Analysis of D2D Underlaid Cellular Networks: SIR Meta Distribution and Mean Local Delay

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***Abstract*—We study the performance of D2D communication underlaying cellular wireless network in terms of the meta distribution of the signal-to-interference ratio (SIR), which is the distribution of the conditional SIR distribution given the locations of the wireless nodes. Modeling D2D transmitters and base stations as Poisson point processes (PPPs), moments of the conditional SIR distribution are derived in order to calculate analytical expressions for the meta distribution and the mean local delay of the typical D2D receiver and cellular downlink user. It turns out that for D2D users, the total interference from the D2D interferers and base stations is equal in distribution to that of a single PPP, while for downlink users, the effect of the interference from the D2D network is more complicated. We also derive the region of transmit probabilities for the D2D users and base stations that result in a finite mean local delay and give a simple inner bound on that region. Finally, the impact of increasing the base station density on the mean local delay, the meta distribution, and the density of users reliably served is investigated with numerical results.**

***Index Terms*—Poisson point process, Meta distribution, Mean local delay, Success probability, D2D communication, Cellular network.**

I. Introduction

A. Motivation

THE goal of fifth generation (5G) cellular wireless networks is to overcome the fundamental challenges of existing cellular networks, including higher data rates, lower latency, and reduced energy consumption [1]. Device-to-device (D2D) communication is an important technology, which is used in LTE-Advanced and will be adopted in the next-generation cellular wireless networks to cope with 5G requirements [2]. D2D communication provides opportunities for proximity-based services, for example media sharing and local advertisement [3], [4]. Underlay D2D communication promises increased spectrum efficiency, reduced latency, and large throughput [5], [6]. On the other hand, due to the coexistence of D2D and cellular communications in the same spectrum, the underlaid D2D signals become a new source of interference [4]. To analyze the trade-offs involved, the meta distribution of the signal-to-interference ratio (SIR), which is a key performance metric in wireless networks [7], needs to be studied. The meta distribution provides detailed information about the distribution of the success probabilities of individual links in each network realization [7] while the standard (mean) success probability is the average of the success probabilities of the individual links in each network realization. As such, the meta distribution is a much sharper and refined metric than the standard success probability, which is easily obtained as the average over the meta distribution. Another important performance metric in wireless networks is delay. One of the main components of the delay is the local delay. To analyze the delay in D2D networks, the mean local delay and the trade-offs involved need to be studied for both D2D and cellular users.

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B. Related Work

The analytical tractability, in particular the simple form of the probability generating functional (PGFL) of the Poisson point process (PPP) have made the PPP by far the most popular spatial model [8], [9]. There exists a growing body of literature using the PPP to model the irregular spatial structure of D2D users and base stations (BSs) [3], [4], [10], [11]. In [4], underlaid D2D communication in a single cell is considered. Analytical expressions of the coverage probabilities of both D2D and cellular links and the sum rate of D2D users are derived, and power control algorithms are proposed to maximize these performance metrics. In [10] cognitive and energy-harvesting D2D communication in cellular networks is investigated where BSs, cellular users, and cognitive D2D transmitters form independent homogeneous PPPs. Using the PGFL of the PPP, the outage probability for a D2D receiver and a cellular user are derived for this network. Mode selection and power control in D2D underlaid cellular networks with Poisson distributed users and BSs are studied in [11], where the mode selection decision considers both D2D link and cellular link quality. The network performance in terms of the signal-to-interference-plus-noise-ratio (SINR) and outage probability for both cellular and D2D users is investigated. The authors in [3] focus on spectrum access and mode selection in D2D communications in cellular networks where users form a PPP and BSs a triangular lattice.

The standard success (or coverage) probability provides limited information on the network performance since it is, in fact, just the mean of a certain random variable, namely the conditional success probability (or SIR distribution) given the underlying point processes. The meta distribution, in contrast, provides the entire distribution of that random variable and thus a much sharper characterization of the network performance. It is a function of two parameters—the target reliability and the target SIR (or data rate)—in contrast to the standard success probability or coverage, which is a function of the target SIR only. Using this metric, questions such as "What fraction of users in the network attain the desired link reliability for a given target SIR?" can be answered. The standard success probability, in contrast, answers only questions like "What fraction of users achieve the given target SIR?".

The meta distribution is introduced in and derived for Poisson bipolar and cellular networks exactly and approximately in [7]. To derive the analytical expressions for the meta distribution, moments of the conditional success probability are first calculated. Specifically, the variance of the success probability and the mean local delay are studied. [12] focuses on the analysis of the meta distribution of the SIR for both the cellular network uplink and downlink with fractional power control. The moments of the meta distribution for both scenarios are calculated. The exact analytical expression, bounds, and the beta approximation of the meta distribution are also provided. The mean local delay is in itself an important performance metric. It is the mean number of transmission attempts for a node to successfully transmit a packet to its target receiver. The general framework for the mean local delay can be traced back to [13], where the mean local delay is derived for ad hoc networks with slotted ALOHA. It shows that in some cases the mean local delay exhibits an interesting phase transition phenomenon called *wireless contention phase transition*, *i.e.*, the mean local delay is finite when certain model parameters are below a threshold and infinite above. To prevent this phenomenon different solutions are also discussed. In [14]–[16], local delay analyses have been carried out for different scenarios. In [14], the local delay is studied in static and highly mobile Poisson networks with ALOHA for different cases of nearest-neighbor communication. The mean local delay in static Poisson ALOHA networks is also analyzed in [15]. First, the results are derived for the interference-free case, then, both interference and noise are considered in the analyses. In [16], the mean local delay is studied for different scenarios, including the interference-free case, the noise-free case, and the case where both noise and interference are considered. In that paper, optimum ALOHA transmit probability which minimizes the mean local delay is also derived. The effect of power control and medium access (MAC) protocols on the local delay are investigated in [17]–[19]. With mean and peak power constraints at each node, in [17], for Rayleigh fading, an ALOHA-type (on-off) power control strategy is introduced to minimize the mean local delay. This is also done in [18] for different fading statistics. In [19], the mean local delay is studied for different MAC protocols. Optimal parameters which minimize the mean local delay are also derived. Moreover, the local delay is derived for out-of-band D2D networks with distributed caching in [20], [21]. When device locations have a PPP distribution, for distributed caching network, the mean local delay is derived in [20] and [21]. In [20] users are static while in [21] the effect of user mobility is investigated.

C. Contributions and Paper Organization

The standard coverage (or success) probability gives only limited information about wireless networks (not just Poisson networks). Fundamentally, the conditional success probability is a random variable (which we denote by $P_{\rm s}$), and merely characterizing the mean of this random variable means that a significant amount of valuable information is lost. For example, if 50% of the users achieve 5% reliability and 50% achieve 99% reliability, the mean (standard) coverage probability is 52%. If 100% of the users achieve 52% reliability, the mean (standard) coverage probability is also 52%. However, the two scenarios are very different in terms of the user experiences.

Related, the wireless industry talks about the performance of the "5% user", which is the performance level that 95% of the users achieve. The meta distribution directly reveals that information, while the standard coverage/success probability does not reveal any information about it. In the first case above, the "5% user" achieves a reliability of a mere 5%, while in the second case it achieves 52%. This is a huge difference in wireless network performance that is completely hidden if only the standard mean success probability is considered.

In this paper, we focus on the meta distribution and the mean local delay in in-band D2D networks. After calculating the moments of the conditional success probability, we give accurate approximations of the meta distribution by matching first and second moments of a beta distribution. Furthermore, we derive the mean local delay, which is the -1-st moment of the conditional success probability, for both D2D and downlink receivers. Finally, the effect of increasing the base station density on the mean local delay, meta distribution, and the density of users reliably served is investigated.

The rest of the paper is organized as follows: In the next section the system model is introduced. Moments of the conditional success probability, the meta distribution, and the mean local delay for D2D receivers, downlink users, and overall (combined) users are calculated in Section III. Section IV presents numerical results; it verifies the accuracy of the beta approximation results by Monte-Carlo simulation and explores the effect of increasing the BS intensity. A summary and conclusions are provided in Section V.

II. System Model

A. Network Model

We consider underlay D2D communication within a cellular network, where the D2D transmitters are distributed according to a homogeneous PPP $\Phi_{\rm D}$ of intensity $\lambda_{\rm D}$ and the base stations form an independent homogeneous PPP $\Phi_{\rm C}$ of intensity $\lambda_{\rm C}$. Moreover, we assume that each D2D transmitter has a dedicated receiver located at distance d in a random direction, *i.e.*, the D2D users form a *Poisson bipolar network* [9]. For the cellular network, our focus is on the downlink with nearest-BS association where downlink cellular users form an arbitrary stationary and ergodic point process of intensity $\lambda_{\rm U} \gg \lambda_{\rm C}$, such that each BS has at least one user in its cell. We assume an ALOHA-type channel access scheme for both

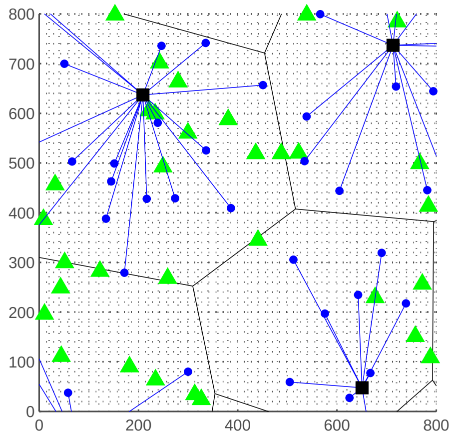


Fig. 1. A realization of the network model for $\lambda_{\mathrm{C}} = 2\,\mathrm{BS/km^2}$ and $\lambda_{\mathrm{D}} = \lambda_{\mathrm{U}} = 50\,\mathrm{UE/km^2}$. The black squares represent the BSs, the blue dots represent the cellular UEs, blue lines indicate the downlink connections, and green triangles represent D2D transmitters. For clarity we omit plotting D2D receivers, each of which is randomly located at distance d from its transmitter. The UEs are assumed to form a homogeneous PPP.

D2D and cellular networks[1]; D2D transmitters and BSs are active independently with probability p_{D} and p_{C}, respectively, and they transmit with constant powers P_{D} and P_{C}. As in [7], [22]–[24], the effect of thermal noise is neglected, *i.e.*, our focus is on the interference-limited regime.

A realization of this network model is shown in Fig. 1.

B. Channel Model

The channel (power) gain between receiver x and transmitter y is given by $h_{xy}\ell(x-y)$ where h_{xy} models the small-scale (multipath) fading and $\ell(x-y)$ represents the large-scale path loss. We assume that all fading coefficients h_{xy} are iid exponential with unit mean (Rayleigh fading) and $\ell(x) = \|x\|^{-\alpha}$, where α is the path loss exponent. In order to distinguish between D2D transmitters and BSs, the superscript $^{\mathrm{c}}$ is used for the fading coefficients in channels sourced at the BSs.

C. Interference and SIR

Due to the stationarity of the PPPs, we can condition on having a user at the origin, which becomes the typical user under expectations over the point processes [9]. To analyze the typical D2D receiver, we further condition on that user at the origin to be a D2D receiver, and vice versa when analyzing the typical cellular user.

