be considered as a noiseless link with another interference distribution. Therefore, although the analytical results in the paper can become more cumbersome if noise is included, the analysis will be the same.

2) Achieving the Strategies: Although this paper shows that in many cases the SNOPC and NEPC strategies are ALOHA-type random on-off policy, closed-form expressions for the transmit probability and transmit power of these strategies only exist in some special cases, *e.g.,* in bipolar networks. In general, efficient numerical approaches are needed to specify the strategies, especially for large static networks (case 3).

Moreover, while deriving the desired power control strategies, we first assume the availability of certain amount of knowledge and decide on power control strategies based on it. In practice, the required knowledge may not be available at the beginning. Instead, it may have to be collected gradually at the transmitter. For example, the knowledge at the transmitters may be estimated by the past successful or unsuccessful transmission attempts. In such a scenario, it is desirable to have an adaptation approach, similar to the one in [28], which iteratively updates the strategy according to the knowledge available at any given time. The development of such algorithms is another interesting problem to be explored in the future.

References

[1] V. Kawadia and P. Kumar, "Principles and protocols for power control in wireless ad hoc networks," *IEEE J. Sel. Areas Commun.*, vol. 23, no. 1, pp. 76–88, Jan. 2005.

[2] V. Mhatre, K. Papagiannaki, and F. Baccelli, "Interference mitigation through power control in high density 802.11 WLANs," in *Proc. 2007 IEEE International Conf. Comput. Commun.*, pp. 535–543.

[3] D. Rajan, A. Sabharwal, and B. Aazhang, "Delay-bounded packet scheduling of bursty traffic over wireless channels," *IEEE Trans. Inf. Theory*, vol. 50, no. 1, pp. 125–144, Jan. 2004.

[4] G. Foschini and Z. Miljanic, "A simple distributed autonomous power control algorithm and its convergence," *IEEE Trans. Veh. Technol.*, vol. 42, no. 4, pp. 641–646, Nov. 1993.

[5] R. Cheng and S. Verdu, "Gaussian multiaccess channels with ISI: capacity region and multiuser water-filling," *IEEE Trans. Inf. Theory*, vol. 39, no. 3, pp. 773–785, May 1993.

[6] Z. Shen, J. Andrews, and B. Evans, "Optimal power allocation in multiuser OFDM systems," in *Proc. 2003 IEEE Global Telecommun. Conf.*, vol. 1, pp. 337–341.

[7] T. Holliday, A. Goldsmith, P. Glynn, and N. Bambos, "Distributed power and admission control for time varying wireless networks," in *Proc. 2004 IEEE Global Telecommun. Conf.*, vol. 2, pp. 768–774.

[8] P. Hande, S. Rangan, M. Chiang, and X. Wu, "Distributed uplink power control for optimal SIR assignment in cellular data networks," *IEEE/ACM Trans. Netw.*, vol. 16, no. 6, pp. 1420–1433, Dec. 2008.

[9] J. Huang, R. Berry, and M. Honig, "Distributed interference compensation for wireless networks," *IEEE J. Sel. Areas Commun.*, vol. 24, no. 5, pp. 1074–1084, May 2006.

[10] A. MacKenzie and S. Wicker, "Game theory in communications: motivation, explanation, and application to power control," in *Proc. 2001 IEEE Global Telecommun. Conf.*, vol. 2, pp. 821–826.

[11] F. Meshkati, H. Poor, S. Schwartz, and N. Mandayam, "An energy-efficient approach to power control and receiver design in wireless data networks," *IEEE Trans. Commun.*, vol. 53, no. 11, pp. 1885–1894, Nov. 2005.

[12] V. Srivastava, J. Neel, A. Mackenzie, R. Menon, L. Dasilva, J. Hicks, J. Reed, and R. Gilles, "Using game theory to analyze wireless ad hoc networks," *IEEE Commun. Surveys Tutorials*, vol. 7, no. 4, pp. 46–56, fourth quarter 2005.

[13] K. Akkarajitsakul, E. Hossain, D. Niyato, and D. I. Kim, "Game theoretic approaches for multiple access in wireless networks: a survey," *IEEE Commun. Surveys Tutorials*, vol. 13, no. 3, pp. 372–395, Mar. 2011.

[14] E. Altman and Z. Altman, "S-modular games and power control in wireless networks," *IEEE Trans. Automatic Control*, vol. 48, no. 5, pp. 839–842, May 2003.

[15] M. Dinitz, "Distributed algorithms for approximating wireless network capacity," in *Proc. 2010 IEEE International Conf. Comput. Commun.*, pp. 1–9.

[16] E. Asgeirsson and P. Mitra, "On a game theoretic approach to capacity maximization in wireless networks," in *Proc. 2011 IEEE International Conf. Comput. Commun.*, pp. 3029–3037.

