Bandwidth- and Power-Efficient Routing in Linear Wireless Networks

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***Abstract*—The goal of this paper is to establish which practical routing schemes for wireless networks are most suitable for power-limited and bandwidth-limited communication regimes. We regard channel state information (CSI) at the receiver and point-to-point capacity-achieving codes for the additive white Gaussian noise (AWGN) channel as practical features, interference cancellation (IC) as possible, but less practical, and synchronous cooperation (CSI at the transmitters) as impractical. We consider a communication network with a single source node, a single destination node, and $N-1$ intermediate nodes placed equidistantly on a line between them. We analyze the minimum total transmit power needed to achieve a desired end-to-end rate for several schemes and demonstrate that multihop communication with spatial reuse performs very well in the power-limited regime, even without IC. However, within a class of schemes not performing IC, single-hop transmission (directly from source to destination) is more suitable for the bandwidth-limited regime, especially when higher spectral efficiencies are required. At such higher spectral efficiencies, the gap between single-hop and multihop can be closed by employing IC, and we present a scheme based upon backward decoding that can remove all interference from the multihop system with an arbitrarily small rate loss. This new scheme is also used to demonstrate that rates of $O(\log N)$ are achievable over linear wireless networks even without synchronous cooperation.**

***Index Terms*—Cooperation, network information theory, relay networks, routing, wireless networks.**

I. Introduction

If we assume that the typical deployment phases of cellular wireless networks are indicative of how other types of wireless networks might evolve, we should expect an initial phase of coverage growth, in which the geographical size of the network is increasing, followed by a phase of throughput growth, in which the density of the network is increasing. The coverage growth phase is typically characterized by a relative abundance of radio bandwidth compared to the required throughput and large signal attenuations due to the large distances between transmitters and receivers. This scenario corresponds to low spectral efficiencies and low signal-to-noise ratios (SNRs), and the main challenge is minimizing the energy per bit used by the network. In contrast, the throughput growth phase involves a relative scarcity of bandwidth, lower attenuations due to denser inter-node spacing, higher spectral efficiencies, and higher SNRs. Communication schemes suitable for this phase should offer good tradeoffs between energy per bit and spectral efficiency. The former set of conditions is commonly referred to as the power-limited regime, and the latter is called the bandwidth-limited regime [1]. Since many communication schemes that perform relatively well in one regime might be a poor choice for the other, cross-regime performance comparisons can help identify the most efficient transmission approach for each deployment phase. This can be particularly useful if upgrading the nodes as the network matures is difficult or impossible and planning for later phases must be done at design time.

Of particular interest are wireless *ad hoc* networks, which consist of nodes that can serve as relays, i.e., assist transmission of the messages without being either the source or the destination for the data. The problem of finding upper and lower bounds on the transmission rates achievable over such networks has been recently studied in [2]–[8] under various assumptions on the network topologies and node capabilities. Gupta and Kumar [2], [3] and Xie and Kumar [4] considered planar networks with N nodes and multiple source–destination pairs and characterized attainable transport rates (in bit-meters per second) for finite and infinite N. For networks with a single source and destination, perfect channel state information (CSI) at all nodes, and perfect synchronization, Gastpar and Vetterli [5] showed that the achievable rate is logarithmic in N if the distance from each relay node to the source and destination is lower-bounded, and without such a bound it can grow linearly in N [3]. Recently, a more general problem of rates achievable over a multiple-relay channel was considered by Xie and Kumar [6] and by Kramer *et al.* [7], [8].

Routing is an important special case of relaying in the sense of [3]–[8]. In this paper, we determine which routing schemes for wireless *ad hoc* networks based on capacity-achieving point-to-point codes are best suited in the power-limited and bandwidth-limited regimes. As a reference scheme, we also consider single-hop transmission consisting of direct transmission between source and destination, which we simply view as a special case of multihop with no intermediate nodes. Although most of the work mentioned above focuses on "order-of" results, we are interested in the actual capacities and power and bandwidth efficiencies, i.e., results that include the pre-constants. To still have a tractable problem, we consider a one-dimensional chain of nodes, a so-called linear network, with equidistant nodes. This case is obviously a simplification,

Manuscript received April 1, 2005; revised February 13, 2006. This work was supported in part by NASA under Grant NNG-05GH73G, NSF under Grants CCR02-05310 , ECS03-29766, and CCF05-15012, and the U.S. Army under Grant DAAD16-02-C-0057.