[1]ALOHA does not imply that BSs randomly choose to be active or not in a given RB. BS use a certain RB if the load in the cell is such that the RB is needed, and this event can be assumed to be reasonably independent from one cell to the next. As a result, the BS activity in the RB considered follows ALOHA. Moreover, our analysis also applies to other channel access schemes, including CDMA (with orthogonal spreading codes), FHMA (frequency hopping multiple access), and OFDMA. For instance, consider FHMA, where the entire frequency band is divided into N sub-bands, and each transmitter independently chooses one of the sub-bands uniformly at random. In this case, the results revert back to ones which are derived for ALOHA simply by letting $p = 1/N$.

The total interference for the typical D2D receiver[2] at time t is given by

$$\begin{aligned} I_{0,\mathrm{D2D}}(t) = P_{\mathrm{D}} &\sum_{x\in\Phi_{\mathrm{D}}\setminus\{x_0\}} \mathbf{1}(x\in\Phi_{\mathrm{D}}(t))h_x(t)\|x\|^{-\alpha} \\ &+ P_{\mathrm{C}} \sum_{x\in\Phi_{\mathrm{C}}} \mathbf{1}(x\in\Phi_{\mathrm{C}}(t))h_x^{\mathrm{c}}(t)\|x\|^{-\alpha}, \end{aligned} \tag{1}$$

where $h_x(t)$ is the fading from D2D transmitter located at x to the typical D2D receiver at time t and $h_x^{\mathrm{c}}(t)$ denotes the fading from BS located at x to the typical D2D receiver, respectively. $\Phi_{\mathrm{D}}(t)$ and $\Phi_{\mathrm{C}}(t)$ are the D2D transmitters and BSs that transmit at time instant t. $\mathbf{1}(.)$ is the indicator function. The SIR at this receiver follows as

$$\mathrm{SIR}_{0,\mathrm{D2D}}(t) = \frac{P_{\mathrm{D}}h_0(t)d^{-\alpha}}{I_{0,\mathrm{D2D}}(t)}. \tag{2}$$

Similarly, conditioning on the user at the origin to be a cellular user, the interference at the typical cellular user is

$$\begin{aligned} I_{0,\mathrm{Cellular}}(t) = P_{\mathrm{D}} &\sum_{x\in\Phi_{\mathrm{D}}} \mathbf{1}(x\in\Phi_{\mathrm{D}}(t))h_x(t)\|x\|^{-\alpha} \\ &+ P_{\mathrm{C}} \sum_{x\in\Phi_{\mathrm{C}}\setminus\{x_0\}} \mathbf{1}(x\in\Phi_{\mathrm{C}}(t))h_x^{\mathrm{c}}(t)\|x\|^{-\alpha}, \end{aligned} \tag{3}$$

where x_0 is the serving BS, which is the nearest BS to the origin. As in (2), the SIR is

$$\mathrm{SIR}_{0,\mathrm{Cellular}}(t) = \frac{P_{\mathrm{C}}h_0^{\mathrm{c}}(t)\|x_0\|^{-\alpha}}{I_{0,\mathrm{Cellular}}(t)}. \tag{4}$$

D. The Meta Distribution

The meta distribution $\bar{F}_{P_{\mathrm{s}}}(x)$ is the complementary cumulative distribution function (CCDF) of the random variable

$$P_{\mathrm{s}}(\theta) \triangleq \mathbb{P}(\mathrm{SIR}_0 > \theta \mid \Phi, \mathrm{tx}), \tag{5}$$

which is the CCDF of the conditional SIR of the typical user given the points processes and conditioned on the desired transmitter to be active. Hence the meta distribution is formally given by [7]

$$\bar{F}_{P_{\mathrm{s}}}(x) \triangleq \mathbb{P}^0(P_{\mathrm{s}}(\theta) > x), \quad x\in[0,1]\,. \tag{6}$$

Here $\mathbb{P}^0$ is the Palm measure (conditioning on the receiver at the origin 0 and on the corresponding transmitter to be active). Since all point processes in the model are ergodic, the meta distribution can be interpreted as the fraction of the active links whose conditional success probabilities are greater than x.

We denote the b-th moment of P_{s} by M_b, *i.e.*, $M_b \triangleq \mathbb{E}^0(P_{\mathrm{s}}^b)$, $b\in\mathbb{C}$. The exact meta distribution can be calculated by applying the Gil-Pelaez theorem [25] after deriving the imaginary moments of $P_{\mathrm{s}}(\theta)$, *i.e.*, M_{jt}, $j \triangleq \sqrt{-1}$, $t\in\mathbb{R}^+$. A much simpler alternative approach is to approximate the meta distribution with the beta distribution, which requires only the first and second moments. In [7], it is shown that for both Poisson bipolar and cellular networks, this approximation is very accurate. In this paper, we verify the accuracy by simulation, for both typical D2D and downlink receivers.

[2]Strictly speaking, this receiver only becomes typical once the expectation over the point processes is taken.

E. The Mean Local Delay

Generally, the per-link delay consists of four types of delays, namely the processing delay, the queueing delay, the transmission delay, and the propagation delay. In most wireless networks, the processing delay and the propagation delay are negligible compared to the queueing delay and the transmission delay. The main component of the transmission delay is the retransmission delay which is closely related to the number of retransmissions of a packet. This type of delay is often called local delay [26].

Specifically, the local delay, denoted by L, is defined as the number of transmission attempts needed until a packet is successfully received (decoded) over a wireless link. Thus, the mean local delay can be written as

$$\mathbb{E}\left[L\right] = \mathbb{E}\left[\mathbb{E}\left[L \mid \Phi\right]\right] \overset{(a)}{=} \mathbb{E}\left[\frac{1}{P_{\mathrm{s}}}\right] = M_{-1}. \tag{7}$$

In the above equation, the inner expectation is obtained by averaging over the fading and the ALOHA. (a) in (7) is obtained since, conditioned on Φ, L is geometrically distributed with parameter P_{s}, *i.e.*, letting $L_\Phi = (L \mid \Phi)$, we have

$$\mathbb{P}\left(L_\Phi = k\right) = (1 - P_{\mathrm{s}})^{k-1} P_{\mathrm{s}}, \quad k \in \mathbb{N}, \tag{8}$$

where P_{s} is the conditional success probability given in (5). This follows from the fact that the transmission success events are conditionally independent given Φ. As we see in (7), the mean local delay is the -1-st moment of the conditional success probability, *i.e.*, M_{-1}.

III. Analytical Results

In this section, exact analytical expressions for the b-th moments M_b, $b \in \mathbb{C}$, are derived, followed by closed-form approximations of the meta distribution for both typical D2D and downlink receivers.

A. Meta Distribution for the Typical D2D Receiver

The first main result yields the b-th moment of P_{s} for the typical D2D receiver, for any $b \in \mathbb{C}$.

Theorem 1 (Moments for D2D receivers). *For the typical D2D receiver, the b-th moment of the conditional success probability is*

$$\begin{aligned} &M_{b,\mathrm{D2D}} \\ &= \exp\Bigg(-\pi\theta^\delta d^2 \frac{\pi\delta}{\sin(\pi\delta)} b\Big[p_{\mathrm{D}}\lambda_{\mathrm{D}}\, {}_2F_1(1-b, 1-\delta; 2; p_{\mathrm{D}}) \\ &\quad + p_{\mathrm{C}}\lambda_{\mathrm{C}}\left(\frac{P_{\mathrm{C}}}{P_{\mathrm{D}}}\right)^\delta {}_2F_1(1-b, 1-\delta; 2; p_{\mathrm{C}})\Big]\Bigg), \quad b \in \mathbb{C}. \end{aligned} \tag{9}$$

Proof: We first express the conditional success probability by substituting (2) in (5).

$$\begin{aligned} P_{\mathrm{s,D2D}}(\theta) &= \mathbb{P}\left(h_0(t) > \frac{\theta d^\alpha}{P_{\mathrm{D}}} I_{0,\mathrm{D2D}}(t) \mid \Phi_{\mathrm{D}}, \Phi_{\mathrm{C}}, \mathrm{tx}\right) \\ &\overset{(a)}{=} \mathbb{E}\Bigg[\exp\Big\{-\theta d^\alpha \sum_{x \in \Phi_{\mathrm{D}} \setminus \{x_0\}} \mathbf{1}(x \in \Phi_{\mathrm{D}}(t)) h_x(t) \|x\|^{-\alpha}\Big\} \\ &\quad \cdot \exp\Big\{-\theta d^\alpha \frac{P_{\mathrm{C}}}{P_{\mathrm{D}}} \sum_{x \in \Phi_{\mathrm{C}}} \mathbf{1}(x \in \Phi_{\mathrm{C}}(t)) h_x^{\mathrm{c}}(t) \|x\|^{-\alpha}\Big\}\Bigg] \\ &\overset{(b)}{=} \prod_{x \in \Phi_{\mathrm{D}} \setminus \{x_0\}} \left(\frac{p_{\mathrm{D}}}{1 + \theta d^\alpha \|x\|^{-\alpha}} + 1 - p_{\mathrm{D}}\right) \\ &\quad \cdot \prod_{x \in \Phi_{\mathrm{C}}} \left(\frac{p_{\mathrm{C}}}{1 + \theta d^\alpha \frac{P_{\mathrm{C}}}{P_{\mathrm{D}}} \|x\|^{-\alpha}} + 1 - p_{\mathrm{C}}\right), \end{aligned} \tag{10}$$

where (a) follows from the unit mean exponential distribution of $h_0(t)$, and (b) is obtained by taking the expectation with respect to $h_x(t)$, $h_x^{\mathrm{c}}(t)$, and the ALOHA channel access scheme. Next we take the expectation over the point processes to obtain the b-th moment.

