Non-Orthogonal Multiple Access (NOMA) in Uplink Poisson Cellular Networks With Power Control

Yanan Liang, Student Member, IEEE, Xu Li, Member, IEEE, and Martin Haenggi, Fellow, IEEE

Abstract—This paper develops an analytical framework for multi-cell uplink NOMA systems based on stochastic geometry. We propose two scenarios for the clustering of NOMA UEs and derive the Laplace transform of the inter-cell interference taking into account uplink power control. We utilize two different ordering techniques, namely mean signal power (MSP-) and instantaneous signal-to-intercell-interference-and-noise-ratio (ISINR-) based, for the successive interference cancellation process at the BSs. For each technique, we present a signal-to-interference-and-noise-ratio (SINR) analysis and derive the transmission success probabilities for the NOMA UEs. We show that uplink power control, which generally reduces the signal power disparity between UEs, does not necessarily degrade the NOMA performance. We discuss how UE clustering and the power control exponent impact this finding. ISINR-based ordering, which jointly considers path loss, fading, inter-cell interference, and noise, is generally superior to MSP-based ordering. Moreover, we show that the advantage of NOMA vanishes when the target SINR exceeds a certain threshold. A comparison of the two UE clustering scenarios indicates that excluding the UEs which are relatively far from the serving BS may improve the NOMA performance.

Index Terms—Cellular networks, non-orthogonal multiple access, multi-cell, uplink power control, stochastic geometry.

I. INTRODUCTION

In state-of-the-art wireless communication systems like the long-term evolution (LTE), a variety of orthogonal multiple access (OMA) technologies such as orthogonal frequency division multiple access (OFDMA) and single-carrier frequency division multiple access (SC-FDMA) are utilized [1]. While OMA avoids intra-cell interference and retrieves users’ signals with a relatively low complexity, the connectivity and data rates are limited by the number of orthogonal resources. To address the requirements of very high data rates and a large number of devices in next-generation (5G) wireless networks [2], non-orthogonal multiple access (NOMA) has received extensive research attention. Contrary to OMA, NOMA serves multiple user equipments (UEs) on the same time-frequency resource block (RB) and thus has the potential to improve the spectral efficiency. The set of UEs served by the same base station (BS) is the NOMA UE cluster. Based on the way messages of the UE cluster are superposed, NOMA is categorized into power-domain NOMA (PD-NOMA) and code-domain NOMA (CD-NOMA). The focus of our work is on PD-NOMA. At the receiver side, successive interference cancellation (SIC) is exploited to separate the messages of the UE cluster [3, 4].

A. Related Work

Extensive studies have been conducted on NOMA-based networks. The performance in terms of outage probability and ergodic sum rate of a downlink NOMA system with randomly deployed UEs is first studied in [5, 6] investigates the impact of user pairing on the performance of a two-user downlink NOMA system. It is demonstrated that the performance gain of NOMA over OMA can be enlarged by pairing UEs with disparate channel conditions. Furthermore, [7] investigates a dynamic power control scheme to achieve a performance gain over OMA and a good tradeoff between user fairness and system throughput. Closed-form expressions of the outage probability and the achievable sum data rate are derived for a two-user uplink NOMA system in [8], which introduces a power back-off scheme to distinguish multiplexing UEs. The outage probability of uplink NOMA systems serving an arbitrary number of UEs is investigated in [9]. The aforementioned works consider NOMA in single cells and ignore the impact of inter-cell interference, which has a drastic negative impact on the NOMA performance.

Recently, stochastic geometry has been applied as an analytical approach to characterize the performance of multi-cell NOMA systems while accounting for inter-cell interference. The performance of downlink multi-cell NOMA has been well explored [10, 11]. In the uplink, however, the analysis is much more complex since the spatial distribution and channel statistics of UEs in interfering cells need to be characterized. Furthermore, the utilization of power control poses significant challenges on the interference analysis. [10, 14, 15] focus on uplink NOMA. Specifically, [10] analyzes the uplink NOMA performance in multi-cell scenarios assuming that the point processes of interfering UEs for each UE in a NOMA cluster are homogeneous Poisson point processes (PPPs) with the same density as the BSs, which is pessimistic since there is no guarantee that one UE is served by each BS under nearest-BS association. [14] extends the model of uplink inter-cell interferers for orthogonal multiple access (OMA) [16], which is approximated as a general PPP with an intensity that depends on the distance from the origin, to the NOMA

The work was supported in part by the National Key R&D Program of China under Grant 2018YFC0826303 and the Fundamental Research Funds for the Central Universities under Grant 2016JBZ003.

Yanan Liang and Xu Li are with the School of Electronic and Information Engineering, Beijing Jiaotong University, 100044, Beijing, China. (Email: {14111052, xli}@bjtu.edu.cn)

Martin Haenggi is with the Department of Electrical Engineering, University of Notre Dame, Notre Dame, IN 46556 USA. (Email: mhaenggi@nd.edu)
case and proposes two simplified models for the point process for the spatial locations of inter-cell interferers in uplink NOMA (which we also use in this paper). The moments of the conditional success probability are derived, and an expression for the SIR meta distribution is given. However, does not take into account the SIC chain in the signal-to-interference-ratio analysis and thus overestimates the coverage. In contrast, presents a framework to analyze uplink multi-cell NOMA systems using the Matérn cluster process (MCP), where NOMA UEs are assumed to reside in a disk of fixed radius centered at their serving BS (i.e., the shape and area of the Voronoi cells are ignored for the user placement). The Laplace transform of the intra-cluster and inter-cluster interference are derived, based on which the rate coverage is obtained. However, this model leads to the unrealistic situation where UEs of a NOMA cluster in the typical cell are actually served by other BSs due to the random shape of the Voronoi cells.

In terms of power control, assumes the same average received power for UEs, i.e., full power control, while assume unit transmit power for UEs, i.e., no power control. In uplink NOMA systems, the impact of power control is double-edged. On the one hand, it increases the mean received powers of NOMA UEs (by compensating for the path loss) . On the other hand, it may increase the inter-cell and intra-cell interference (the UEs near the cell edge will use higher power, which increases the intra-cell interference, and some of them may be relatively close to the receiving BSs in adjacent cells, which increases the inter-cell interference) and reduce the difference between NOMA UEs compared to the case of no power control, which degrades the NOMA performance. Therefore, the overall impact of power control in multi-cell uplink NOMA systems needs to be explored in detail.

B. Contributions

In this work we use stochastic geometry to study a large multi-cell uplink NOMA system that takes into account the inter-cell and intra-cell interference, the SIC chain, and power control. We analyze and compare two different UE clustering models, i.e., two location-based schemes to place NOMA UEs in a cluster. With the utilization of a fractional power control scheme, we investigate the performance of mean signal power-based ordering, which is equivalent to distance-based ordering and instantaneous signal-to-intercell-interference-and-noise-ratio-based ordering for the SIC process of NOMA UEs. To the best of our knowledge, an analytical work that compares both ordering techniques in uplink NOMA does not exist. Results for an equivalent OMA system are also given to benchmark the gains attained by NOMA. The main contributions of this work are summarized as follows:

- We propose two scenarios for NOMA UE clustering. In the first scenario, the NOMA UEs are distributed uniformly on the in-disk, which is the largest disk centered at the BS that fits inside the Voronoi cell (cf. Fig. 1). Note that the radius of the in-disk is a random variable.

This scenario is referred to as the VC (abbreviation of “Voronoi cell”) scenario in the following analysis. In the other scenario, the NOMA UEs are distributed uniformly in the Voronoi cell of their serving BS (cf. Fig. 2), which is referred to as the VC (abbreviation of “Voronoi cell”) scenario.

- We show that, counterintuitively, uplink power control, which generally reduces the signal power disparity between UEs, does not necessarily degrade the NOMA performance. We discuss how UE clustering and the power control exponent impact this finding. Also, we explore what happens if we artificially enhance or over-compensate for the path loss by setting the power control exponent to or to .