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Communicated by M. Chiang, Guest Editor.
Digital Object Identifier 10.1109/TIT.2006.874520

but it constitutes an important special case of more general two-dimensional networks. Assuming only a single route is active in a multihop network, point-to-point coding is used, and there is no interference between nodes, placing all intermediate nodes on a line at equal intervals is the best case in terms of throughput and energy consumption (the same is not true for the general relay channel, see [8]). The same linear network model is used in [9]–[13].

We adopt a fairly conservative view of which schemes can be regarded as practical. We require that the multihop transmission be based entirely on point-to-point coding for the additive white Gaussian noise (AWGN) channel. This implies that each node fully decodes the original message based on the signal received from the preceding node, re-encodes it, and forwards it to the following node. The decoding operation must rely on the decoder for the AWGN channel, and all interference from all nodes transmitting simultaneously with the preceding node is regarded as additional Gaussian noise. As more difficult to implement, but still practical, we also consider canceling a known interference from the received signal before decoding.

Among the techniques that are frequently encountered in the literature but cannot be integrated into the above framework are synchronous cooperation and sliding-window decoding. Synchronous cooperation (used in, e.g., [3]–[5]), which is analogous to beamforming performed by several transmit antennas controlled by a single transmitter, gives the transmitter full control over how the signals add up at the receiver's antenna. This includes maximum ratio transmission, in which several copies of the same signal transmitted from different antennas add up in amplitude at the receiver, providing large power savings, and active interference cancellation (IC), in which two signals sent from two different nodes add up to zero at a selected receiver, and neither of them interferes with that receiver. However, whenever the antennas attempting to synchronously cooperate are controlled by separate transmitters, providing precise timing and phase synchronization between them is extremely difficult. Sliding-window decoding, which was originally introduced by Carleial [14] and generalized to the multiple-relay channel in [4], [6], involves determining the most likely message using not only the signal from the preceding node received in the current slot, but also from the upstream transmissions received in past slots. However, since in general the codebooks used at each hop are different, the complexity of a sliding-window decoder is considerably larger than that of a standard point-to-point decoder.

There are two main contributions of this paper. First, we demonstrate that multihop with spatial reuse, but without IC, achieves excellent performance in the power-limited regime, but due to excess interference its performance suffers in the bandwidth-limited regime. In fact, above certain rates, single-hop communication performs significantly better. Second, we present an IC scheme based on backward decoding [15]–[17], capable of canceling all interference in a linear multihop network at an arbitrarily small rate loss. This technique, called recursive backward IC, can significantly improve the performance of multihop transmission in the bandwidth-limited regime at a complexity and delay cost. Additionally, the new scheme shows that the capacity of a linear wireless network without synchronous cooperation is $O(\log N)$, i.e., of the same order as the capacity of a network with synchronous cooperation. The possibility of extending block Markov encoding and backward decoding to the multiple-relay channel using nested blocks was first mentioned briefly in [7].

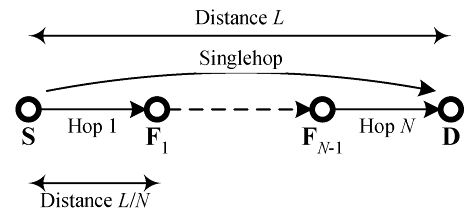

Fig. 1. Single-hop and multihop communication over a regular linear network.

The remainder of this paper is organized as follows. Section II describes the operation of the regular linear network. Section III defines the concept of a communication scheme for this network and describes our methods for evaluating performance. Section IV presents and evaluates two communication schemes not requiring IC, namely single-hop and multihop with spatial reuse. Section V introduces the multihop scheme with recursive backward IC. Finally, Section VI presents some concluding remarks.

II. System Model

A. A Wireless Linear Network Model

The communication system under consideration is illustrated in Fig. 1. It consists of a source node $\boldsymbol{S}$ and a destination node $\boldsymbol{D}$, separated by a distance L, and $N-1$ intermediate relay nodes $\boldsymbol{F}_i$, $i = 1, \dots, N-1$, placed equidistantly on the line from $\boldsymbol{S}$ to $\boldsymbol{D}$. The nodes share a band of radio frequencies allowing for a signaling rate of W complex-valued symbols per second. The objective of the system is the reliable delivery of bits generated at the source node $\boldsymbol{S}$ at a bandwidth-normalized rate (henceforth just called the rate) of R bits per second per hertz (i.e., RW bits per second) to the destination node using coded transmission and consuming the least possible total transmission power P_T. We place no restriction on how this total power is allocated among nodes. The nodes comprising the system operate in half-duplex, i.e., they are incapable of simultaneous transmission and reception. Additionally, the source node $\boldsymbol{S}$ does not receive, and the destination node $\boldsymbol{D}$ does not transmit.