$$\begin{aligned} &M_{b,\mathrm{D2D}} = \mathbb{E}\Big[P_{\mathrm{s,D2D}}(\theta)^b\Big] \\ &= \mathbb{E}\Bigg[\prod_{x \in \Phi_{\mathrm{D}} \setminus \{x_0\}} \left(\frac{p_{\mathrm{D}}}{1 + \theta d^\alpha \|x\|^{-\alpha}} + 1 - p_{\mathrm{D}}\right)^b\Bigg] \\ &\quad \cdot \mathbb{E}\Bigg[\prod_{x \in \Phi_{\mathrm{C}}} \left(\frac{p_{\mathrm{C}}}{1 + \theta d^\alpha \frac{P_{\mathrm{C}}}{P_{\mathrm{D}}} \|x\|^{-\alpha}} + 1 - p_{\mathrm{C}}\right)^b\Bigg] \\ &\overset{(a)}{=} \exp\left(-\lambda_{\mathrm{D}} \int \left[1 - \left(\frac{p_{\mathrm{D}}}{1 + \theta d^\alpha \|x\|^{-\alpha}} + 1 - p_{\mathrm{D}}\right)^b\right] \mathrm{d}x\right) \times \\ &\exp\left(-\lambda_{\mathrm{C}} \int \left[1 - \left(\frac{p_{\mathrm{C}}}{1 + \theta d^\alpha \frac{P_{\mathrm{C}}}{P_{\mathrm{D}}} \|x\|^{-\alpha}} + 1 - p_{\mathrm{C}}\right)^b\right] \mathrm{d}x\right) \\ &\overset{(b)}{=} \exp\Bigg(-\pi\theta^\delta d^2 \frac{\pi\delta}{\sin(\pi\delta)} b\Big[p_{\mathrm{D}}\lambda_{\mathrm{D}}\, {}_2F_1(1-b, 1-\delta; 2; p_{\mathrm{D}}) \\ &\quad + p_{\mathrm{C}}\lambda_{\mathrm{C}}\left(\frac{P_{\mathrm{C}}}{P_{\mathrm{D}}}\right)^\delta {}_2F_1(1-b, 1-\delta; 2; p_{\mathrm{C}})\Big]\Bigg), \end{aligned} \tag{11}$$

where (a) follows from Slivnyak's theorem [9] and the PGFL of the general PPP of intensity measure Λ, which is given by

$$\mathbb{E}\left[\prod_{x \in \Phi} f(x)\right] = \exp\left\{-\int_{\mathbb{R}^2} [1 - f(x)] \Lambda(\mathrm{d}x)\right\}. \tag{12}$$

In step (b) we use

$$\begin{aligned} &\int_{\mathbb{R}^2} \left[1 - \left(\frac{p}{1 + \theta' \|x\|^{-\alpha}} + 1 - p\right)^b\right] \mathrm{d}x \\ &\qquad = \pi\theta'^\delta \frac{\pi\delta}{\sin(\pi\delta)} \sum_{k=1}^{\infty} \binom{b}{k} \binom{\delta-1}{k-1} p^k \\ &\qquad = \pi\theta'^\delta \frac{\pi\delta}{\sin(\pi\delta)} p b\, {}_2F_1(1-b, 1-\delta; 2; p), \end{aligned} \tag{13}$$

where $\delta = 2/\alpha$ and $\theta' = \theta d^\alpha$. (13) is obtained by using Appendix A and Eqns. (3) and (4), all from [7]. The term $\sum_{k=1}^{\infty} \binom{b}{k}\binom{\delta-1}{k-1} p^k$, called *diversity polynomial* in [27], is expressed in terms of the Gaussian hypergeometric function ${}_2F_1$ in (13). ∎

From this result we observe that from a D2D receiver view, the network behaves as a Poisson bipolar network, where the moments have exponential form. Let Φ and Φ' be two PPPs interfering nodes with intensities λ and λ' and transmit powers

P and P'. The interferences stemming from these two sets are equal in distribution if $\lambda P^\delta = \lambda' P'^\delta$. With this in mind, BSs with intensity $\lambda_{\rm C}$ and transmit power $P_{\rm C}$ can be modeled by a PPP with intensity $\lambda_{\rm C}(\frac{P_{\rm C}}{P_{\rm D}})^\delta$ and power $P_{\rm D}$. This can also be derived by using Corollary 5.4 of [9]. Using this result and assuming that both sets of transmitters (BSs and D2D transmitters) have the same transmit probability, the above equations revert back to the known results for Poisson bipolar networks [7]. Moreover, $M_{b,\rm D2D}$ can be factorized into a term without BS interference ($\lambda_{\rm C}=0$) and a term without D2D interference ($\lambda_{\rm D}=0$).

Applying the Gil-Pelaez theorem [25] to the imaginary moments M_{jt}, $t\in\mathbb{R}$, the exact meta distribution of the typical D2D receiver is

$$\bar{F}_{P_{\rm s,D2D}}(x)=\frac{1}{2}+\frac{1}{\pi}\int_0^\infty \frac{\Im\big(e^{-jt\log x}M_{jt,\rm D2D}\big)}{t}{\rm d}t, \quad (14)$$

where $\Im(z)$ is the imaginary part of $z\in\mathbb{C}$. While this expression is exact, it is too unwieldy to gain direct insight from it, and it is cumbersome to evaluate numerically. Hence we proceed by approximating the meta distribution by a beta distribution by matching first and second moments, which are easily obtained from the general result in (9):

$$M_{1,\rm D2D}=\exp\left(-\pi\theta^\delta d^2\frac{\pi\delta}{\sin(\pi\delta)}\left[p_{\rm D}\lambda_{\rm D}+p_{\rm C}\lambda_{\rm C}\left(\frac{P_{\rm C}}{P_{\rm D}}\right)^\delta\right]\right), \quad (15)$$

$$M_{2,\rm D2D}=\exp\bigg(-\pi\theta^\delta d^2\frac{\pi\delta}{\sin(\pi\delta)}\bigg[(2p_{\rm D}+(\delta-1){p_{\rm D}}^2)\lambda_{\rm D} + \big(2p_{\rm C}+(\delta-1){p_{\rm C}}^2\big)\lambda_{\rm C}\left(\frac{P_{\rm C}}{P_{\rm D}}\right)^\delta\bigg]\bigg). \quad (16)$$

By matching the mean and variance of the beta distribution with $M_{1,\rm D2D}$ and $M_{2,\rm D2D}-M_{1,\rm D2D}^2$, the approximate meta distribution of the typical D2D receiver follows as

$$\bar{F}_{P_{\rm s,D2D}}(x)\approx 1-I_x\left(\frac{M_{1,\rm D2D}\beta}{1-M_{1,\rm D2D}},\beta\right),\qquad x\in[0,1], \quad (17)$$

where

$$\beta\triangleq\frac{(M_{1,\rm D2D}-M_{2,\rm D2D})\,(1-M_{1,\rm D2D})}{M_{2,\rm D2D}-M_{1,\rm D2D}^2}$$

and $I_x(a,b)$ is the regularized incomplete beta function

$$I_x(a,b)\triangleq\frac{\int_0^x t^{a-1}(1-t)^{b-1}dt}{B(a,b)} \quad (18)$$

and $B(a,b)$ denotes the beta function.

B. Meta Distribution for the Typical Downlink Receiver

In the following, the b-th moment of $P_{\rm s}$ for the typical downlink receiver is derived, for any $b\in\mathbb{C}$.

Theorem 2 (Moments for downlink receivers). *The b-th moment of the conditional success probability for the typical downlink receiver is*

$$M_{b,\rm Cellular}=\left(1+\frac{\lambda_{\rm D}}{\lambda_{\rm C}}\theta^\delta\left(\frac{P_{\rm D}}{P_{\rm C}}\right)^\delta\frac{\pi\delta}{\sin(\pi\delta)}p_{\rm D}b\,{}_2F_1(1-b,1-\delta;2;p_{\rm D}) -\sum_{k=1}^\infty\binom{b}{k}(-p_{\rm C}\theta)^k\frac{\delta}{k-\delta}\,{}_2F_1(k,k-\delta;k-\delta+1;-\theta)\right)^{-1}, \quad b\in\mathbb{C}. \quad (19)$$

Proof: We first express the conditional success probability for the typical downlink receiver.

$$\begin{aligned}P_{\rm s,Cellular}(\theta)&=\mathbb{P}\left(h_0^{\rm c}(t)>\frac{\theta\|x_0\|^\alpha}{P_{\rm C}}I_{0,\rm Cellular}(t)\mid\Phi_{\rm D},\Phi_{\rm C},{\rm tx}\right)\\&=\prod_{x\in\Phi_{\rm D}}\left(\frac{p_{\rm D}}{1+\theta\frac{P_{\rm D}}{P_{\rm C}}\|x_0\|^\alpha\|x\|^{-\alpha}}+1-p_{\rm D}\right)\\&\quad\cdot\prod_{x\in\Phi_{\rm C}\setminus\{x_0\}}\left(\frac{p_{\rm C}}{1+\theta\|x_0\|^\alpha\|x\|^{-\alpha}}+1-p_{\rm C}\right).\end{aligned} \quad (20)$$

Then the b-th moment is obtained by taking the expectation with respect to the point processes.

$$\begin{aligned}&M_{b,\rm Cellular}\\&=\mathbb{E}\Bigg[\prod_{x\in\Phi_{\rm D}}\left(\frac{p_{\rm D}}{1+\theta\frac{P_{\rm D}}{P_{\rm C}}\|x_0\|^\alpha\|x\|^{-\alpha}}+1-p_{\rm D}\right)^b\prod_{x\in\Phi_{\rm C}\setminus\{x_0\}}\left(\frac{p_{\rm C}}{1+\theta\|x_0\|^\alpha\|x\|^{-\alpha}}+1-p_{\rm C}\right)^b\Bigg]\\&=\mathbb{E}_{\Phi_{\rm C}}\Bigg[\mathbb{E}_{\Phi_{\rm D}}\Bigg[\prod_{x\in\Phi_{\rm D}}\left(\frac{p_{\rm D}}{1+\theta\frac{P_{\rm D}}{P_{\rm C}}\|x_0\|^\alpha\|x\|^{-\alpha}}+1-p_{\rm D}\right)^b\Bigg]\prod_{x\in\Phi_{\rm C}\setminus\{x_0\}}\left(\frac{p_{\rm C}}{1+\theta\|x_0\|^\alpha\|x\|^{-\alpha}}+1-p_{\rm C}\right)^b\Bigg]\\&=\mathbb{E}_{\Phi_{\rm C}}\Bigg[\exp\bigg\{-\lambda_{\rm D}\pi\theta^\delta\left(\frac{P_{\rm D}}{P_{\rm C}}\right)^\delta\|x_0\|^2\frac{\pi\delta}{\sin(\pi\delta)}p_{\rm D}b\cdot{}_2F_1(1-b,1-\delta;2;p_{\rm D})\bigg\}\prod_{x\in\Phi_{\rm C}\setminus\{x_0\}}\left(\frac{p_{\rm C}}{1+\theta\|x_0\|^\alpha\|x\|^{-\alpha}}+1-p_{\rm C}\right)^b\Bigg]\\&\overset{(a)}{=}\left(1+\frac{\lambda_{\rm D}}{\lambda_{\rm C}}\theta^\delta\left(\frac{P_{\rm D}}{P_{\rm C}}\right)^\delta\frac{\pi\delta}{\sin(\pi\delta)}p_{\rm D}b\,{}_2F_1(1-b,1-\delta;2;p_{\rm D})-\sum_{k=1}^\infty\binom{b}{k}(-p_{\rm C}\theta)^k\frac{\delta}{k-\delta}\,{}_2F_1(k,k-\delta;k-\delta+1;-\theta)\right)^{-1},\end{aligned}$$

where (a) is calculated by doing the same calculation as which was done in [22] to obtain the PGFL of a relative distance process (RDP), which is defined as

$$\mathcal{R}\triangleq\{x\in\Phi_{\rm C}\setminus\{x_0\}:\|x_0\|/\|x\|\}. \quad (21)$$

To derive (a) we also need the nearest-neighbor distance distribution of PPP [28]. ■

As in Poisson cellular networks, the moments of the conditional success probability for downlink users have the form of a fraction, but the term representing the co-tier interference is different from the cross-tier interference term since cellular interference is different in structure (interferers are further away than the serving BS) than the D2D interference. As $\lambda_{\mathrm{D}} \to 0$ or $P_{\mathrm{D}} \to 0$, equation (19) reverts to the result for Poisson cellular networks (when BSs are active with probability p) which is known from [7, Eqn. (25)]. For $b \in \mathbb{N}$, the infinite sum in (19) becomes finite and has only b terms.