- We show that UE ordering based on ISINR, which takes into account path loss, fading, intercell interference, and noise, is generally superior to MSP-based ordering. However, for a given power control exponent, there may exist a critical level of the target SINR beyond which MSP-based ordering outperforms ISINR-based ordering.

- We also show that the advantage of NOMA vanishes when the target SINR exceeds a certain threshold. Moreover, a critical minimum level of SIC is required for NOMA to outperform OMA. A comparison of the VC and VC scenarios indicates that excluding the UEs which are relatively far from the serving BS may improve the NOMA performance.

The rest of the work is organized as follows. Section II discusses the VC clustering scenario while Section III discusses the VC scenario. For both scenarios, the Laplace transform of the inter-cell interference and the transmission success probabilities of the NOMA UEs are derived. In Section IV, numerical results of the VC and VC scenarios are given, followed by a comparison of the two scenarios. The concluding remarks are presented in Section V.

II. THE VC SCENARIO: UES RESIDE IN THE VORONOI DISKS

A. System Model

We consider an uplink cellular network where BSs are modeled as , where is a homogeneous PPP with intensity . By Slivnyak’s theorem , the BS at the origin becomes the typical BS serving UEs in the typical cell under expectation over . In this work we study the performance of the typical cell.

Denote by the distance between the typical BS and its nearest neighbor. Since is a PPP, the distance follows a Rayleigh distribution with probability density function (pdf)

Each BS serves UEs in one time-frequency RB by multiplexing them in the power domain. The UEs, referred to as NOMA UEs, are distributed uniformly on the in-disk, which is the largest disk centered at the BS that fits inside the

1See for the definition.

2See for a detailed explanation.

3The VC scenario can be viewed as a benchmark scenario for multi-cell uplink NOMA systems.
Voronoi cell. The UEs outside the disk are relatively far from their serving BS and thus are better served on their own RB or using coordinated multi-point (CoMP) transmission. The radius of the in-disk is denoted as $R = \rho/2$.

**Approximation 1.** We assume that in each cell, there are $N$ UEs distributed uniformly in the disk centered at the BS with the same radius $R = \rho/2$. Hence we ignore that the radii of the in-disks in interfering cells are different from that of the typical cell. Using the actual radii would render the analysis prohibitively complex. Then, given $\rho$, the user point process is a Poisson cluster process (PCP) with $N$ points distributed uniformly and independently in each cluster.

The interfering UEs seen from the typical BS are modeled as $\Phi_I = \bigcup_{y \in \Phi} N^y$, where $N^y$ denotes the set of UEs whose serving BS is at location $y$. A realization of the cell at $o$, its in-disk, and the surrounding cells are shown in Fig. 1. In the typical cell, the locations of UEs are denoted as $x_i (1 \leq i \leq N)$, as shown in Fig. 2. The link distances from the typical UE $x_i$ to the typical BS at $o$ are denoted as $R_i$. In the interfering cell whose serving BS is at $y$, the locations of UEs are denoted as $x_{i,y}$, and the link distances are denoted as $R_{i,y}$.

Given $\rho$, the link distances $R_i (1 \leq i \leq N)$ in the typical cell are identically and independently distributed (i.i.d.) with pdf

$$f_{R_i | \rho}(r | \rho) = \frac{8r}{\rho^2}, \quad 0 \leq r \leq \frac{\rho}{2}. \quad (2)$$

The link distances $R_{i,y} (1 \leq i \leq N)$ follow the same distribution as $R_i$.

The standard power-law path loss model with exponent $\alpha > 2$ for signal propagation and the standard Rayleigh fading are used. The power fading coefficients $h_{x_i}$ associated with the user at $x_i$ and the typical BS are exponentially distributed variables with unit mean, i.e., $h_{x_i} \sim \exp(1)$. The same follows for $h_{x_{i,y}}$. We assume $h_{x_i}$ and $h_{x_{i,y}}$ are independent for all $x_i \in N^{o}$ and $x_{i,y} \in \Phi_I$.

We use fractional power control in the form $P_i = P_0 R_i^{\epsilon}$ for the typical UEs and $P_{i,y} = P_0 R_{i,y}^{\epsilon}$ for the interfering UEs, which is one of the most widely used schemes for uplink cellular networks to partially compensate for the path loss. $\epsilon$ denotes the power control exponent. In addition to the usual range $\epsilon \in [0, 1]$, we also consider the cases of $\epsilon < 0$ and $\epsilon > 1$ to explore what happens if we artificially enhance or overcompensate for the path loss. $P_0$ denotes the baseline transmit power when there is no power control. Without loss of generality, we assume $P_0 = 1$. The noise power is denoted as $\sigma^2$.

**B. LT of the Inter-Cell Interference**

**Approximation 2.** In the analysis of the inter-cell interference in the uplink case, we need to consider the distances from the interfering UEs to the typical BS. Since both the distances from the interfering UEs to their serving BS (i.e., $R_{i,y}$) and the distances from other BSs to the typical BS (i.e., $\|y\|$) are random variables, the analysis of the distances from interfering UEs to the typical BS (i.e., $D_i$) is rather difficult. To make the analysis tractable, we use an approximation that the interfering UEs are all located at their serving BS’s location in the analysis of the inter-cell interference.

**Lemma 1.** In the VD scenario under Approximations 1 and 2, the Laplace transform (LT) of the inter-cell interference conditioned on $\rho$ is given as

$$L_{I | \rho}(s) \approx \exp\left(-2\pi\lambda \int_{\rho}^{\infty} \left(1 - (V_i(y))^\epsilon\right)y dy\right) \cdot (V_i(\rho))^N,$$

where $V_i(y) \triangleq \frac{1}{N} \sum_{j \in \Phi} \delta\left(\frac{\|x_j - y\|}{\|x_j - y\|}\right)$, $\delta \triangleq \frac{\rho}{2}$, and $\epsilon > 0$. For the case of $\epsilon = 0$, the LT of $I^*$ is simplified as

$$L_{I^* | \rho}(s) \approx \exp\left(-2\pi\lambda \int_{\rho}^{\infty} \left(1 - \frac{1}{(1 + sy^{-\alpha})^N}\right)y dy\right) \cdot \frac{1}{(1 + sp^{-\alpha})^N}. \quad (4)$$

**Proof.** See Appendix A.

\*Note that our model is different from the MCP model in that $\rho$ is a random variable affected by the shape and area of the Voronoi cells. Our model ensures that the typical NOMA UEs are exclusively served by the typical BS.

\*\*\* uses a similar approximation in downlink NOMA that assumes the UEs are at their serving BS’s location. It is their average location since the UEs are uniformly distributed on the in-disk.
Approximation 3. To simplify \( P \) we use the average power \( P \) to replace \( P_{i,y} \) for all \( 1 \leq i \leq N \) and \( y \in \Phi \) as an approximation, which is given as
\[
P = \int_0^\frac{\pi}{2} x^{\alpha s} \frac{8x}{\rho^2} dx = \frac{2}{\alpha + 2} \left( \frac{\rho}{2} \right) = \frac{\delta}{\epsilon} \left( \frac{\rho}{2} \right)^{\alpha}, \tag{5}
\]
valid for \( \epsilon > -\delta \).

Corollary 1. Assuming average transmit power for the interfering UEs, the LT of the inter-cell interference is approximated as
\[
L_{I_{\text{avg}}}(s) = \exp \left[ - \sum_{k=1}^N \left( \frac{N}{k} \right) W_k(\rho) k_2 F_1 \left( k, k - \delta; k + 1 - \delta; \frac{-sP}{\rho^2} \right) \right] \frac{(1 + sP)^{-k}}{ a(s)}, \tag{6}
\]
where \( W_k(\rho) = \frac{\pi^{\delta}(k+1) \rho^k}{(k-\delta) \Gamma(k-\delta)} \) and \( \epsilon > -\delta \).
Proof. See Appendix [B]
\]

C. Transmission Success Probabilities

Next we analyze the transmission success probabilities of the \( N \) NOMA UEs in the typical cell. SIC is employed for decoding NOMA UEs, which requires the ordering of UEs based on the link quality. We order UEs in such a way that the \( i^{\text{th}} \) UE, which is denoted as \( U_i \), has the \( i^{\text{th}} \) strongest link. Here, we consider two link quality metrics, namely:

- Mean signal power (MSP): The MSP of the typical UEs are \( P' = h_i R_i^{1-\epsilon} \alpha = \frac{1}{\rho^2} \), where \( R_i \) is the signal power at the BS. Therefore, in MSP-based ordering, the typical UEs are indexed according to their ascending ordered distance \( R_i \) with \( R_1 < \ldots < R_N \) under the condition \( \epsilon < 1 \). i.e., the \( i^{\text{th}} \) closest UE from the origin is referred to as \( U_i \).