If at any given time the nodes in a set

$$S_T \subset \{\boldsymbol{S}, \boldsymbol{F}_1, \dots, \boldsymbol{F}_{N-1}\}$$

transmit and the nodes in a set

$$S_R \subset \{\boldsymbol{F}_1, \dots, \boldsymbol{F}_{N-1}, \boldsymbol{D}\}$$

receive, the sequence $y_i[n]$ of baseband-equivalent, discrete-time complex-valued symbols received by node $\boldsymbol{F}_i$, $i \in S_R$, can be expressed as

$$y_i[n] = \sum_{s \in S_T} a_{s,i} x_s[n] + z_i[n] \tag{1}$$

where $x_s[n]$ is the complex-valued symbol transmitted by node s at time n, $a_{s,i}$ is a distance-dependent attenuation factor, and $z_i[n]$ is white Gaussian noise with zero mean and variance $N_0/2$

per dimension. The attenuation factor $a_{s,i}$ depends on the distance $d(s,i)$ between nodes as

$$a_{s,i} = cd(s,i)^{-\alpha/2} \tag{2}$$

which corresponds to $10\alpha\log_{10} d(s,i) - 20\log_{10} c$ decibel power loss. In (2), α is the path loss exponent (typically taking values between 2 and 4), and c is a constant.

To simplify our analysis, we will assume $L = 1$ and $c = 1$ throughout this paper. Different choices for these parameters will only cause scaling of the total transmission power P_T or energy per bit E_b identically for all communication schemes and will not affect the outcome of performance comparisons among them. Finally, we do not impose any delay constraints on the system, and we allow the coded transmission to have an arbitrarily large block length.

B. Communication Schemes

Information from $\boldsymbol{S}$ to $\boldsymbol{D}$ is sent through the linear network in N hops, being sequentially recovered at nodes $\boldsymbol{F}_i$, $i = 1,\ldots,N-1$, before it arrives at $\boldsymbol{D}$. The transmission at each hop is implemented using capacity-achieving codes for the complex-valued AWGN channel. To be precise, we assume that for any positive rate r and block size b there exists a set of messages $M_{r,b}$ with $|M_{r,b}| = \lceil 2^{rb} \rceil$, an encoder $f_{r,b} : M_{r,b} \mapsto \mathbb{C}^b$, and a decoder $g_{r,b} : \mathbb{C}^b \mapsto M_{r,b}$. Let m be a random variable drawn uniformly from $M_{r,b}$, $\boldsymbol{z}$ a b-vector of independent and identically distributed (i.i.d.) zero-mean complex Gaussian random variables with variance $N_0/2$ per dimension, and define $\boldsymbol{x} = f_{r,b}(m)$, $\boldsymbol{y} = \boldsymbol{x} + \boldsymbol{z}$, and $\hat{m} = g_{r,b}(\boldsymbol{y})$. Then for $b \to \infty$, the covariance $\boldsymbol{R}_{xx} \to E_s\boldsymbol{I}_b$ and $\Pr(m \neq \hat{m}) \to 0$, where E_s, N_0, and r are related by $r = \log_2(1 + E_s/N_0)$, and $\boldsymbol{I}_b$ is the identity matrix of dimension b. The existence of such codes is one of the main results of [18].

The communication takes place in time slots of length b. The information generated at $\boldsymbol{S}$ is mapped to a sequence of messages m_n. If node $\boldsymbol{F}_i$ knows a certain m_n, but node $\boldsymbol{F}_{i+1}$ does not, $\boldsymbol{F}_i$ is allowed to transmit it to $\boldsymbol{F}_{i+1}$ using one of the encoders in a single time slot. Node $\boldsymbol{F}_{i+1}$ can then use the signal received during this time slot (and only during this time slot) to recover the message using a corresponding decoder. As an extension, the decoding step can be preceded by IC, assuming the interference is caused by messages already decoded at this node. The remaining interference is regarded as Gaussian noise.