It is worth mentioning that including thermal noise simply adds a constant term to the analytical results. When σ^2 is the variance of the thermal noise, for D2D users, $P_{\mathrm{s,D2D}}$ and $M_{b,\mathrm{D2D}}$ are multiplied by $\exp\left\{-\frac{\theta d^\alpha}{P_{\mathrm{D}}}\sigma^2\right\}$ and $\exp\left\{-\frac{\theta d^\alpha}{P_{\mathrm{D}}} b\sigma^2\right\}$. Similarly, for downlink users, $P_{\mathrm{s,Cellular}}$ is multiplied by $\exp\left\{-\frac{\theta\|x_0\|^\alpha}{P_{\mathrm{C}}}\sigma^2\right\}$. Moreover, without noise, the support of the conditional success probabilities are $[0,1]$, while considering the thermal noise reduces the support of $P_{\mathrm{s,D2D}}$ to $\left[0, \exp\left\{-\frac{\theta d^\alpha}{P_{\mathrm{D}}}\sigma^2\right\}\right]$. For downlink users, the support of $P_{\mathrm{s,Cellular}}$ remains $[0,1]$ since the link distance can be arbitrarily small.

As in (14), the exact meta distribution of the typical downlink receiver is

$$\bar{F}_{P_{\mathrm{s,Cellular}}}(x) = \frac{1}{2} + \frac{1}{\pi}\int_0^\infty \frac{\Im\left(e^{-jt\log x} M_{jt,\mathrm{Cellular}}\right)}{t}\mathrm{d}t. \quad (22)$$

Using the first and the second moments of the conditional success probability

$$M_{1,\mathrm{Cellular}} = \left(1 + \frac{\lambda_{\mathrm{D}}}{\lambda_{\mathrm{C}}}\theta^\delta \left(\frac{P_{\mathrm{D}}}{P_{\mathrm{C}}}\right)^\delta \frac{\pi\delta}{\sin(\pi\delta)} p_{\mathrm{D}} + p_{\mathrm{C}}\theta \frac{\delta}{1-\delta}\, {}_2F_1(1, 1-\delta; 2-\delta; -\theta)\right)^{-1}, \quad (23)$$

$$M_{2,\mathrm{Cellular}} = \Bigg(1 + \frac{\lambda_{\mathrm{D}}}{\lambda_{\mathrm{C}}}\theta^\delta \left(\frac{P_{\mathrm{D}}}{P_{\mathrm{C}}}\right)^\delta \frac{\pi\delta}{\sin(\pi\delta)}[2p_{\mathrm{D}} + (\delta-1){p_{\mathrm{D}}}^2] + 2p_{\mathrm{C}}\theta\frac{\delta}{1-\delta}\, {}_2F_1(1, 1-\delta; 2-\delta; -\theta) - (p_{\mathrm{C}}\theta)^2 \frac{\delta}{2-\delta}\, {}_2F_1(2, 2-\delta; 3-\delta; -\theta)\Bigg)^{-1}, \quad (24)$$

the beta approximation of the meta distribution for the typical downlink receiver follows as

$$\bar{F}_{P_{\mathrm{s,Cellular}}}(x) \approx 1 - I_x\left(\frac{M_{1,\mathrm{Cellular}}\beta'}{1 - M_{1,\mathrm{Cellular}}}, \beta'\right), \quad (25)$$

where β' is given by

$$\beta' = \frac{(M_{1,\mathrm{Cellular}} - M_{2,\mathrm{Cellular}})(1 - M_{1,\mathrm{Cellular}})}{M_{2,\mathrm{Cellular}} - M_{1,\mathrm{Cellular}}^2}. \quad (26)$$

In D2D communications, when D2D transmitters and BSs transmit with the same access probability ($p_{\mathrm{D}} = p_{\mathrm{C}}$) and same power ($P_{\mathrm{D}} = P_{\mathrm{C}}$), interference coming from both D2D transmitters and BSs can be modeled by a single PPP with intensity $\lambda_{\mathrm{D}} + \lambda_{\mathrm{C}}$. However, when D2D users and BSs transmit with different powers, the effect of the interference coming from BSs on the SIR of the typical D2D receiver depends on the power of the desired D2D transmitter. When the desired D2D transmitter uses more power, the effect of the BSs interference on the SIR of the typical D2D receiver will be smaller. Therefore, if we model BSs with a PPP with intensity λ' and transmit power P_{D}, λ' decreases as the D2D desired transmit power increases. This is the same for downlink users.

From Theorem 1 and 2, we can derive an interesting invariance result that shows how the BS density and the transmit powers can be traded off against each other, without changing the meta distributions. To do this, let us write (9) and (19) in the form

$$M_{b,\mathrm{D2D}} = \exp\left(-\lambda_{\mathrm{D}}\left[k_1 + k_2\frac{\lambda_{\mathrm{C}}}{\lambda_{\mathrm{D}}}\left(\frac{P_{\mathrm{C}}}{P_{\mathrm{D}}}\right)^\delta\right]\right)$$

and

$$M_{b,\mathrm{Cellular}} = \left(1 + k_3\left[\frac{\lambda_{\mathrm{C}}}{\lambda_{\mathrm{D}}}\left(\frac{P_{\mathrm{C}}}{P_{\mathrm{D}}}\right)^\delta\right]^{-1} + k_4\right)^{-1},$$

where k_1, k_2, k_3, and k_4 do not depend on the densities and transmit powers. Clearly, if

$$C \triangleq \lambda_{\mathrm{C}}\left(\frac{P_{\mathrm{C}}}{P_{\mathrm{D}}}\right)^\delta$$

is held constant (and all other parameters stay the same), all moments and thus both meta distributions remain unchanged. For example, for $\delta = 1/2$, the effect of doubling the BS density can be compensated for by quadrupling the D2D transmit power.

C. Combined Meta Distribution

The meta distribution of the typical D2D receiver $\bar{F}_{P_{\mathrm{s,D2D}}}(x)$ given in (17) can be interpreted as the fraction of active D2D receivers whose conditional success probabilities are greater than x. $\bar{F}_{P_{\mathrm{s,Cellular}}}(x)$ is the fraction of active downlink users which have a success probability larger than x in each realization of D2D transmitters and BSs. We define the meta distribution of the whole network $\bar{F}_{P_{\mathrm{s,total}}}(x)$ as the fraction of active receivers that achieve the target SIR θ in each realization with a probability that is larger than the target value x. $\bar{F}_{P_{\mathrm{s,total}}}(x)$ is provided in the following corollary.

Corollary 1 (**Meta distribution of the whole network**). *For the overall users, the meta distribution of the SIR is*

$$\bar{F}_{P_{\mathrm{s,total}}}(x) = \frac{\lambda_{\mathrm{D}}p_{\mathrm{D}}}{\lambda_{\mathrm{D}}p_{\mathrm{D}} + \lambda_{\mathrm{C}}p_{\mathrm{C}}}\bar{F}_{P_{\mathrm{s,D2D}}}(x) + \frac{\lambda_{\mathrm{C}}p_{\mathrm{C}}}{\lambda_{\mathrm{D}}p_{\mathrm{D}} + \lambda_{\mathrm{C}}p_{\mathrm{C}}}\bar{F}_{P_{\mathrm{s,Cellular}}}(x). \quad (27)$$

Proof: Let us consider the point process of all active receivers (those who have active transmitters) and focus on the typical receiver of this point process. Then (17) is the meta distribution conditioned on that this typical receiver is a D2D receiver, while (25) is the meta distribution conditioned on that this typical receiver is a cellular receiver (UE). Therefore, the unconditioned meta distribution is

$$\begin{aligned}&\bar{F}_{P_{\mathrm{s,total}}}(x)\\&=\mathbb{P}\left(\text{Rx is D2D}\right)\bar{F}_{P_{\mathrm{s,D2D}}}(x)+\mathbb{P}\left(\text{Rx is UE}\right)\bar{F}_{P_{\mathrm{s,Cellular}}}(x)\\&=\frac{\lambda_{\mathrm{D}}p_{\mathrm{D}}}{\lambda_{\mathrm{D}}p_{\mathrm{D}}+\lambda_{\mathrm{C}}p_{\mathrm{C}}}\bar{F}_{P_{\mathrm{s,D2D}}}(x)+\frac{\lambda_{\mathrm{C}}p_{\mathrm{C}}}{\lambda_{\mathrm{D}}p_{\mathrm{D}}+\lambda_{\mathrm{C}}p_{\mathrm{C}}}\bar{F}_{P_{\mathrm{s,Cellular}}}(x).\end{aligned}$$

■

Since we have $M_b=\int_0^1 bt^{b-1}\bar{F}_{P_{\mathrm{s}}}(t)\mathrm{d}t$, the b-th moment of the conditional success probability of the whole network can be written as

$$\begin{aligned}&M_{b,\mathrm{total}}\\&=\frac{\lambda_{\mathrm{D}}p_{\mathrm{D}}}{\lambda_{\mathrm{D}}p_{\mathrm{D}}+\lambda_{\mathrm{C}}p_{\mathrm{C}}}M_{b,\mathrm{D2D}}+\frac{\lambda_{\mathrm{C}}p_{\mathrm{C}}}{\lambda_{\mathrm{D}}p_{\mathrm{D}}+\lambda_{\mathrm{C}}p_{\mathrm{C}}}M_{b,\mathrm{Cellular}}.\end{aligned}\quad(28)$$

For example the standard success probability of the whole network simply is obtained by substituting $b=1$ in the above equation. It can be interpreted as the success probability of the (overall) typical user or as the fraction of (all) users whose SIRs are larger than θ in each realization of D2D transmitters and BSs.

D. The Mean Local Delay

Another important network performance metric is the mean local delay, defined as the mean number of transmissions until the first success (see Subsection II-E). It is finite when certain parameters of the network, such as the SIR threshold and the medium access probability, are below specific thresholds [13]. In these situations, the fraction of nodes that suffer from long delays is negligible [14]. The mean local delay may exhibit a phase transition called *wireless contention phase transition* [13]. In this case, it becomes infinite at some critical value of a network parameter, which means there is a significant number of links with large delays [15]. From (7) and (9), the mean local delay for the typical D2D user is given by

$$\begin{aligned}&M_{-1,\mathrm{D2D}}\\&=\exp\Bigg(\pi\theta^{\delta}d^2\frac{\pi\delta}{\sin(\pi\delta)}\Big[\lambda_{\mathrm{D}}p_{\mathrm{D}}(1-p_{\mathrm{D}})^{\delta-1}\\&\quad+\lambda_{\mathrm{C}}\left(\frac{P_{\mathrm{C}}}{P_{\mathrm{D}}}\right)^{\delta}p_{\mathrm{C}}(1-p_{\mathrm{C}})^{\delta-1}\Big]\Bigg),\quad p_{\mathrm{D}},p_{\mathrm{C}}<1,\end{aligned}\quad(29)$$

where the phase transition occurs at $p_{\mathrm{D}}=1$ or $p_{\mathrm{C}}=1$.