- Instantaneous signal-to-intercell-interference-and-noise-ratio (ISINR): The ISINR of the typical UEs are \( Z_i = h_i R_i^{1-\epsilon} \alpha = \frac{1}{\rho^2} \). In ISINR-based ordering, the typical UEs are indexed with respect to their descending ordered ISINR \( \hat{Z}_i \), i.e., \( U_i \) has the \( i^{\text{th}} \) largest ISINR.

If SIC is ideal\(^4\) i.e., there is no residual interference, the SINR of \( U_i \) for \( 1 \leq i \leq N \) is
\[
\text{SINR}_i = \frac{h_i R_i^{1-\epsilon} \alpha}{\sum_{j=i+1}^N h_j R_j^{1-\epsilon} \alpha + I^s + \sigma^2}. \tag{7}
\]

\(^4\)In the case of \( \epsilon > 1 \), the typical UEs are indexed according to their descending ordered link distance \( R_i \). In the case of \( \epsilon = 1 \), i.e., full power control, the MSPs of the NOMA UEs are the same, therefore the decoding order does not matter. Without loss of generality, we take into account the distances from the NOMA UEs to the BS and decode the UE nearer to the BS first.

To decode \( U_i \)'s message, the BS needs to successfully decode the messages of UEs whose order indexes are smaller than \( i \). We use \( \theta_i \) to denote the target SINR (i.e., the SINR threshold corresponding to the target rate) of \( U_i \). Then the transmission success (sometimes also called coverage) of \( U_i \) is defined as the joint event
\[
C_i = \bigcap_{k=1}^i \{ \text{SINR}_k > \theta_k \}. \tag{8}
\]

Theorem 1. In the VD scenario, the transmission success probability of \( U_i, 1 \leq i \leq N \) using MSP-based ordering is given as
\[
\mathbb{P}(C_i) = \mathbb{E}_u \left[ \prod_{j=1}^i b_{i,j} e^{-\rho d_i \mu(d_i)} \right], \tag{9}
\]
where
\[
a_k = \begin{cases} 1, & \epsilon \leq 1, \\ 1 + \sum_{k=1}^{k-1} b_{i,j} (1 + \theta_j) \sum_{q=t+1}^{k-1} (1 + \theta_j) R_{q+1}^{1-\epsilon} \alpha R_q^{1-\epsilon} \alpha, & \epsilon > 1, \end{cases} \tag{10}
\]
\[
b_{i,j} = \begin{cases} 1, & \epsilon \leq 1, \\ 1 + \sum_{k=1}^{i} b_{i,j} (1 + \theta_j) \sum_{q=t+1}^{k-1} (1 + \theta_j) R_{q+1}^{1-\epsilon} \alpha R_q^{1-\epsilon} \alpha, & \epsilon > 1, \end{cases} \tag{11}
\]
\[
d_i = \begin{cases} 1, & \epsilon \leq 1, \\ 1 + \sum_{k=1}^{i} b_{i,j} (1 + \theta_j) \sum_{q=t+1}^{k-1} (1 + \theta_j) R_{q+1}^{1-\epsilon} \alpha R_q^{1-\epsilon} \alpha, & \epsilon > 1, \end{cases} \tag{12}
\]
for \( 1 \leq k \leq i, i+1 \leq j \leq N \) and \( 1 \leq i \leq N \).

Proof. See Appendix [C]

Corollary 2. For \( N = 2 \) in the VD scenario, the transmission success probability of \( U_1 \) using MSP-based ordering is given as
\[
\mathbb{P}(C_1) = \int_0^\infty \int_0^\infty F_r(x,r) \mathcal{L}_{I|x}(\theta_1 r^{1-\epsilon}) T_r(x,r) df_r(x) dx, \quad \epsilon < 1, \tag{13}
\]
\[
\mathbb{P}(C_1) = \int_0^\infty \int_0^\infty (-\tau_r) r^2 F_r(x,r) \mathcal{L}_{I|x}(\theta_1 r^{1-\epsilon}) df_r(x) dx, \quad \epsilon > 1, \tag{14}
\]
where \( F_r(x,r) \) and \( T_r(x,r) \) are defined in the Appendix [E].
For U_2, it is given as
\[
\mathbb{P}(C_2) = \begin{cases}
\int_0^\infty \int_0^\infty \int_0^\infty \frac{G_i(x, r_1, r_2)}{1 + \theta_1 r_1^2 + r_2^2} dr_1 dr_2 f_\rho(x) dx, & \epsilon < 1, \\
\int_0^\infty \int_0^\infty \int_0^\infty \frac{G_h(x, r_1, r_2)}{1 + \theta_1 r_1^2 + r_2^2} dr_1 dr_2 f_\rho(x) dx, & \epsilon > 1,
\end{cases}
\]
where \(G_i(x, r_1, r_2) \triangleq 128 \rho_1 \rho_2 x^{-2} e^{-\alpha^2 v_1, r_1, r_2}\) and \(v_1, r_1, r_2 \triangleq \theta_1 r_1^2 + \theta_1(1 + \theta_2)r_2^2\).

**Proof.** See Appendix [D].

For ISINR-based ordering, the transmission success probability of U_i, 1 \leq i \leq N is
\[
\mathbb{P}(C_i) = \mathbb{P}\left[\sum_{j=1}^N \tilde{Z}_j + 1 > \sum_{j=1}^N \tilde{Z}_j + 1 \mid \tilde{Z}_1 > \cdots > \tilde{Z}_N\right]
\]
\[
= \mathbb{E}\left[\prod_{i=1}^N \mathbb{P}(Z_i > \sum_{j=i}^N Z_{j+1}) \right]
\]
\[
= \prod_{i=1}^N \mathbb{P}(Z_i > \sum_{j=i}^N Z_{j+1}) \right)
\]
\[
= \prod_{i=1}^N \mathbb{P}(Z_i > \sum_{j=i}^N Z_{j+1}) \right)
\]
(16)
where (a) follows from the independence of \(\tilde{Z}_i\) for 1 \leq i \leq N and \(f_{\tilde{Z}_1, \ldots, \tilde{Z}_N}(z_1, \ldots, z_N) = n! f_{\tilde{Z}_1}(z_1) \cdots f_{\tilde{Z}_N}(z_N)\) based on order statistics [22].

For \(N \geq 3\), the expressions of \(\mathbb{P}(C_i)\) are too complex. To illustrate the analytical model more briefly and coherently, we focus on \(N = 2\) for ISINR-based ordering.