[17] M. Chiang, P. Hande, T. Lan, and C. W. Tan, *Power Control in Cellular Networks*. NOW: Foundations and Trends in Networking, 2008.

[18] T.-S. Kim and S.-L. Kim, "Random power control in wireless ad hoc networks," *IEEE Commun. Lett.*, vol. 9, no. 12, pp. 1046–1048, Dec. 2005.

[19] M. Elmusrati, N. Tarhuni, and R. Jantti, "Performance analysis of random uniform power allocation for wireless networks in Rayleigh fading channels," *European Trans. Telecommun.*, vol. 20, no. 4, pp. 457–462, 2009.

[20] X. Zhang and M. Haenggi, "Delay-optimal power control policies," *IEEE Trans. Wireless Commun.*, 2012, accepted. Available: http://www.nd.edu/~mhaenggi/pubs/twc12c.pdf

[21] M. Andrews and M. Dinitz, "Maximizing capacity in arbitrary wireless networks in the SINR model: complexity and game theory," in *Proc. 2009 IEEE International Conf. Comput. Commun.*, pp. 1332–1340.

[22] M. Haenggi, J. Andrews, F. Baccelli, O. Dousse, and M. Franceschetti, "Stochastic geometry and random graphs for the analysis and design of wireless networks," *IEEE J. Sel. Areas Commun.*, vol. 27, no. 7, pp. 1029–1046, Sep. 2009.

[23] M. Haenggi, *Stochastic Geometry for Wireless Networks*. Cambridge University Press, 2012.

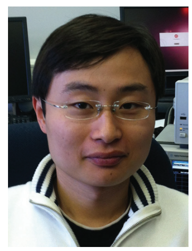
[24] ——, "On distances in uniformly random networks," *IEEE Trans. Inf. Theory*, vol. 51, no. 10, pp. 3584–3586, Oct. 2005.

[25] M. Haenggi and R. K. Ganti, *Interference in Large Wireless Networks*. NOW: Foundations and Trends in Networking, 2008.

[26] F. Baccelli, B. Blaszczyszyn, and P. Muhlethaler, "An Aloha protocol for multihop mobile wireless networks," *IEEE Trans. Inf. Theory*, vol. 52, no. 2, pp. 421–436, Feb. 2006.

[27] M. Haenggi, "A geometric interpretation of fading in wireless networks: theory and applications," *IEEE Trans. Inf. Theory*, vol. 54, no. 12, pp. 5500–5510, Dec. 2008.

[28] Z. Tong and M. Haenggi, "Optimizing spatial reuse by dynamic power control," in *2012 IEEE International Conf. Commun.*



Xinchen Zhang (S'10) received the B. Eng. (with honors) in Information Engineering from Shanghai Jiao Tong University (SJTU) in 2008. He was a visiting student at University of New Mexico (UNM) in 2007. Since 2008, he has been working towards the Ph.D. degree in the Department of Electrical Engineering at University of Notre Dame (ND), Notre Dame, IN, USA, where he worked on various topics including signal processing, circuit design and MAC design. His current research interests include power control, channel-access schemes and system performance evaluation in wireless networks.

Martin Haenggi (S'95, M'99, SM'04) is a Professor of Electrical Engineering and a Concurrent Professor of Applied and Computational Mathematics and Statistics at the University of Notre Dame, Indiana, USA. He received the Dipl.-Ing. (M.Sc.) and Dr.sc.techn. (Ph.D.) degrees in electrical engineering from the Swiss Federal Institute of Technology in Zurich (ETH) in 1995 and 1999, respectively. After a postdoctoral year at the University of California in Berkeley, he joined the University of Notre Dame in 2001. In 2007-08, he spent a Sabbatical Year at the University of California at San Diego (UCSD). For both his M.Sc. and his Ph.D. theses, he was awarded the ETH medal, and he received a CAREER award from the U.S. National Science Foundation in 2005 and the 2010 IEEE Communications Society Best Tutorial Paper award.

He served as a member of the Editorial Board of the *Elsevier Journal of Ad Hoc Networks* from 2005-08, as a Guest Editor for the IEEE JOURNAL ON SELECTED AREAS IN COMMUNICATIONS in 2009, as an Associate Editor for the IEEE TRANSACTIONS ON MOBILE COMPUTING (TMC) from 2008-2011 and for the *ACM Transactions on Sensor Networks* from 2009-2011, and as a Distinguished Lecturer for the IEEE Circuits and Systems Society in 2005-06. Presently he is a Steering Committee Member of TMC. He is a co-author of the monograph *Interference in Large Wireless Networks* (NOW Publishers, 2009) and the author of the textbook *Stochastic Geometry for Wireless Networks* (Cambridge University Press, 2012). His scientific interests include networking and wireless communications, with an emphasis on ad hoc, cognitive, cellular, sensor, and mesh networks.