To derive the mean local delay for the typical downlink cellular user, we first define the function $G(p_{\mathrm{D}},p_{\mathrm{C}})$ as

$$\begin{aligned}G(p_{\mathrm{D}},p_{\mathrm{C}})\triangleq{}&1-\frac{\lambda_{\mathrm{D}}}{\lambda_{\mathrm{C}}}\theta^{\delta}\left(\frac{P_{\mathrm{D}}}{P_{\mathrm{C}}}\right)^{\delta}\frac{\pi\delta}{\sin(\pi\delta)}p_{\mathrm{D}}(1-p_{\mathrm{D}})^{\delta-1}\\&-p_{\mathrm{C}}\theta\frac{\delta}{1-\delta}(1+\theta(1-p_{\mathrm{C}}))^{-1}\\&\cdot{}_2F_1\left(1,1;2-\delta;\frac{\theta(1-p_{\mathrm{C}})}{1+\theta(1-p_{\mathrm{C}})}\right).\end{aligned}\quad(30)$$

Then we have

$$M_{-1,\mathrm{Cellular}}=\frac{1}{G(p_{\mathrm{D}},p_{\mathrm{C}})},\quad(p_{\mathrm{D}},p_{\mathrm{C}})\in\mathcal{S},\quad(31)$$

where the region $\mathcal{S}\subset[0,1]^2$ of finite mean local delay is given by

$$\mathcal{S}\triangleq\left\{(p_{\mathrm{D}},p_{\mathrm{C}})\in[0,1]^2\colon G(p_{\mathrm{D}},p_{\mathrm{C}})>0\right\}.\quad(32)$$

Based on the network parameters, the mean local delay can be finite or infinite. There is a boundary for the network parameters which separates the regions of finite and infinite mean local delay. For example, when other parameters of the network are fixed, for small values of p_{D} the mean local delay can be finite, but as p_{D} increases, at a certain threshold the mean local delay of downlink users becomes infinite (this value of p_{D} makes the denominator of $M_{-1,\mathrm{Cellular}}$ zero). The threshold is called the critical value, and since at the critical value the mean local delay changes from finite to infinite we say a *phase transition* or *wireless contention phase transition* occurs. Here the phase transition occurs at the boundary $\partial\mathcal{S}\triangleq\{p_{\mathrm{D}},p_{\mathrm{C}}\in[0,1]\colon G(p_{\mathrm{D}},p_{\mathrm{C}})=0\}$. Using the following inequality

$$(1+\theta(1-p_{\mathrm{C}}))^{-1}{}_2F_1\left(1,1;2-\delta;\frac{\theta(1-p_{\mathrm{C}})}{1+\theta(1-p_{\mathrm{C}})}\right)<1,\quad(33)$$

an inner bound of $\mathcal{S}$ is provided by $\{p_{\mathrm{D}},p_{\mathrm{C}}\in[0,1]\colon f(p_{\mathrm{D}},p_{\mathrm{C}})<1\}$, where

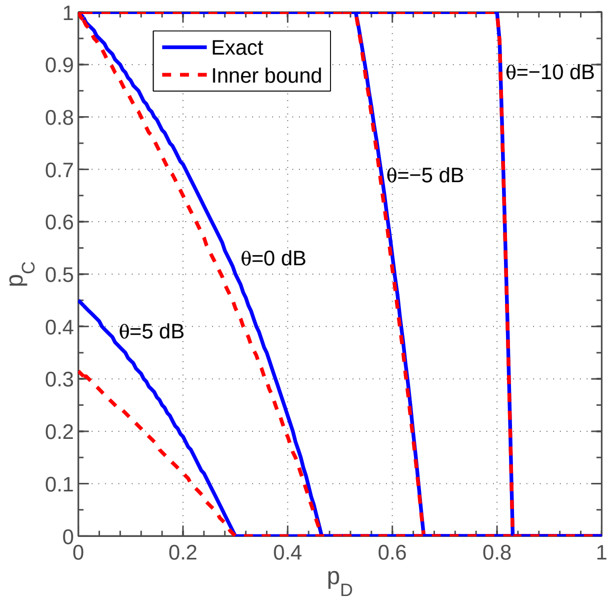
$$f(p_{\mathrm{D}},p_{\mathrm{C}})=\frac{\theta\delta}{1-\delta}p_{\mathrm{C}}+\frac{\lambda_{\mathrm{D}}}{\lambda_{\mathrm{C}}}\theta^{\delta}\left(\frac{P_{\mathrm{D}}}{P_{\mathrm{C}}}\right)^{\delta}\frac{\pi\delta}{\sin(\pi\delta)}p_{\mathrm{D}}(1-p_{\mathrm{D}})^{\delta-1}.\quad(34)$$

In Fig. 2, $\partial\mathcal{S}$ and the proposed inner bound are shown for two different sets of parameters. As we see, as $\theta\to 0$, the inner bound becomes tighter, and the boundary of the region $\partial\mathcal{S}$ approaches a vertical line, since $\theta^{\delta}\gg\theta$ and thus the term in p_{D} becomes dominant in the function f. In other words, for small target SIRs, BSs can transmit with any probability as long as the D2D transmitters' medium access probability p_{D} is less than the critical value. In Fig. 3, the mean local delay (31) is illustrated as a function of p_{D} for different values of p_{C}. As p_{D} approaches its critical value $p_{\mathrm{D,critical}}$, this means that a significant fraction of the links suffer from high delay, which is certainly an undesirable operating regime.

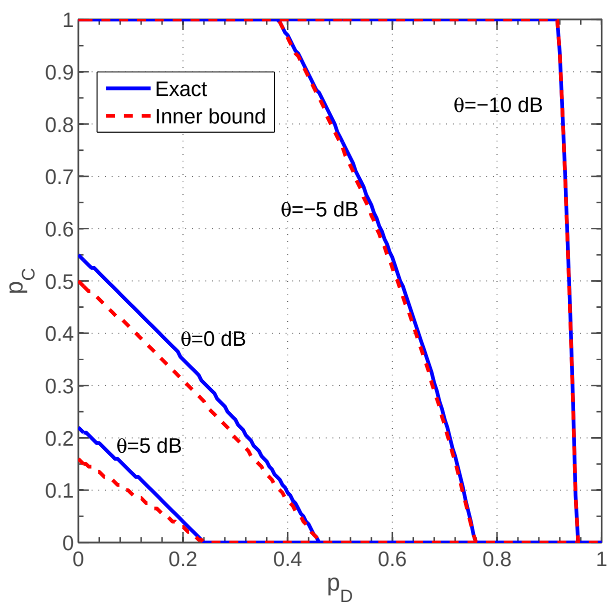
In Fig. 4, the critical probability p_{critical} is illustrated as a function of the SIR threshold θ for $\alpha=3,4$ when both D2D transmitters and BSs transmit with the same probability ($p_{\mathrm{D}}=p_{\mathrm{C}}=p$). For $p<p_{\mathrm{critical}}(\theta)$, the mean local delay is finite while at $p=p_{\mathrm{critical}}(\theta)$ the phase transition occurs. As expected, the critical probability decreases as θ increases. For low target SIRs the desired signal at the receiver is stronger when path loss exponent is 3, so the critical probability for $\alpha=3$ is higher than for $\alpha=4$. On the other hand, for high thresholds, the critical probability for $\alpha=4$ is larger since the interferers are more severely attenuated. Moreover, it can be observed that for large target SIRs, the slope of $p_{\mathrm{critical}}(\theta)$ is $-\delta$, which is formally stated in the next result.

Corollary 2. *When $p_{\mathrm{D}}=p_{\mathrm{C}}=p$ and $\theta\to\infty$,*

$$p_{\mathrm{critical}}(\theta)\sim\left[\left(\frac{\lambda_{\mathrm{D}}}{\lambda_{\mathrm{C}}}\left(\frac{P_{\mathrm{D}}}{P_{\mathrm{C}}}\right)^{\delta}+1\right)\frac{\pi\delta}{\sin(\pi\delta)}\right]^{-1}\theta^{-\delta}.\quad(35)$$



(a) $\frac{\lambda_{\mathrm{D}}}{\lambda_{\mathrm{C}}} = 10$, $\frac{P_{\mathrm{C}}}{P_{\mathrm{D}}} = 100$, and $\alpha = 4$.



(b) $\frac{\lambda_{\mathrm{D}}}{\lambda_{\mathrm{C}}} = 25$, $\frac{P_{\mathrm{C}}}{P_{\mathrm{D}}} = 200$, and $\alpha = 3$

Fig. 2. Boundaries $\partial\mathcal{S}$ of the region $\mathcal{S}$ of finite mean local delay and the proposed inner bound.

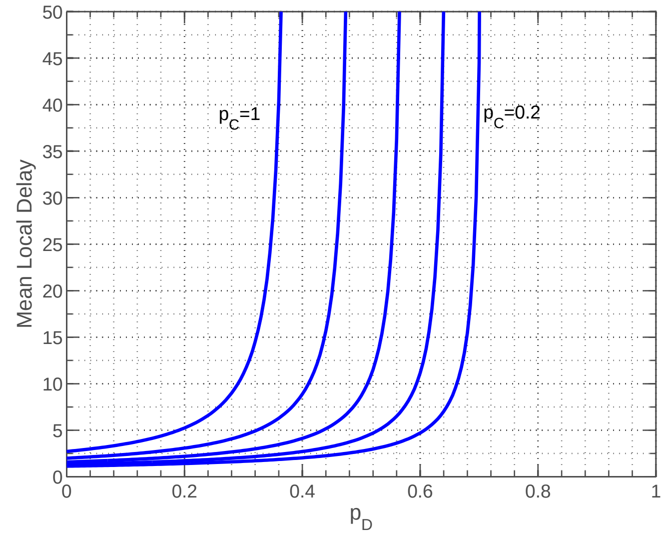


Fig. 3. The mean local delay of downlink receivers (31) as a function of p_{D} for $p_{\mathrm{C}} = 0.2, 0.4, 0.6, 0.8, 1$ when $\frac{\lambda_{\mathrm{D}}}{\lambda_{\mathrm{C}}} = 25$, $\frac{P_{\mathrm{C}}}{P_{\mathrm{D}}} = 200$, $\theta = -5\,\mathrm{dB}$, and $\alpha = 3$.