**Theorem 2.** For \(N = 2\) in the VD scenario, the transmission success probabilities of U_1 and U_2 using ISINR-based ordering for \(\theta_1 \geq 1\) are given as
\[
\mathbb{P}(C_1) = \int_0^\infty \int_0^\infty 2J(\theta_1 z + \theta_1, x)K(z, x) dz f_\rho(x) dx,
\]
(17)
\[
\mathbb{P}(C_2) = \int_0^\infty \int_0^\infty 2J(\theta_1 z + \theta_1, x)K(z, x) dz f_\rho(x) dx,
\]
(18)
while for \(\theta_1 < 1\) they are given as
\[
\mathbb{P}(C_1) = \int_0^\infty \int_0^\infty 2J(\theta_1 z + \theta_1, x)K(z, x) dz + \int_0^\infty 2J(z, x)K(z, x) dz f_\rho(x) dx,
\]
(19)
\[
\mathbb{P}(C_2) = \int_0^\infty \int_0^\infty 2J(\theta_1 z + \theta_1, x)K(z, x) dz + \int_0^\infty 2J(z, x)K(z, x) dz f_\rho(x) dx,
\]
(20)
where
\[
J(z, x) = \int_0^\infty 8r x^{-2} \exp(-z \sigma^2 r^{(1-\epsilon)\alpha}) \mathcal{L}_{I^* |z}(z r^{(1-\epsilon)\alpha}) dr,
\]
(21)
\[
K(z, x) = \int_0^\infty 8r^{1+\epsilon(1-\epsilon)} x^{-2} \exp(-z \sigma^2 r^{(1-\epsilon)\alpha})
\]
\[
\cdot \left(\sigma^2 \mathcal{L}_{I^* |z}(z r^{(1-\epsilon)\alpha}) - \mathcal{L}_{I^* |z}(z r^{(1-\epsilon)\alpha})\right) dr,
\]
(22)
in which \(\mathcal{L}_{I^* |z}(z r^{(1-\epsilon)\alpha})\) is the correction factor relative to the distribution in the Crofton cell.

**Proof.** See Appendix [F].

\section{D. NOMA Gain}

In the OMA case, the LT of the inter-cell interference \(I^o\) is obtained by assuming \(N = 1\), i.e., \(\mathcal{L}_{I^o |z}(s) = \mathcal{L}_{I^o |z}(s) |_{N=1}\).

Let \(\theta\) denote the target SINR of the OMA UE. The transmission success probability of the typical OMA UE is
\[
\mathbb{P}(C) = \mathbb{P}\left[\frac{h_1 R_1^{(1-\epsilon)\alpha}}{I^o + \sigma^2} > \theta\right]
\]
\[
= \mathbb{E}\left[\mathbb{E} \left[\mathcal{L}_{I^o |\rho}(\theta R_1^{(1-\epsilon)\alpha})\right]\right]
\]
\[
= \int_0^\infty \int_0^\infty 8r x^{-2} \int_0^x \int_0^{\theta r^{(1-\epsilon)\alpha}} \mathcal{L}_{I^o |z}(z r^{(1-\epsilon)\alpha}) dz f_\rho(x) dx.
\]
(23)

We use \(\mathcal{R}_{NOMA}\) and \(\mathcal{R}_{OMA}\) to denote the achievable transmission sum rate of NOMA and OMA, respectively, and define the uplink NOMA gain \(G\) as
\[
G \triangleq \mathcal{R}_{NOMA} - \mathcal{R}_{OMA}
\]
\[
= \sum_{i=1}^N \mathbb{P}(C_i) \log(1 + \theta_i) - \mathbb{P}(C) \log(1 + \theta).
\]
(24)

For \(N = 2\), \(\mathbb{P}(C_1)\) and \(\mathbb{P}(C_2)\) are given in [13] and [15] for MSP-based ordering and in [17] [20] for ISINR-based ordering.

\section{III. THE VC SCENARIO: UEs RESIDE IN THE VORONOI CELLS}
\subsection{A. System Model}

In the VC scenario, we also model the BSs as \(\Phi_b = \Phi \cup \{o\}\), in which \(\Phi \subset \mathbb{R}^2\) is a homogeneous PPP with intensity \(\lambda\). By Slivnyak’s theorem, the BS at the origin becomes the typical BS under expectation over \(\Phi_b\). Assume that \(N\) NOMA UEs are served on one RB by each BS. A realization of the cell at \(o\), the UEs in it, and the surrounding cells are shown in Fig. [3] The UEs in the typical cell are denoted as \(x_i, 1 \leq i \leq N\), as depicted in Fig. [4]. The link distances from UE \(x_i\) to the typical BS are denoted as \(R_i\). From the results in [16], the \(R_i\) are distributed as
\[
f_{R_i}(r) = 2a\pi \lambda r \exp(-a\pi \lambda r^2), \quad r > 0,
\]
(25)
where \(a\) is the correction factor relative to the distribution in the Crofton cell. The value \(a = 9/7\) is used in the following analysis.

To characterize the inter-cell interference, we consider two approximative models based on the pair correlation function between the interfering UEs and the typical BS [14].
As in Section II-A, we assume the standard power-law path loss model with exponent $\alpha > 2$ for signal propagation and the standard Rayleigh fading. Fractional power control in the form $P = P^\epsilon$ is used for all UEs. In addition to the usual interval $\epsilon \in [0,1]$, we also consider the cases of $\epsilon < 0$ and $\epsilon > 1$ to explore what happens if we artificially enhance or overcompensate for the path loss.

B. LT of the Inter-Cell Interference

**Lemma 2.** In the VC scenario, the LT of the inter-cell interference is approximated as

$$L_I^*(s) \approx \exp \left( -2\pi N \lambda \int_0^\infty \left( 1 - e^{-b\pi \lambda z^2} \right) \left( 1 - \mu_I(z, s) \right) z dz \right),$$

for Model 1 and

$$L_I^*(s) \approx \exp \left( -2\pi \lambda \int_0^\infty \left( 1 - e^{-b\pi \lambda z^2} \right) \left( 1 - \mu_N(z, s) \right) z dz \right),$$

for Model 2, where

$$\mu_{\alpha}(z, s) \triangleq \int_0^z \frac{2\pi \lambda y e^{-a\pi \lambda y^2}}{(1 - e^{-a\pi \lambda y^2})(1 + s y^\alpha z^{-\alpha})^\alpha} dy.$$  \hspace{1cm} (30)

**Proof.** See Appendix C. \hfill \Box

C. Transmission Success Probabilities

**Theorem 3.** In the VC scenario, the transmission success probability of $U_i$, $1 \leq i \leq N$ using MSP-based ordering is given as

$$P(C_i) = \mathbb{E}_{R_1 \ldots \hat{R_i} \ldots R_N} \left[ \prod_{k=1}^{i-1} a_k \prod_{j=i+1}^{N} b_k e^{-\delta_k^2 d_k} \right] L_I^*(d_i).$$  \hspace{1cm} (31)

where $a_k$, $b_k$, and $d_k$ are given in (10), (11), and (12), respectively.

**Proof.** The proof is similar to that of Theorem 1 (see Appendix C). \hfill \Box

**Corollary 3.** For $N = 2$ in the VC scenario, the transmission success probability of $U_1$ using MSP-based ordering for $\epsilon < 1$ is given as

$$P(C_1) = \begin{cases} \int_0^{2e^{-\theta_1 \pi^2 \frac{z}{l}}} L_I^*\left( \theta_1 + \frac{y}{l} \right) \xi_i(x, \alpha \pi r^2, \theta_1) 2a \pi \lambda e^{-a \pi \lambda r^2} dr, & \epsilon < 1, \\ \int_0^{2e^{-\theta_1 \pi^2 \frac{z}{l}}} L_I^*\left( \theta_1 + \frac{y}{l} \right) (-\xi_i(x, \alpha \pi r^2, \theta_1)) 2a \pi \lambda e^{-a \pi \lambda r^2} dr, & \epsilon > 1, \end{cases}$$

$$\frac{e^{-\theta_1 \pi^2 \frac{z}{l}} L_I^\epsilon(\theta_1)}{1 + \theta_1}, \quad \epsilon = 1,$$ \hspace{1cm} (32)

where

$$\xi_i(y, z) \triangleq \delta_i y \int_0^{1} e^{-\delta_i - y v^{\epsilon} - \delta_i} 1 + z v \, dv.$$  \hspace{1cm} (33)
For $U_2$, it is given as
\[
\mathbb{P}(C_2) = \begin{cases} 
\int_0^\infty \int_0^\infty M(r_1, r_2) e^{-\sigma^2 r_2 v_r(r_1, r_2)} L_1^I(v_r(r_1, r_2)) \, dr_2 \, dr_1, & \epsilon < 1, \\
\int_0^\infty \int_0^\infty M(r_2, r_1) e^{-\sigma^2 r_1 v_r(r_2, r_1)} L_1^I(v_r(r_2, r_1)) \, dr_1 \, dr_2, & \epsilon > 1,
\end{cases}
\]
(34)
where $M(r_1, r_2) \equiv 8(a^2 \lambda)^2 r_1 r_2 e^{-\alpha^2 \lambda(r_1^2 + r_2^2)}$ and $v_r(r_1, r_2)$ is defined in Corollary 2.