Suppose that P_0 and β_0 denote, respectively, the transmit power of $\boldsymbol{S}$ and the fraction of time $\boldsymbol{S}$ transmits, and P_i, β_i, $i = 1,\ldots,N-1$, denote the analogous parameters for the relays $\boldsymbol{F}_i$. Let $P_T = \sum P_i\beta_i$ be the total average power. Then the rate r_i achieved at hop $i = 1,\ldots,N$ is

$$r_i = \beta_i \log_2\left(1 + \frac{P_{i-1}N^\alpha}{WN_0 + \sum_{s=0}^{N-1}\gamma_{s,i-1}P_sN^\alpha|i-s|^{-\alpha}}\right) \tag{3}$$

where $\gamma_{s,i-1} = 1$ if nodes $\boldsymbol{F}_s$ and $\boldsymbol{F}_{i-1}$ transmit simultaneously and the receiver at $\boldsymbol{F}_i$ cannot cancel out the interfering signal from $\boldsymbol{F}_s$, and otherwise ,$\gamma_{s,i-1} = 0$. The achievable end-to-end rate is the minimum of the rates achievable at each of the N hops

$$R = \min_{i=1,\ldots,N} r_i. \tag{4}$$

Communication schemes described in the following sections, depending on the processing performed at each node, will differ in the values of $\beta_i, \gamma_{s,i-1}$, and in the values of the optimal power allocation $P_i^*(R)$ that minimizes the total transmit power P_T for a given R.

Let $R^*(P_T)$ denote the highest rate achieved with total transmit power P_T for a certain scheme. Then the following simple properties hold.

Property 1: The maximum achievable rate $R^*(P_T)$ is strictly increasing in P_T.

Proof: Suppose that for a given P_T, $\{P_0^*,\ldots,P_{N-1}^*\}$ is the power allocation that achieves the maximum rate $R^*(P_T)$. Suppose we are now allowed to use αP_T transmitted power, where $\alpha > 1$. Let us choose a (possibly suboptimal) power allocation $\{\alpha P_0^*,\ldots,\alpha P_{N-1}^*\}$. Then it is easy to see that the rate at every hop (3), as well as the overall rate (4), has increased. But since the rate $R^*(\alpha P_T)$ can only be greater than or equal to this new rate, $R^*(P_T)$ is strictly increasing in P_T. □

Property 2: The power allocation that achieves the maximum rate $R^*(P_T)$ forces the rates at all hops r_i to be equal.

Proof: Let $\{P_0^*,\ldots,P_{N-1}^*\}$ be the optimum power allocation for a certain $R^*(P_T)$. Suppose that for a certain hop i, $r_i > R^*(P_T)$. Since r_i is continuous and increasing in P_{i-1}, we can replace P_{i-1}^* by a slightly smaller value, for which $r_i \geq R^*(P_T)$ still holds. Since all other r_j, $j \neq i-1$, are nonincreasing in P_{i-1}, the new power allocation achieves the same or a higher rate R at a lower power P_T. But this contradicts Property 1. Hence, at all hops, $r_i = R^*(P_T)$. □

Property 3: The power allocation that achieves the maximum rate $R^*(P_T)$ is unique.

Proof: Let $\{P_0^*,\ldots,P_{N-1}^*\}$ be a power allocation that achieves $R^*(P_T)$ for a certain P_T. Suppose that a different power allocation $\{\tilde{P}_0^*,\ldots,\tilde{P}_{N-1}^*\}$ also achieves $R^*(P_T)$. By Property 2, both allocations yield $r_i = R^*(P_T)$. Suppose that for a certain i, $P_i^* = \alpha_i\tilde{P}_i^*$ with $\alpha_i < 1$. Then both power allocations can yield the same value of r_{i+1} in (3) only if, for some other $j \neq i$, $P_j^* = \alpha_j\tilde{P}_j^*$, and $\alpha_i > \alpha_j$. By applying this argument recursively, we can construct an infinite chain of inequalities $\alpha_{i_1} > \alpha_{i_2} > \alpha_{i_3} > \cdots$. But since there is only a finite number of nodes, this will eventually lead to a contradiction. Hence $P_i^* = \tilde{P}_i^*$ for all i. □

Property 4: Let $\{P_0^*(P_T),\ldots,P_{N-1}^*(P_T)\}$ denote the power allocation achieving $R^*(P_T)$. Then each $P_i^*(P_T)$ is nondecreasing in P_T.