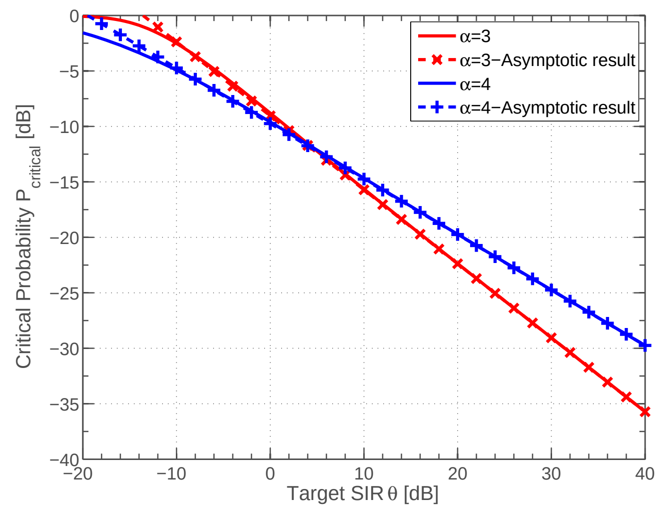


Fig. 4. Critical probability $p_{\mathrm{critical}}(\theta)$ and its asymptotic approximation (35) (in dB) as a function of target SIR θ when $p_{\mathrm{D}} = p_{\mathrm{C}}$ for $\frac{\lambda_{\mathrm{D}}}{\lambda_{\mathrm{C}}} = 50$, $\frac{P_{\mathrm{C}}}{P_{\mathrm{D}}} = 100$, and path loss exponent 3 and 4.

Proof: Since $G(0,0) = 1$, there exist a $p > 0$ such that $G(p,p) > 0$, for any fixed θ. Conversely, for any fixed $p > 0$, $G(p,p)$ becomes negative as $\theta \to \infty$. Therefore, $p_{\mathrm{critical}} \to 0$ as $\theta \to \infty$, and we have $p = o(1)$ necessarily if $p \le p_{\mathrm{critical}}$. Hence for any $p \le p_{\mathrm{critical}}$, as $\theta \to \infty$,

$$G(p,p) \sim 1 - \frac{\lambda_{\mathrm{D}}}{\lambda_{\mathrm{C}}} \left(\frac{P_{\mathrm{D}}}{P_{\mathrm{C}}}\right)^{\delta} \frac{\pi\delta}{\sin(\pi\delta)} p\theta^{\delta} - \frac{\delta}{1-\delta} p\theta \, {}_2F_1\left(1, 1-\delta; 2-\delta; -\theta\right) \tag{36}$$

$$= 1 - \frac{\lambda_{\mathrm{D}}}{\lambda_{\mathrm{C}}} \left(\frac{P_{\mathrm{D}}}{P_{\mathrm{C}}}\right)^{\delta} \frac{\pi\delta}{\sin(\pi\delta)} p\theta^{\delta} - \delta p\theta \int_0^1 t^{-\delta}(1+\theta t)^{-1} \mathrm{d}t$$
$$\stackrel{(a)}{=} 1 - \frac{\lambda_{\mathrm{D}}}{\lambda_{\mathrm{C}}} \left(\frac{P_{\mathrm{D}}}{P_{\mathrm{C}}}\right)^{\delta} \frac{\pi\delta}{\sin(\pi\delta)} p\theta^{\delta} - \delta p\theta^{\delta} \int_0^{\theta} u^{-\delta}(1+u)^{-1} \mathrm{d}u$$
$$\stackrel{(b)}{=} 1 - \left(\frac{\lambda_{\mathrm{D}}}{\lambda_{\mathrm{C}}} \left(\frac{P_{\mathrm{D}}}{P_{\mathrm{C}}}\right)^{\delta} + 1\right) \frac{\pi\delta}{\sin(\pi\delta)} p\theta^{\delta}$$

In (36) the identity

$${}_2F_1(a,b;c;z) \equiv (1-z)^{-a} \, {}_2F_1\left(a, c-b; c; \frac{z}{z-1}\right) \tag{37}$$

is applied. (a) follows from the substitution $\theta t = u$ and (b) from $\int_0^{\infty} u^{-\delta}(1+u)^{-1}\mathrm{d}u = \frac{\pi}{\sin(\pi\delta)}$. Finally, (35) is obtained by equating $G(p_{\mathrm{critical}}, p_{\mathrm{critical}}) = 0$. ∎

IV. Numerical Results

In this section, we first confirm the accuracy of the beta approximation by comparing the approximations (17) and (25) with simulation results[3]. Next, the effect of the BS intensity λ_{C} on the mean local delays of both D2D and cellular users is investigated. Moreover, the meta distribution of the SIR is used to study the effect of BS intensity on overall users. Finally, the density of users served is introduced as a better network-wide metric to investigate the effect of increasing λ_{C}.

A. Accuracy of the Beta Approximation

Fig. 5 shows the simulated meta distribution and the beta approximation for both D2D receivers and downlink users, for different sets of parameters. It can be seen that the beta distribution is an excellent approximation for the meta distribution, and it is apparent that the standard success probabilities provide only limited information on the network performance. In Fig. 5(b), although $M_{1,\mathrm{D2D}} \approx M_{1,\mathrm{Cellular}}$, for some reliability values x we have $\bar{F}_{P_{\mathrm{s,D2D}}}(x) > \bar{F}_{P_{\mathrm{s,Cellular}}}(x)$ and for others we have $\bar{F}_{P_{\mathrm{s,D2D}}}(x) < \bar{F}_{P_{\mathrm{s,Cellular}}}(x)$.

Now that its accuracy is verified, we will use the beta approximation to obtain the numerical results in Subsections IV-C and IV-D.

B. Mean Local Delay as a Function of BS Density

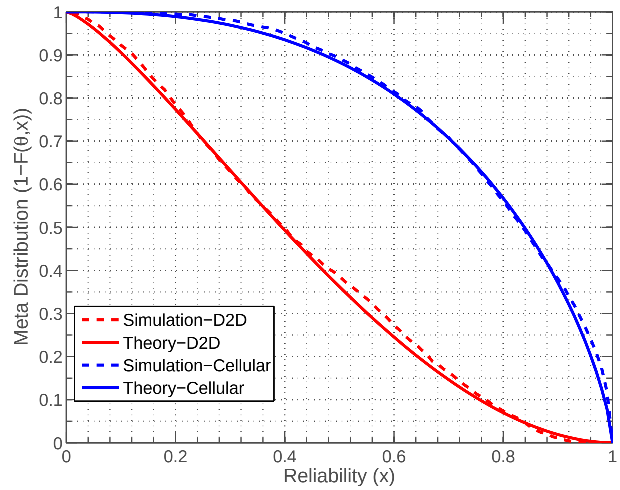
Based on (29) and (31), increasing the BS intensity λ_{C} increases the mean local delay for D2D receivers while reducing it for the downlink users. Fig. 6 illustrates this behavior for two different sets of access probabilities. If we just consider the mean local delay of D2D users, it is obvious that it is minimized at $\lambda_{\mathrm{C}} = 0$ because there is the least interference in this case. This can also be understood by (29), which shows that $M_{-1,\mathrm{D2D}}$ is strictly increasing in λ_{C}. On the other hand, when we consider the mean local delay of cellular users, we see that $\lambda_{\mathrm{C}} = \infty$ minimizes $M_{-1,\mathrm{Cellular}}$. Although $M_{-1,\mathrm{Cellular}}(\lambda_{\mathrm{C}})$ is strictly decreasing, after a while, the effect of increasing BS intensity on $M_{-1,\mathrm{Cellular}}$ is negligible and the function becomes flat. Therefore, considering both D2D and downlink cellular users, a finite value of λ_{C} should be chosen that offers a good trade-off, so that the mean local delays of both D2D and cellular users are close to optimum.

Obviously, $\lambda_{\mathrm{C}} = 0$ and $\lambda_{\mathrm{C}} = \infty$ are not good choices in terms of fairness because $\lambda_{\mathrm{C}} = 0$ means no cellular communications and $\lambda_{\mathrm{C}} = \infty$ means no D2D communications. In order to define the combined (overall) mean local delay for the whole network, using (28), we write $M_{-1,\mathrm{total}}$ as

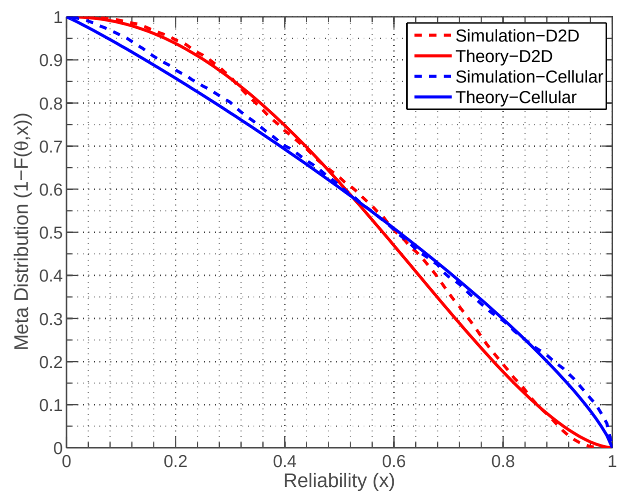
$$M_{-1,\mathrm{total}} = \frac{\lambda_{\mathrm{D}} p_{\mathrm{D}}}{\lambda_{\mathrm{D}} p_{\mathrm{D}} + \lambda_{\mathrm{C}} p_{\mathrm{C}}} M_{-1,\mathrm{D2D}} + \frac{\lambda_{\mathrm{C}} p_{\mathrm{C}}}{\lambda_{\mathrm{D}} p_{\mathrm{D}} + \lambda_{\mathrm{C}} p_{\mathrm{C}}} M_{-1,\mathrm{Cellular}} \quad (38)$$

where the first weight is the fraction of active D2D users and the second weight is the fraction of active downlink users, so if we have more active D2D users, they will have priority

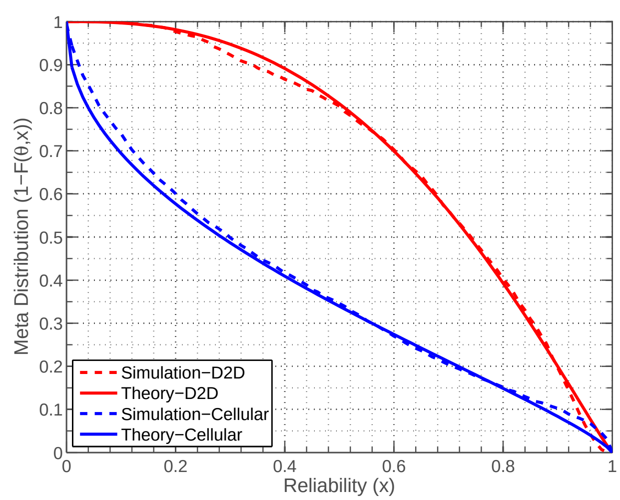
[3]In principle, the exact analytical expressions (14) and (22) could be used for the comparison, but it turns out that simulation results are easier to obtain than a numerical evaluation of the exact expressions.



(a) $\lambda_{\mathrm{C}} = 8\,\mathrm{km}^{-2}$, $\frac{P_{\mathrm{C}}}{P_{\mathrm{D}}} = 400$, $p_{\mathrm{D}} = 0.3$, $p_{\mathrm{C}} = 0.7$, and $\theta = -5\,\mathrm{dB}$. The means are $M_{1,\mathrm{D2D}} \approx 0.41$ and $M_{1,\mathrm{Cellular}} \approx 0.77$.