\textbf{Proof.} See Appendix [1] \hfill \Box

\textbf{Theorem 4.} For $N = 2$ in the VC scenario, the transmission success probabilities of $U_1$ and $U_2$ using ISINR-based ordering for $\theta_1 \ge 1$ are given as
\[
\mathbb{P}(C_1) = \int_0^{\theta_1} 2J'(\theta_1 z + \theta_1)K'(z)\, dz, \quad (35)
\]
\[
\mathbb{P}(C_2) = \int_0^{\theta_1} 2J'(\theta_1 z + \theta_1)K'(z)\, dz, \quad (36)
\]
while for $\theta_1 < 1$ they are given as
\[
\mathbb{P}(C_1) = \int_{\frac{\theta_1}{\theta_2}}^{\theta_1} 2J'(\theta_1 z + \theta_1)K'(z)\, dz \quad + \int_{\frac{\theta_1}{\theta_2}}^\infty 2J'(z)K'(z)\, dz, \quad (37)
\]
\[
\mathbb{P}(C_2) = \int_{\min(\theta_2, \theta_1)}^{\theta_2} 2J'(\theta_1 z + \theta_1)K'(z)\, dz \quad + \int_{\min(\theta_2, \theta_1)}^\infty 2J'(z)K'(z)\, dz, \quad (38)
\]
where
\[
J'(z) = \int_0^\infty \exp(-\alpha^2 r^2 \frac{z}{r^2}) L_1^I(\alpha \sqrt{z} \frac{z}{r^2}) 2a\pi \lambda e^{-a\pi \lambda r^2} \, dr, \quad (39)
\]
\[
K'(z) = \int_0^\infty \exp(-\alpha^2 r^2 \frac{z}{r^2}) \left( \sigma^2 L_1^I(\alpha \sqrt{z} \frac{z}{r^2}) - \mathcal{L}_1^I(\alpha \sqrt{z} \frac{z}{r^2}) \right) \cdot 2a\pi \lambda e^{-a\pi \lambda r^2} \, dr, \quad (40)
\]
in which $\mathcal{L}_1^I(s) = dL_1^I(s) / ds$.

\textbf{Proof.} See Appendix [1] \hfill \Box

\textbf{D. NOMA Gain}

In the OMA case, the LT of the inter-cell interference $I^\circ$ is obtained by assuming $N = 1$, i.e., $L_1^I|_{x=0} = L_1^I(s)|_{N=1}$. Thus the transmission success probability of the typical OMA UE is given as
\[
\mathbb{P}(C) = \mathbb{P} \left[ \frac{h_1 h_2}{P^\circ + \sigma^2} \right] > \theta
\]
\[
= \int_0^{\theta} e^{-\theta \alpha^2 r(\theta^2 + \alpha^2)} \mathcal{L}_1^I(\theta \alpha^2 r^2) 2a\pi \lambda e^{-a\pi \lambda r^2} \, dr. \quad (41)
\]
Similar as in the VD scenario, the NOMA gain given in (24) is considered. For $N = 2$, $\mathbb{P}(C_1)$ and $\mathbb{P}(C_2)$ for the VC scenario are given in (32) and (34) assuming MSP-based ordering and in (35)-(38) assuming ISINR-based ordering.

\textbf{IV. NUMERICAL RESULTS}

\textbf{A. The VD Scenario}

First, we show that the simplifying assumptions (Approximations 1-3) we make in the analysis of the VD scenario are sensible and result in tight approximative results. Fig. 5 denotes the transmission success probabilities of the NOMA UEs for $N = 2$ under different simplifying assumptions in the VD scenario. $\lambda = 1$, $\epsilon = 1$, $\alpha = 4$, $\sigma^2 = 0.01$, $\theta_1 = \theta_2 = \theta$ are assumed.

\textbf{Fig. 5.} Comparison of the analytical and simulation results for the transmission success probabilities of the NOMA UEs for $N = 2$ under different approximations. It is indicated that Approximation 2 (assuming the interfering UEs at their serving BS's locations), leads to a slightly larger deviation than the other approximations. Overall, the assumptions are mild and sensible.

Assuming perfect SIC and the same target SINR for the NOMA UEs, we explore the impact of $\epsilon$ on NOMA systems. Fig. 6 depicts the transmission success probabilities of the NOMA UEs using MSP- and ISINR-based ordering for different values of $\epsilon \in [0, 1]$ assuming $N = 2$. The curves correspond to the analytical results given in Corollary 2 and Theorem 2 while the markers correspond to the simulation results. It is shown that the transmission success probabilities of $U_1$ and $U_2$ deteriorate with the increase of $\epsilon$ for all values
of \( \theta \) assuming MSP-based ordering. IS\( \overline{\text{S}} \)N\( \overline{\text{R}} \)-based ordering shows a similar trend except that \( \epsilon = 0.5 \) exhibits a marginal performance advantage over \( \epsilon = 0 \) and \( \epsilon = 1 \) for \( U_2 \) when \( \theta \) is lower than 0 dB.

Fig. 7 depicts the NOMA gain for different \( \epsilon \) assuming MSP-based ordering and \( N = 2 \). For a given \( \epsilon \), the NOMA gain increases gradually with the increase of \( \theta \) at first, then starts to sharply decline after reaching the maximum value, and finally drops below zero when \( \theta \) exceeds a certain threshold, which means that the advantage of NOMA vanishes. For example, for \( \epsilon = 1 \) (i.e., full power control), NOMA is superior to OMA for \( \theta < -1 \) dB. For \( \epsilon = 0 \) (i.e., no power control), NOMA outperforms OMA if \( \theta \) is below 5 dB. Moreover, the NOMA gain of \( \epsilon = 0 \) shows a significant advantage over \( \epsilon = 1 \) for a given \( \theta \), which is intuitive based on the results in Fig. 6(a).

Furthermore, we consider the case of \( \epsilon < 0 \) and \( \epsilon > 0 \) to explore the result if we artificially enhance or oversubscribe for the path loss. It is shown that using a negative \( \epsilon \) improves the NOMA gain for \( \theta > 2 \) dB. Moreover, the NOMA gain gets more evident for a smaller \( \epsilon \) given \( \epsilon < 0 \). The case of \( \epsilon = 1.2 \) exhibits a marginal improvement of the NOMA gain compared to \( \epsilon = 1 \) for \( \theta > -6 \) dB. Further increasing \( \epsilon \) does not lead to a performance enhancement.

For the VD scenario, we take values of \( \theta = -5, 0, 5 \) dB and plot the NOMA gain vs. \( \epsilon \) for the cases of \( N = 2 \) and \( N = 3 \). As demonstrated in Fig. 9 there exists an optimal \( \epsilon \) for each \( \theta \) to achieve the maximum NOMA gain. Moreover, the optimal value of \( \epsilon \) decreases as \( \theta \) increases. For example, for \( N = 2 \) using IS\( \overline{\text{S}} \)N\( \overline{\text{R}} \)-based ordering, the optimal value of \( \epsilon \) for \( \theta = 5 \) dB is \(-0.5\), which indicates that artificially enhancing the path loss improves the NOMA performance. In comparison, the optimal \( \epsilon \) is 0.6 for \( \theta = -5 \) dB, which implies that partial compensation for the path loss results in the optimal performance. Furthermore, the optimal \( \epsilon \) for IS\( \overline{\text{S}} \)N\( \overline{\text{R}} \)-based ordering is larger than for MSP-based ordering. Similar conclusions can be drawn for \( N = 3 \). Moreover, it is shown in Fig. 9 that IS\( \overline{\text{S}} \)N\( \overline{\text{R}} \)-based ordering is generally superior to MSP-based ordering. Note that the variation of the NOMA gain with respect to \( \epsilon \) is not completely smooth in Fig. 8 which is due to the fact that the impacts of \( \epsilon \) on the transmission success probabilities of NOMA UEs are intricate, as shown in [9]-[12] and [17]-[22].