Proof: Analogous to the proof of Property 3, it can be shown from (3) that, if $P_i^*(P_T)$ is decreasing for some P_T and i, then some other $P_j^*(P_T)$ must be decreasing even faster. Using this fact recursively shows that assuming $P_i^*(P_T)$ to be decreasing leads to a contradiction. □

Property 5: The maximum achievable rate $R^*(P_T)$ is continuous for $P_T > 0$.

Proof: Again let $\{P_0^*(P_T), \ldots, P_{N-1}^*(P_T)\}$ denote the power allocation achieving $R^*(P_T)$. Using the fact that $P_T = \sum \beta_i P_i^*(P_T)$ for every P_T, we can write

$$\begin{aligned} 0 &= \lim_{\delta \to 0^+} ((P_T + \delta) - P_T) \\ &= \lim_{\delta \to 0^+} \left(\sum_i \beta_i P_i^*(P_T + \delta) - \sum_i \beta_i P_i^*(P_T) \right) \\ &= \lim_{\delta \to 0^+} \sum_i \beta_i \left(P_i^*(P_T + \delta) - P_i^*(P_T) \right). \end{aligned}$$

By Property 4, the $P_i^*(P_T)$ are nondecreasing, and so all terms $P_i^*(P_T + \delta) - P_i^*(P_T)$ must be nonnegative. But if a sum of nonnegative terms converges to zero, each term separately must also converge to zero. Hence,

$$\lim_{\delta \to 0^+} P_i^*(P_T + \delta) = P_i^*(P_T)$$

and $P_i^*(P_T)$ is right-continuous. Using the limit $\delta \to 0^-$ instead, we can also show that $P_i^*(P_T)$ is left-continuous. Finally, since r_i in (3) are continuous in P_i and the minimum of the continuous functions in (4) is itself continuous, $R^*(P_T)$ is continuous in P_T. □

We use $P_T^*(R)$ to denote the minimum transmit power necessary to achieve $R > 0$. For the rates that can be achieved with finite power, $P_T^*(R)$ is simply the inverse of $R^*(P_T)$, and for all other rates we define $P_T^*(R) \triangleq +\infty$. An even more useful characterization of the achievable power–throughput tradeoff can be obtained by looking at the energy spent by the entire network per information bit, i.e., $E_b^*(R) = P_T^*(R)/RW$, and its inverse, $R^*(E_b)$, where we define $E_b^*(0) \triangleq \lim_{R \to 0} E_b^*(R)$. These two functions are the main tools used in the remainder of the paper for comparing different communication schemes. In order to distingush between the power–rate functions for different schemes we will adopt the notation of replacing the asterisk in $E_b^*(R)$ with an acronym of the scheme name.

III. Performance Evaluation Methods for Communication Schemes

A. Power-Limited Regimes

A power-limited communication regime is the preferred way of sending information over systems in which transmitter power is much more costly than bandwidth. Since bandwidth is in abundance, communication in this regime is characterized by low SNRs, very low signal power spectral densities, and negligible interference power. It is fairly easy to distinguish among schemes in this regime by comparing their $E_b(R)$ in the neighborhood of zero bandwidth efficiency R.

Definition 1: A communication scheme $\mathcal{X}$ is *more power efficient* than scheme $\mathcal{Y}$ if there exists an $R_0 > 0$ such that, for all $0 \leq R < R_0$, we have $E_b^{\mathcal{X}}(R) \leq E_b^{\mathcal{Y}}(R)$ and this inequality is strict for at least one such R.

Since $E_b^{\mathcal{X}}(R)$ and $E_b^{\mathcal{Y}}(R)$ are continuous in the neighborhood of $R = 0$, two simple tests for checking if $\mathcal{X}$ is more power efficient than $\mathcal{Y}$ can be used. The first test simply involves checking if $E_b^{\mathcal{X}}(0) < E_b^{\mathcal{Y}}(0)$—if this condition is satisfied, then $\mathcal{X}$ is indeed more power efficient. If the first test yields $E_b^{\mathcal{X}}(0) = E_b^{\mathcal{Y}}(0)$, the second test is to check if

$$\partial E_b^{\mathcal{X}}(R)/\partial R|_{R=0} < \partial E_b^{\mathcal{Y}}(R)/\partial R|_{R=0}.$$

This second test is essentially the wideband slope introduced in [19], where the significance of $\partial E_b(R)/\partial R$ for evaluating the performance in the power-limited regime is thoroughly analyzed.