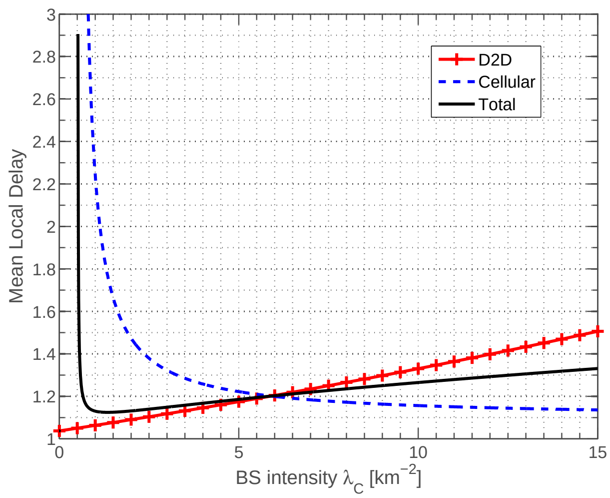


(b) $\lambda_{\mathrm{C}} = 6\,\mathrm{km}^{-2}$, $\frac{P_{\mathrm{C}}}{P_{\mathrm{D}}} = 100$, $p_{\mathrm{D}} = 0.2$, $p_{\mathrm{C}} = 0.6$, and $\theta = 0\,\mathrm{dB}$. The means are $M_{1,\mathrm{D2D}} \approx 0.56$ and $M_{1,\mathrm{Cellular}} \approx 0.57$.

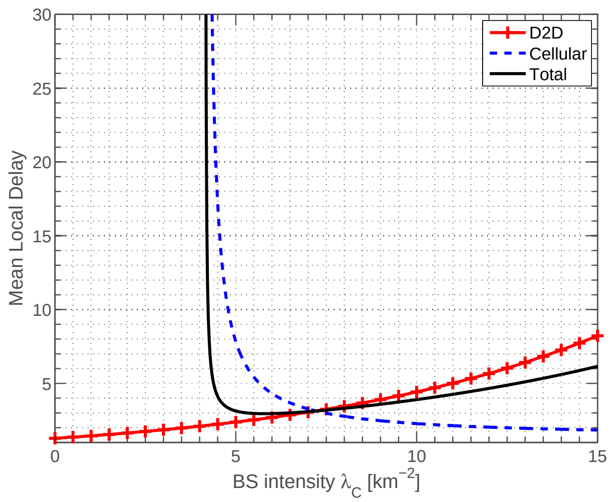


(c) $\lambda_{\mathrm{C}} = 2\,\mathrm{km}^{-2}$, $\frac{P_{\mathrm{C}}}{P_{\mathrm{D}}} = 100$, $p_{\mathrm{D}} = 0.3$, $p_{\mathrm{C}} = 0.7$, and $\theta = 0\,\mathrm{dB}$. The means are $M_{1,\mathrm{D2D}} \approx 0.69$ and $M_{1,\mathrm{Cellular}} \approx 0.36$.

Fig. 5. Meta distribution as a function of reliability x for $d = 50\,\mathrm{m}$, $\lambda_{\mathrm{D}} = 50\,\mathrm{km}^{-2}$, and $\alpha = 4$.



(a) $p_{\mathrm{D}} = 0.1$, $p_{\mathrm{C}} = 0.3 \Rightarrow \lambda_{\mathrm{C,critical}} = 0.5\,\mathrm{km}^{-2}$, $\lambda_{\mathrm{C,equal}} = 5.9\,\mathrm{km}^{-2}$.



(b) $p_{\mathrm{D}} = 0.5$, $p_{\mathrm{C}} = 0.8 \Rightarrow \lambda_{\mathrm{C,critical}} = 4.2\,\mathrm{km}^{-2}$, $\lambda_{\mathrm{C,equal}} = 7.2\,\mathrm{km}^{-2}$.

Fig. 6. Effect of BS intensity λ_{C} on the mean local delays of D2D users (29), downlink users (31), and total users (38) for $d = 50\,\mathrm{m}$, $\lambda_{\mathrm{D}} = 50\,\mathrm{km}^{-2}$, $\frac{P_{\mathrm{C}}}{P_{\mathrm{D}}} = 100$, $\theta = -5\,\mathrm{dB}$ and $\alpha = 4$. Optimal λ_{C} that minimizes $M_{-1,\mathrm{total}}$ is between $\lambda_{\mathrm{C,critical}}$ (the critical value of λ_{C} for finite mean local delay) and $\lambda_{\mathrm{C,equal}}$ (the value of λ_{C} where the mean local delay for cellular and D2D users is the same).

over downlink cellular users. In Fig. 6, the mean local delays of D2D, downlink (cellular), and total users are shown for different access probabilities. As we see, for these network parameters, λ_{C} that minimizes $M_{-1,\mathrm{total}}$ is between $\lambda_{\mathrm{C,critical}}$ (the critical value of λ_{C} for finite mean local delay) and $\lambda_{\mathrm{C,equal}}$ (the value of λ_{C} where the mean local delay for cellular and D2D users is the same).

It is worth mentioning that an increase in λ_{C} can be compensated for by decreasing P_{C} or increasing P_{D}, such that $\lambda_{\mathrm{C}}(P_{\mathrm{C}}/P_{\mathrm{D}})^{\delta}$ is kept constant (see the invariance result at the end of Subsection III-B). With such compensation, the mean local delays at both cellular and D2D users remain the same.

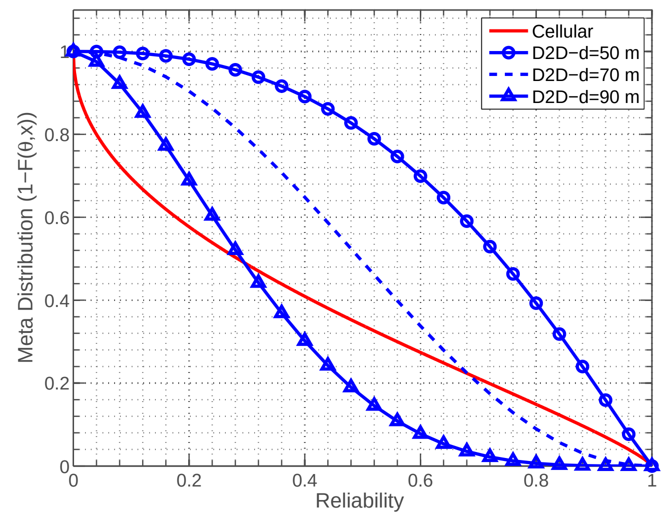


Fig. 7. The effect of different D2D link distances on the meta distribution for $\lambda_{\mathrm{C}} = 2\,\mathrm{km}^{-2}$, $\lambda_{\mathrm{D}} = 50\,\mathrm{km}^{-2}$, $\theta = 0\,\mathrm{dB}$, $\frac{P_{\mathrm{C}}}{P_{\mathrm{D}}} = 100$, $p_{\mathrm{D}} = 0.3$, $p_{\mathrm{C}} = 0.7$, and $\alpha = 4$.

C. Meta Distribution as a Function of D2D Link Distance and BS Density

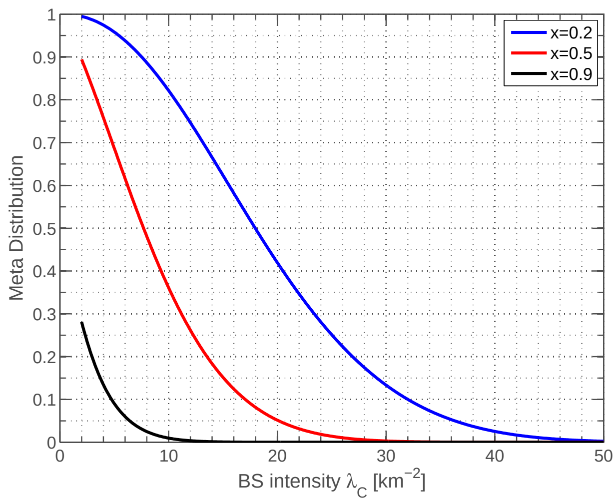
In Fig. 7, the meta distribution is shown for D2D users for three different link distances and BS density $\lambda_{\mathrm{C}} = 2\,\mathrm{km}^{-2}$. For comparison, the meta distribution of the downlink users is also shown. The average link distance of the cellular downlink is $1/(2\sqrt{\lambda_{\mathrm{C}}}) \approx 353\,\mathrm{m}$, which is much bigger than the D2D distances.

Fig. 8 shows the meta distribution as a function of λ_{C} for D2D receivers, downlink users, and in combination.

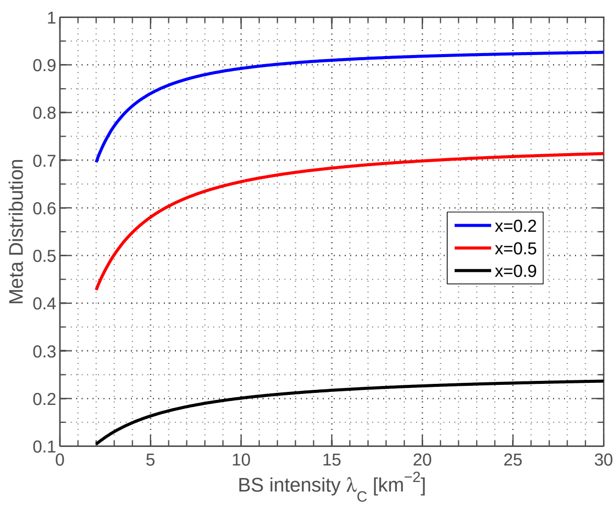
From (9), it is easily seen that as $\lambda_{\mathrm{C}} \to \infty$ (while keeping the other parameters fixed), for any $b \in \mathbb{R}^+$, we have $M_{b,\mathrm{D2D}} \to 0$. In [7], the Markov inequality is used to obtain an upper bound for the meta distribution. By applying this inequality we have $\bar{F}_{P_{\mathrm{s,D2D}}}(x) \leq \frac{M_{b,\mathrm{D2D}}}{x^b} \to 0$ which can be seen in Fig. 8(a). Moreover, in Fig. 8(c) we see that a local minimum exists in each curve. In other words, there is a worst-case BS density that minimizes the fraction of overall users that achieve θ with reliability x.