Fig. 8 depicts the achievable sum rate with different \( \theta_i \) for \( U_i \) assuming \( N = 2 \) and \( \epsilon = 0 \) for the VD scenario. The product \((1 + \theta_1)(1 + \theta_2) = (1 + \theta)^2\) is fixed for all cases to ensure the same target sum rate for the NOMA UEs. We observe that setting \( \theta_1 > \theta_2 \) does not lead

\[ \text{Here we obtain the optimal } \epsilon \text{ for a given } N, \theta \text{ and ordering technique through simulation results. It is rather unlikely that an exact analytical expression of the optimal } \epsilon \text{ that maximizes the NOMA gain can be found.} \]

The same applies for Fig. 14.
to a performance gain for any value of $\theta$. $\theta_1 = \theta_2$ achieves the optimal performance for $\theta < 2$ dB while $\theta_1 = 1/2 \theta_2$ leads to the highest sum rate for $\theta \in [2, 10]$ dB. Furthermore, $\theta_1 = 1/4 \theta_2$ achieves the highest sum rate for $\theta \in [10, 12]$ dB while $\theta_1 = 1/2 \theta_2$ outperforms the other cases for $\theta > 12$ dB. This indicates that using a smaller rate for $U_1$ than $U_2$ leads to a better overall performance in the high-$\theta$ regime.

B. The VC Scenario

First, we show that the approximative models of inter-cell interferers we use in the analysis of the VC scenario are sensible. Fig. 10 depicts the analytical results for Model 1 and 2 and the simulation results assuming $N = 2$, $\epsilon = 1$, $\lambda = 1$, $\alpha = 4$, $\sigma^2 = 0.01$ and $\theta_1 = \theta_2 = \theta$. It is shown that the results of Model 1 are lower bounds for Model 2 while both models lead to upper bounds on the simulation results. The analytical and the simulation results match closely for both ordering schemes.

As in Sec. IV-A, we explore the impact of $\epsilon$ assuming perfect SIC and the same target SINR for the NOMA UEs. Fig. 11 depicts the transmission success probabilities of $U_1$ and $U_2$ using MSP- and ISINR-based ordering for $\epsilon \in [0, 1]$. It is shown that the performance of both UEs decline with the increase of $\theta$. Moreover, the pace of deterioration accelerates as $\epsilon$ increases. Furthermore, it is interesting to find that in the VC scenario, counterintuitively, using power control (i.e., assuming $\epsilon > 0$), which generally reduces signal power disparity between NOMA UEs, does not necessarily degrade the performance. Assuming MSP-based ordering, $\epsilon = 0.5$ results in the best performance for $U_1$ if $\theta$ is lower than -5 dB while $\epsilon = 0$ provides the optimal performance when $\theta$ exceeds 5 dB. As for $U_2$, $\epsilon = 0.5$ results in the best performance for $\theta$ lower than 0 dB while $\epsilon = 0$ provides the optimal performance for $\theta > 0$ dB. For the case of ISINR-based ordering, $\epsilon = 1$, $\epsilon = 0.5$ and $\epsilon = 0$ results in the best performance for $U_1$ when $\theta$ is lower than -10 dB, within $[-10, -2]$ dB, and larger than -2 dB, respectively.

Fig. 12 depicts the NOMA gain for MSP-based ordering assuming $N = 2$. It is shown that the variation of the NOMA gain with respect to $\theta$ in the VC scenario exhibits a similar trend as in Fig. 7 for the VD scenario. The advantage of NOMA vanishes when the target SINR exceeds a certain threshold. For example, for $\epsilon = 1$, NOMA is superior to OMA for $\theta < -6$ dB. For $\epsilon = 0$, NOMA outperforms OMA if $\theta$ is below 6 dB. Moreover, it is shown that using $\epsilon = -0.5$ leads to the optimal NOMA gain for $\theta > -1$ dB. In contrast, the NOMA gain of $\epsilon = 0$ exceeds all the other cases for $\theta \in [-5, -1]$ dB while $\epsilon = 0.5$ is optimal for $\theta < -5$ dB. Furthermore, we compare the achievable sum rate of NOMA using MSP-based and ISINR-based ordering for $\epsilon = -0.5, 0, 0.5$. As depicted in Fig. 13 ISINR-based...
ordering is superior to MSP-based ordering for $\epsilon = 0, 0.5$ for all values of $\theta$. However, MSP-based ordering outperforms ISINR-based ordering as $\theta$ exceeds 10 dB for $\epsilon = -0.5$.

For the VC scenario, we also take values of $\theta = -5, 0, 5$ dB and plot the NOMA gain vs. $\epsilon$ for the cases of $N = 2$ and $N = 3$. As depicted in Fig. 14, the optimal value of $\epsilon$ for ISINR-based ordering is larger than for MSP-based ordering for $\theta = -5, 0$ dB, which indicates that the NOMA gain improves more for ISINR-based ordering in response to power control in comparison to MSP-based ordering. In contrast, the optimal $\epsilon$ for both ordering schemes are approximately equal for $\theta = 5$ dB. Furthermore, the optimal $\epsilon$ decreases as $\theta$ increases, which is similar to the observations of the VD scenario in Fig. 9. Based on the comparison of Fig. 9 and Fig. 14 we see that the optimal $\epsilon$ for the VD and VC scenarios are close for a given $\theta$ and a given ordering scheme, which indicates that the NOMA gain varies in a similar manner in response to power control for the two clustering scenarios.
we have characterized the Laplace transform of the inter-cell interference with the assumption of fractional power control. Two kinds of UE ordering techniques are analyzed, namely MSP- and ISINR-based ordering, and the transmission success probabilities of the NOMA UEs taking into account the SIC chain are derived. We show that uplink power control does not necessarily reduce the NOMA performance and identify the selection of the power control exponent given different clustering scenarios and target SINR constraints. Also, we show that ISINR-based ordering is generally superior to MSP-based ordering. Nevertheless, MSP-based ordering could provide a marginal performance gain over ISINR-based ordering for large $\theta$ if we artificially enhance the path loss, i.e., use negative $\epsilon$, in the VC clustering scenario. Moreover, we show that the advantage of NOMA vanishes when the target SINR exceeds a certain threshold. Furthermore, a critical minimum level of SIC is required for NOMA to outperform OMA.

The results highlight the importance of choosing network parameters such as the power control exponent and the UE ordering technique, depending on the network objective and the target SINR constraint. A comparison of the two UE clustering scenarios indicates that excluding the UEs that are relatively far from the serving BS from the clustering process may improve the NOMA performance.

## Appendix A

### Proof of Lemma 1

Based on Approximations 1 and 2, the inter-cell interference at the typical BS originating from $\Phi_i$ is given as

$$ I^* \approx \sum_{y \in \Phi} \sum_{\|y\| > \rho} P_i y h_i, y \|y\|^{-\alpha} \sum_{\|y\| = \rho} \sum_{i=1}^{N} P_i y h_i, y \|y\|^{-\alpha} \sum_{i=1}^{N} R_i y h_i, y \|y\|^{-\alpha} + \sum_{i=1}^{N} R_i y h_i, y \|y\|^{-\alpha} \sum_{i=1}^{N} R_i y h_i, y \rho^{-\alpha}, \quad (42) $$

where $I^*_m$ is the interference from BS $m$. 