B. Bandwidth-Limited Regimes

A bandwidth-limited communication regime is needed whenever bandwidth is scarce and much more costly than transmit power. This regime is characterized by higher SNRs, higher signal spectral densities, and high susceptibility to interference. By analogy to the power-limited case, we can adopt the following definition.

Definition 2: A communication scheme $\mathcal{X}$ is *more bandwidth efficient* than scheme $\mathcal{Y}$ if there exists an $R_0 < \infty$ such that, for all $R > R_0$, we have $E_b^{\mathcal{X}}(R) \leq E_b^{\mathcal{Y}}(R)$ and this inequality is strict for at least one such R.

Unfortunately, Definition 2 for the bandwidth-limited regime is not as useful as Definition 1 for the power-limited regime, since the smallest R_0 for which the condition in Definition 2 is satisfied might itself be very large. In such a case, a supposedly less bandwidth-efficient scheme could still achieve large savings in E_b at practical rates below R_0. Hence, when comparing communication schemes, we will be more interested in identifying rate regions in which one of them has the lowest E_b.

C. Limits on Communication Over Linear Networks

The ability to assess the performance of any particular communication scheme is most useful if it can be compared to theoretical upper bounds. A simple bound based on the max-flow min-cut principle [20] can be derived using the broadcast cut, i.e., the cut separating the source node $\boldsymbol{S}$ from all remaining nodes $\boldsymbol{F}_i$ and $\boldsymbol{D}$. We allow the source node to use all available transmission power P_T and all channel time, while letting all remaining nodes exchange information for free (the relay nodes thus become additional antennas of the destination node). The rate is then upper-bounded by the capacity of the resulting single-input multiple-output channel, i.e.,

$$R \leq \log_2 \left(1 + \frac{P_T}{WN_0} \sum_{i=1}^{N} (i/N)^{-\alpha} \right). \tag{5}$$

The finite sum in (5) can be further upper-bounded by an infinite sum, which converges for $\alpha > 1$ and is proportional to the Riemann Zeta function $\zeta(\alpha)$ [21]

$$R < \log_2 \left(1 + \frac{P_T}{WN_0} N^{\alpha} \zeta(\alpha) \right). \tag{6}$$

It follows that, given the power P_T, the rate R can asymptotically scale at most as fast as $\alpha \log_2 N$. To make the bound (6) useful for the power-efficient regime, we can substitute $P_T = RWE_b$, solve for $E_b(R)$, and take the limit as $R \to 0$ to obtain

$$\frac{E_b(0)}{N_0} > \frac{N^{-\alpha}\ln 2}{\zeta(\alpha)}. \tag{7}$$

It is interesting to note that the above bound holds for an arbitrary communication scheme, including those performing synchronous cooperation, since the broadcast cut involves only a single transmitter (see, for example, [5]).

IV. Communication Schemes Without Interference Cancellation

A. Single-Hop Transmission

The simplest communication scheme defined for $N = 1$, called *single-hop*, involves just a direct transmission from $\boldsymbol{S}$ to $\boldsymbol{D}$. All power and channel time is allocated to the source node, which uses capacity-achieving coding with complex Gaussian-distributed symbols. Its power efficiency–bandwidth efficiency characteristic is captured by the well-known capacity formula for the complex-valued AWGN channel

$$R = \log_2\left(1 + \frac{RE_b}{N_0}\right) \tag{8}$$

and its inverse

$$\frac{E_b^{\mathrm{SH}}(R)}{N_0} = \frac{2^R - 1}{R}. \tag{9}$$

At first glance, single-hop transmission only seems to be useful as a reference for more advanced schemes. Indeed, it performs poorly in the power-limited regime with $E_b^{\mathrm{SH}}(0)/N_0 = \ln 2$ (-1.59 dB). However, it will soon be apparent that it performs surprisingly well in the bandwidth-limited regime relative to the other schemes we consider.