D. Density of Users with Reliability Constraint

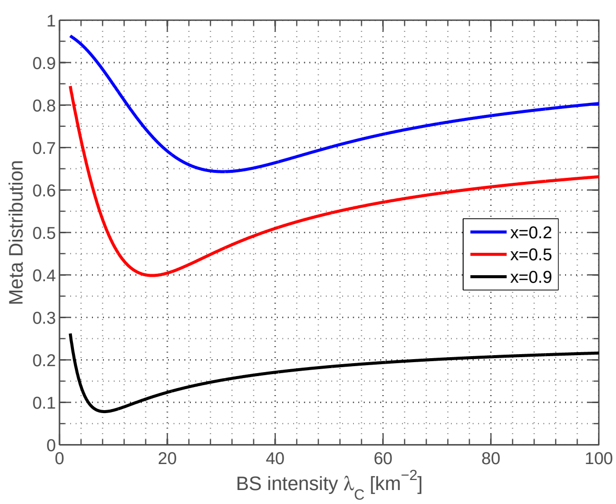
The meta distribution Fig. 8(c) yields the fraction of users that are served with a reliability constraint x. This fraction can be used to evaluate the *density of users* that achieve an SIR of θ with reliability x, given by $(\lambda_{\mathrm{D}}p_{\mathrm{D}} + \lambda_{\mathrm{C}}p_{\mathrm{C}})\bar{F}_{P_{\mathrm{s,total}}}(x)$. When the network parameters are the same as in Fig. 8(c), the effect of the BS intensity on the density of users served is illustrated in Fig. 9. While Fig. 8(c) indicates that $\lambda_{\mathrm{C}} = 0$ results in the largest *fraction* of users, clearly having no BSs is not a good choice in terms of the *density* of users. As we saw earlier in Fig. 8(b), the meta distribution of the downlink users $\bar{F}_{P_{\mathrm{s,Cellular}}}(x)$ approaches a constant at some moderate BS densities. Thus, for λ_{C} not too small, the density of users served increases in proportion to λ_{C}, according to $p_{\mathrm{C}}\bar{F}_{P_{\mathrm{s,Cellular}}}(x)$. In this regime, however, there will be almost no D2D transmissions that satisfy the reliability constraint.



(a) Meta distribution of the D2D receivers (17) as a function of λ_C for $x = 0.2, 0.5, 0.9$ (from top to bottom).



(b) Meta distribution of downlink UEs (25) as a function of λ_C for $x = 0.2, 0.5, 0.9$ (from top to bottom).



(c) Meta distribution of combined users (27) as a function of λ_C for $x = 0.2, 0.5, 0.9$ (from top to bottom).

Fig. 8. Meta distribution of the typical D2D receiver (17), downlink UE (25), and total users (27) as a function of BS intensity λ_C for $d = 50\,\mathrm{m}$, $\lambda_\mathrm{D} = 50\,\mathrm{km}^{-2}$, $\frac{P_\mathrm{C}}{P_\mathrm{D}} = 100$, $\theta = 0\,\mathrm{dB}$, $p_\mathrm{D} = 0.2$, $p_\mathrm{C} = 0.6$, and $\alpha = 4$.

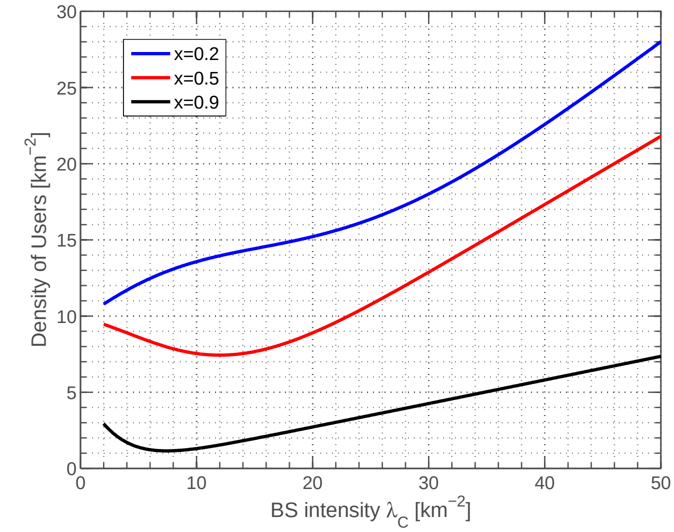


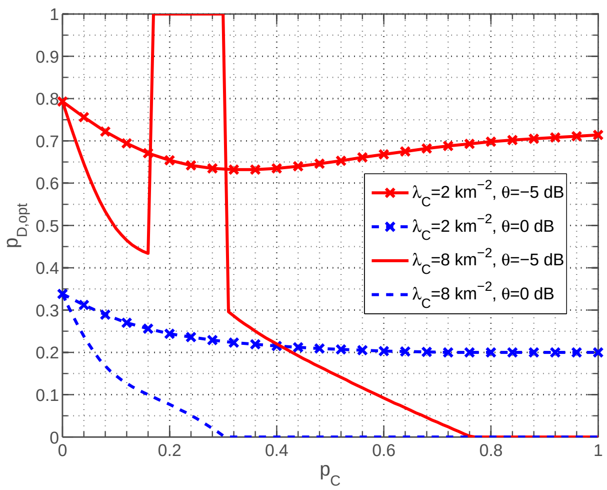
Fig. 9. Density of users served as a function of BS intensity λ_C for $x = 0.2, 0.5, 0.9$ (from top to bottom) when $d = 50\,\mathrm{m}$, $\lambda_\mathrm{D} = 50\,\mathrm{km}^{-2}$, $\frac{P_\mathrm{C}}{P_\mathrm{D}} = 100$, $\theta = 0\,\mathrm{dB}$, $p_\mathrm{D} = 0.2$, $p_\mathrm{C} = 0.6$, and $\alpha = 4$.

Lastly, we assume that a fixed fraction p_C of the BSs are active and ask what transmit probability p_D maximizes the density of users that are served with reliability at least x. This is an important question if the cellular users are considered primary users and the question is how many underlaid D2D links can be established while maintaining the reliability in all links. To answer this question, $p_{\mathrm{D,opt}}(p_\mathrm{C})$ and the resulting user density are shown in Fig. 10 as functions of p_C for different BS densities and target SIRs. In Fig. 10(a), when $\lambda_\mathrm{C} = 8\ \mathrm{km}^{-2}$ and $\theta = -5$ dB, there is a discontinuity in $p_{\mathrm{D,opt}}$. For these parameters, density of overall, D2D, and downlink users are shown in Fig. 11. When $p_\mathrm{C} = 0.25$, the local maximum inside $(0, 1)$ happens to be a bit smaller than the value at $p_\mathrm{D} = 1$, which is why $p_{\mathrm{D,opt}}$ jumps to 1 for this value of p_C. Moreover, since the density of overall users served is relatively flat, if p_D was always set to the value of the local maximum in $(0, 1)$, the overall density would not be affected much. Also for other values of p_C, we can, without much harm, set $p_\mathrm{D} = 1$ instead of $p_{\mathrm{D,opt}}$. For larger values of θ, however, this is may no longer be the case, *i.e.*, setting $p_{\mathrm{D,opt}} = 1$ would result in a highly suboptimal user density.

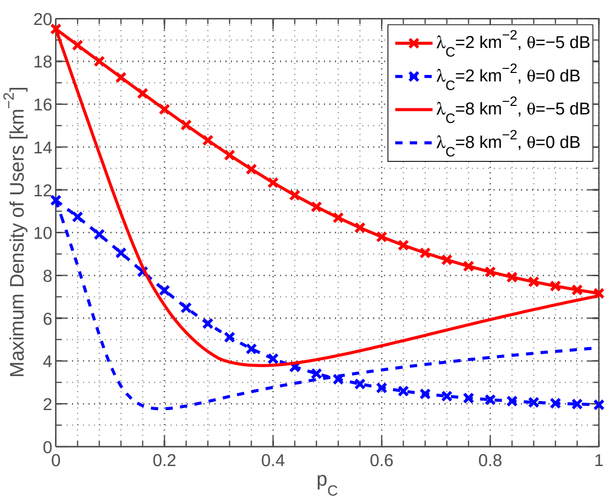
V. Summary and Conclusions

The meta distribution is a fine-grained key performance metric of wireless systems. In this paper, we focus on D2D underlaid cellular wireless networks with ALOHA channel access and provide closed-form expressions of all moments on the conditional success probability for both cellular and D2D users, which are then used to approximate the meta distribution. The accuracy of the approximation is confirmed by simulations.

We also derived the exact mean local delay in closed form for both types of users and analyzed the set of transmit probabilities $(p_\mathrm{D}, p_\mathrm{C})$ for D2D and cellular user that result in a finite mean local delay.



(a) $p_{\mathrm{D,opt}}$ as a function of p_{C}.



(b) Maximum density of users served as a function of p_{C}, achieved as $p_{\mathrm{D}} = p_{\mathrm{D,opt}}$.

Fig. 10. $p_{\mathrm{D,opt}}$ as a function of p_{C} and the resulting user density when $d = 80\,\mathrm{m}$, $\lambda_{\mathrm{D}} = 50\,\mathrm{km}^{-2}$, $\frac{P_{\mathrm{C}}}{P_{\mathrm{D}}} = 400$, $\alpha = 4$, and $x = 0.5$.

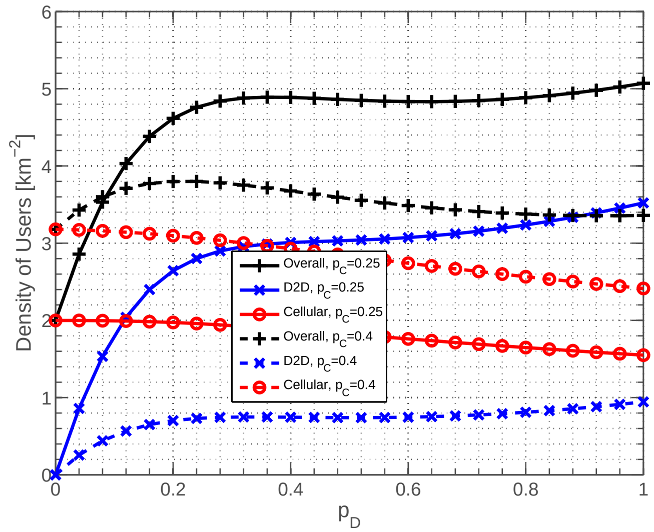


Fig. 11. Density of overall, D2D, and downlink users as functions of p_{D} when p_{C} is fixed for $d = 80\,\mathrm{m}$, $\lambda_{\mathrm{D}} = 50\,\mathrm{km}^{-2}$, $\lambda_{\mathrm{C}} = 8\,\mathrm{km}^{-2}$, $\frac{P_{\mathrm{C}}}{P_{\mathrm{D}}} = 400$, $\alpha = 4$, $x = 0.5$, $\theta = -5$ dB.

The analytical results are used to study the effect of the BS intensity on the performance of the typical receiver and overall users in terms of the mean local delay and the meta distribution, and to determine the network parameters that result in the maximum density of users reliably served.

From a D2D receiver's view, the total interference coming from BSs and D2D transmitters can be modeled by a single Poisson point process, and the network behaves as a Poisson bipolar network. On the other hand, for downlink cellular users, when there are no D2D users, the moments of the conditional SIR distribution and thus the mean local delay do not depend on the BS density because although increasing λ_{C} increases the interference, the average downlink distance is also reduced. When D2D users communicate with each other, the effect of the interference from D2D network on downlink cellular users is more complicated. The results show that for both D2D receivers and downlink cellular users moments remain unchanged if $\lambda_{\mathrm{C}} \left(\frac{P_{\mathrm{C}}}{P_{\mathrm{D}}}\right)^{\delta}$ is held constant and all other parameters stay the same.

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