### C. Comparison of the VD and VC Scenarios

Fig. 15 depicts a comparison of the NOMA gain for NOMA systems with $N = 2, 3, 4$ under the VD and VC scenarios. $\epsilon = 0$ is selected to obtain the best performance for MSP-based ordering. For the VD scenario, $N = 4$ results in the best performance for $\theta < -5$ dB while $N = 3$ provides the optimal performance for $\theta \in [-5, 0]$ dB. $N = 2$ outweights $N = 4$ and $N = 3$ for $\theta > 0$ dB. As for the VC scenario, $N = 4$ provides the optimal performance for $\theta < 3$ dB while $N = 2$ provides the optimal performance for $\theta > 3$ dB. Generally, for a given $N$, the NOMA gain in the VD scenario largely exceeds its counterpart in the VC scenario. Nevertheless, the NOMA gain in the VC scenario is higher than the VD scenario for a relatively large $\theta$, namely $0$ dB for $N = 4$ and $3$ dB for $N = 3$. However, it is noteworthy that the NOMA gain falls under $0$ when $\theta$ exceeds $5$ dB, which means that NOMA is more beneficial for small $\theta$. Therefore, it can be concluded that the VD clustering scenario is preferable in the application of uplink NOMA in multi-cell scenarios. [13] came to a similar conclusion for downlink NOMA.

We then explore the impact of imperfect SIC on NOMA. Fig. 16 depicts the achievable sum rate vs. $\beta$ for $\theta = 1, 3$ dB assuming MSP-based ordering, $N = 2$, $\epsilon = 0$, and the same $\theta$ for the NOMA UEs. Since OMA does not use SIC, the corresponding sum rate plots are independent of $\beta$. The figure shows the existence of a maximum $\beta$ until which a NOMA system with a particular $\theta$ outperforms the corresponding OMA system for both the VD and the VC scenarios. This highlights that a critical minimum level of SIC is required for NOMA to outperform OMA. Moreover, the threshold for the VD scenario is lower than for the VC scenario, which indicates that the VD scenario is more susceptible to the RI. We also observe that the decrease in the sum rate as a function of $\beta$ is steeper for a larger $\theta$ highlighting its increased susceptibility to the RI.

### V. Conclusion

In this paper, we have developed a theoretical framework to analyze the performance of multi-cell uplink NOMA systems under two different UE clustering scenarios, namely the VD and VC scenarios. Using some mild simplifying assumptions,
where $I_{nn}^*$ denotes the inter-cell interference which originates from UEs in the non-nearest cells while $I_{n}^*$ denotes the interference that stems from UEs in the nearest cell\textsuperscript{[10]}

The LT of $I^*$ given $\rho$ is thus given as

$$L_{I^*|\rho}(s) \approx L_{I_{nn}^*|\rho}(s) \cdot L_{I^*_n}(s).$$  
(43)

$L_{I_{nn}^*|\rho}(s)$ is obtained as

$$L_{I_{nn}^*|\rho}(s) = E \left[ \exp \left( -s \sum_{i=1}^{N} \rho^{R_{i,nn}^{*}} y_i \right) \right].$$

\begin{align*}
\overset{(a)}{=} & \quad E_{\Phi} \left[ \prod_{y \in \Phi} \mathbb{E}_{R_{i,y}} \left[ \prod_{i=1}^{N} \mathbb{E}_{R_{i,\rho}} \left[ \left( \frac{\frac{5}{12}}{1 + sz^{sx}} y^{-\alpha} \right) \right] \right] \right] \\
\overset{(b)}{=} & \quad E_{\Phi} \left[ \prod_{y \in \Phi} \left( \frac{\frac{5}{12}}{1 + sz^{sx}} y^{-\alpha} \right) \right] \\
\overset{(c)}{=} & \quad \exp \left( -2\pi \lambda \int_{\rho}^{\infty} \left( 1 - \left( \frac{\frac{5}{12}}{1 + sz^{sx}} y^{-\alpha} \right) \right) y dy \right),
\end{align*}

where (a) follows from $h_{i,y} \sim \exp(1)$ and the independence between $h_{i,y}$, (b) follows since $R_{i,y}$ are i.i.d. for $1 \leq i \leq N$, (c) follows from mapping $\Phi$ to one dimension and applying the probability generating functional (pgfl) of the PPP \textsuperscript{[20]}.

The integral $A$ is calculated as

$$A = \int_{0}^{\frac{5}{12}} \frac{8\pi}{\rho^{2}} \frac{1}{1 + sz^{sx} y^{-\alpha}} \frac{dz}{\rho^2}$$

\begin{align*}
\overset{(d)}{=} & \quad 4\delta \frac{\frac{5}{12}}{\epsilon \rho^{2}} \int_{0}^{\frac{5}{12}} \frac{1}{1 + sz^{sx} y^{-\alpha}} u \frac{u^{\frac{5}{12}} - 1}{u^{\frac{5}{12}} - 1} \frac{du}{u^{\frac{5}{12}}} \\
\overset{(e)}{=} & \quad 2F_1 \left( 1, \frac{\delta}{\epsilon}; 1 + \frac{\delta}{\epsilon} - s \left( \frac{\frac{5}{12}}{\rho^{2}} \right) y^{-\alpha} \right),
\end{align*}

where (d) follows from the substitution $u = \frac{5}{12}$ and $\epsilon \neq 0$, (e) follows from $\epsilon > 0$ and $\int_{0}^{\frac{5}{12}}(\frac{1}{1 + z\alpha})dz = \frac{\frac{5}{12}}{\mu^2} 2F_1(i, \mu; 1 + \mu; -\beta l, \mu > 0)$ \textsuperscript{[23]} Eqn. 3.194.1.

$L_{I^*|\rho}(s)$ is obtained as

$$L_{I^*|\rho}(s) = E \left[ \exp \left( -s \sum_{i=1}^{N} \rho^{R_{i,\rho}^{*}} h_{i,\rho} y_i \right) \right].$$

\begin{align*}
\overset{(f)}{=} & \quad \left( \int_{0}^{\frac{5}{12}} \frac{f_{R_{i,\rho}}(x)}{1 + sz^{sx} \rho^{-\alpha}} \right) N \\
\overset{(g)}{=} & \quad 2F_1 \left( 1, \frac{\delta}{\epsilon}; 1 + \frac{\delta}{\epsilon} - s \left( \frac{\frac{5}{12}}{\rho^{2}} \right) \rho^{-\alpha} \right),
\end{align*}

where (f) follows since $h_{i,\rho}$ is i.i.d. and $R_{i,\rho}$ are i.i.d., (g) follows from (45) by replacing $y$ with $\rho$.

Combining (46) and (44), we obtain (3).

**APPENDIX B**

**PROOF OF COROLLARY 1**

Assuming that the interfering UEs use transmit power $\bar{P}$, the LT of $I_{nn}^*$ is approximated as

$$L_{I_{nn}^*|\rho}(s) \approx \exp \left( -2\pi \lambda \int_{0}^{\infty} \left( 1 - \left( \frac{1}{1 + s P y^{-\alpha}} \right)^N \right) y dy \right)$$

\begin{align*}
\overset{(a)}{=} & \quad \exp \left( -\pi \lambda \delta \int_{0}^{\rho^{\alpha}} \left( 1 - \left( \frac{1}{1 + s \bar{P} y^{-\alpha}} \right)^N \right) u^{-\delta - 1} du \right) \\
\overset{(b)}{=} & \quad \exp \left( -\pi \lambda \delta \sum_{j=1}^{\infty} (\frac{\bar{P}}{\lambda}) (\frac{\bar{P}}{\lambda})^{N} \right) \int_{0}^{\rho^{\alpha}} \frac{u^{-\delta - 1}}{1 + s \bar{P} u^{\alpha}} du \\
\overset{(c)}{=} & \quad \exp \left( -\sum_{k=1}^{\infty} (\frac{\bar{P}}{\lambda})^{k} F_k \left( k, k - \delta; k + 1 - \delta; s \bar{P} \rho^{-\alpha} \right) \right),
\end{align*}

where (a) follows from the substitution $u = y^{-\alpha}$ and $\delta = 2/\alpha$, (b) follows from the binomial expansion of $\left( 1 - \frac{1}{1 + s \bar{P} u^{\alpha}} \right)^N$, and (c) follows from \textsuperscript{[23]} Eqn. 3.194.1.

$L_{I^*|\rho}(s)$ is approximated as

$$L_{I^*|\rho}(s) \approx \mathbb{E} \left[ \exp \left( -s \sum_{i=1}^{N} \rho^{R_{i,\rho}^{*}} \right) \right] = \frac{1}{(1 + s P \rho^{-\alpha})^N}.$$  
(48)

Combining (47) and (48), we obtain (6).

**APPENDIX C**

**PROOF OF THEOREM 1**

Assuming $\epsilon \leq 1$,

$$P(C_i) = \mathbb{P} \left[ \frac{h_1 \hat{R}_1^{(1-\epsilon)\alpha}}{\sum_{j=2}^{N} h_j \hat{R}_j^{(1-\epsilon)\alpha}} + I^* + \sigma^2 > \theta_1, \ldots \right]$$

$$\overset{(a)}{=} \mathbb{P} \left[ \sum_{j=2}^{N} h_j \hat{R}_j^{(1-\epsilon)\alpha} + I^* + \sigma^2 > \theta_1 \right]$$

\begin{align*}
\overset{(a)}{=} & \quad \mathbb{E} \left[ \int_{0}^{\infty} dh_N \cdots \int_{0}^{\infty} dh_i \int_{0}^{\infty} \theta_1(h_N^{(1-\epsilon)\alpha} + I^* + \sigma^2) dh_i \\
& \quad \times \cdots \times \int_{0}^{\infty} \theta_1(h_j^{(1-\epsilon)\alpha} + I^* + \sigma^2) dh_j \right] dh_1 e^{-h_1} \cdots e^{-h_N},
\end{align*}

where (a) follows from $h_i \sim \exp(1)$ and the independence between $h_i$ for $1 \leq i \leq N$. Applying the definition of the Laplace transform, we obtain (9).

For $\epsilon > 1$,

$$P(C_i) = \mathbb{P} \left[ \frac{h_N \hat{R}_N^{(1-\epsilon)\alpha}}{\sum_{j=1}^{N-1} h_j \hat{R}_j^{(1-\epsilon)\alpha} + I^* + \sigma^2} + \frac{h_{N+1-\epsilon} \hat{R}_{N+1-\epsilon}^{(1-\epsilon)\alpha}}{\sum_{j=1}^{N} h_j \hat{R}_j^{(1-\epsilon)\alpha} + I^* + \sigma^2} > \theta_1 \right].$$  
(50)

Following similar derivations as in (49), we obtain (9).

\textsuperscript{[10]}The nearest cell refers to the cell whose serving BS is the nearest neighbor of the typical BS. The other interfering cells are referred to as the non-nearest cells. In \textsuperscript{[12]}, we denote the location of the nearest BS, which is at distance $\rho$ from the typical BS, as $y_{\rho}$.
\[
E(\rho, r_1) \triangleq \int_{-\rho}^{\rho} f_{R_2}(r_2) \frac{1}{1 + \theta_r r_1^{-1/(1-\alpha)} r_2^{-1/(1-\alpha)}} dr_2
\]

where (a) follows from the substitution \( v = r_2^{-(1-\epsilon)/(1-\alpha)} \) and \( \delta_v = (2/(1-\epsilon)/\alpha) \).

Assuming \( \epsilon < 1 \), from (53) we have
\[
P(C_1) = \mathbb{E}_\rho \left[ \mathbb{E}_{\hat{R}_1, \hat{R}_2} \left[ e^{-\sigma_2 \hat{R}_1^{(1-\alpha)}} L_{I*}(\hat{R}_1^{(1-\alpha)}) \right] \right]
\]

where (b) follows from \( f_{\hat{R}_1, \hat{R}_2}(r_1, r_2) = 2 f_{\hat{R}_1}(r_1) f_{\hat{R}_2}(r_2) \) based on order statistics (54). Substituting (51) in (52), we get (53). Similarly,
\[
P(C_2) = \mathbb{E}_\rho \left[ \mathbb{E}_{\hat{R}_1, \hat{R}_2} \left[ e^{-\sigma_2 \hat{R}_1^{(1-\alpha)}} L_{I*}(\hat{R}_1^{(1-\alpha)}) \right] \right]
\]

Applying the joint pdf of \( \hat{R}_1 \) and \( \hat{R}_2 \) and some simplifications, we obtain (15). Using similar derivations, we obtain the results for \( \epsilon > 1 \) and \( \epsilon = 1 \) as given in (13) and (15).

\section*{APPENDIX F}

\section*{IMPROT OF IMPERFECT SIC}

Considering imperfect SIC, the SINR of \( U_i \) for \( 1 \leq i \leq N \) is
\[
\text{SINR}_i = \frac{h_i R_i^{(1-\alpha)}}{\sum_{j=1}^{N} h_j R_j^{(1-\alpha)} + \beta \sum_{q=1}^{N-1} h_q R_q^{(1-\alpha)} + I^* + \sigma^2}
\]

where \( \beta \in [0, 1] \) is the fraction of residual interference (RI) from UEs decoded before \( U_i \). \( \beta = 0 \) means perfect SIC, while \( \beta = 1 \) corresponds to no SIC at all. Using similar derivations as in the perfect SIC case, the transmission success probabilities of the NOMA UEs can be derived. As an example, here we give the expression of \( P(C_2) \) considering imperfect SIC using MSP-based ordering for the VD scenario with \( N = 2 \). \( P(C_2) \) is given in (55) for \( \theta_1 \theta_2 \beta < 1 \), where
\[
w_1(r_1, r_2) = \frac{1}{1 + \theta_1 r_1^{-1/(1-\alpha)} r_2^{-1/(1-\alpha)}},
\]
\[
u_1(r_1, r_2) = \frac{1}{1 + \theta_2 r_1^{-1/(1-\alpha)} r_2^{-1/(1-\alpha)}},
\]

For \( \theta_1 \theta_2 \beta \geq 1 \), \( P(C_2) = 0 \). The numerical results are given in Fig. 16.
APPENDIX G

PROOF OF LEMMA 2

For Model 1, the inter-cell interference at the typical BS originating from $\Psi_1$ is denoted as $I^*$ and given by

$$I^* = \sum_{x \in \Psi_1} P_x h_x \|x\|^{-\alpha} \sum_{x \in \Psi_1} R_x^* h_x D_x^{-\alpha}. \quad (62)$$

The LT of $I^*$ is given as

$$\mathcal{L}_{I^*}(s) = \mathbb{E} \left[ \exp \left( -s \sum_{x \in \Psi_1} R_x^* h_x D_x^{-\alpha} \right) \right].$$

Using similar derivations as in (63), we obtain (29).

For Model 2, the LT of the inter-cell interference at the typical BS is given as

$$\mathcal{L}_{I^*}(s) = \mathbb{E} \left[ \exp \left( -s \sum_{x \in \Psi_1} \sum_{i=1}^N R_x^* h_x D_x^{-\alpha} \right) \right]. \quad (64)$$

Using similar derivations as in (63), we obtain (29).

APPENDIX H

PROOF OF COROLLARY 3

Define

$$F(r_1) = \int_{r_1}^{\infty} \frac{f_{R_1}(r_2)}{1 + \theta_1 r_2^{1-\alpha}(1-\epsilon)} dr_2 \quad (65)$$

where (a) follows from the substitution $v = (\frac{r}{r_1})^{1-\epsilon}$ and $\delta_x = \frac{2}{1-\epsilon}$. $\theta_1 < 1$, from [31] we have

$$\mathbb{P}(C_1) = \mathbb{P} \left[ \frac{h_1 R_1^{1-\epsilon}(1-\alpha)}{h_2 R_2^{1-\epsilon}(1-\alpha) + I^* + \sigma^2} > \theta_1 \right] \quad (66)$$

where (b) follows from $h_1 \sim \exp(1), h_2 \sim \exp(1)$ and the independence of $h_1$ and $h_2$. $\delta_2 = \frac{2}{1-\epsilon}$.

Using similar derivations, we obtain (37). Similarly, we can obtain the results for $\epsilon > 1$ and $\epsilon = 1$ as given in [32] and [34].

REFERENCES