B. Multihop With Spatial Reuse

Multihop transmission is the most natural extension of single-hop transmission that can take advantage of the reduced attenuation between closely spaced relay nodes. In multihop transmission, each node utilizes capacity-achieving point-to-point codes to forward the most recently decoded message to its nearest neighbor in the direction of $\boldsymbol{D}$. Each codeword is received, decoded, and retransmitted by each relay $\boldsymbol{F}_i$, $i = 1, \dots, N-1$, until it is finally received and decoded at $\boldsymbol{D}$. To facilitate parallel transmission of several packets through the network, the available bandwidth is reused between transmitters, with a minimum separation of K nodes between simultaneously transmitting nodes ($2 \le K \le N$). When decoding the message, nodes $\boldsymbol{F}_i$ and $\boldsymbol{D}$ regard all signals not originating from the preceding node as Gaussian interference.

Suppose that $\boldsymbol{S}$ transmits with power P_0 and the relays $\boldsymbol{F}_i$, $i = 1, \dots, N-1$, with power P_i, respectively, so that $P_T = \frac{1}{K}\sum P_i$. Then the achievable end-to-end rate is the minimum of the rates achievable at each of the N hops

$$R = \min_{i=0,\dots,N-1} \left\{\frac{1}{K}\log_2\left(1 + \frac{P_i N^\alpha}{WN_0 + \sum_{s\in S_i} P_s N^\alpha |i+1-s|^{-\alpha}}\right)\right\} \tag{10}$$

where S_i denotes the set of nodes transmitting simultaneously with node $\boldsymbol{F}_i$, i.e.,

$$S_i = \{s \in \{0, 1, \dots, N-1\} | s \ne i \text{ and } K \text{ divides } i - s\}. \tag{11}$$

An optimal power allocation maximizing R is nontrivial to compute for arbitrary N, K, and P_T. However, for a large N and $K \ll N$, all nodes except those close to $\boldsymbol{S}$ and $\boldsymbol{D}$ have the same distances to their primary interferers, and so we can approximate the optimal power allocation as uniform. Though suboptimal, the uniform allocation yields the useful lower bound on (10)

$$R > \frac{1}{K}\log_2\left(1 + \frac{P_T K N^{\alpha-1}}{WN_0 + P_T K N^{\alpha-1} Z(K,\alpha)}\right) \tag{12}$$

where

$$Z(K,\alpha) = \sum_{s=1}^{\infty}(sK+1)^{-\alpha} + \sum_{s=1}^{\infty}(sK-1)^{-\alpha}. \tag{13}$$

Similarly, we also assume the uniform power allocation when $P_T \to 0$, since the vanishing interference terms in (10) will make all hops symmetric.

1) Performance of Multihop in the Power-Limited Regime: The derivations presented in the Appendix demonstrate that, under the optimal power allocation, the multihop system described by (10) in the power-limited regime has

$$\frac{E_b^{\mathrm{MH}}(0)}{N_0} = N^{-\alpha+1}\ln 2 \tag{14}$$

and

$$\frac{E_b'^{\mathrm{MH}}(0)}{N_0} = (\ln 2)^2 N^{1-\alpha} K\left(\frac{1}{2} + \frac{1}{N}\sum_{i=0}^{N-1}\sum_{s\in S_i}|i+1-s|^{-\alpha}\right). \tag{15}$$

Based on (14) it is clear that multihop can take advantage of the increased node density, and that it provides $10(\alpha-1)\log_{10} N$ decibel power savings relative to single-hop. The $10(\alpha-1)$ decibel gain resulting from a tenfold increase in node density falls just short of the 10α decibel gain indicated by the outer bound (7).

Since (14) does not depend on the spatial reuse parameter K, the size of the network N is the primary factor in deciding which multihop variant is most power efficient—higher N always means higher power efficiency. However, among schemes with the same N but different K, the most power-efficient scheme can be identified by the least value of $E_b'(0)$ in (15). By numerically evaluating (15) for $N \ge 3$, $2 \le K \le N$, and $2 \le \alpha \le 4$, we determined that in each case $K = 3$ is the optimal choice for the power-limited regime. The power efficiency–bandwidth efficiency characteristics for low rates R and for selected values of N and K are plotted in Fig. 2.

2) Performance of Multihop, $K = N$, in the Bandwidth-Limited Regime: To examine the bandwidth efficiency of multihop, we need to distinguish between two cases: $K = N$ and $K < N$. In the first case, each transmission takes place in a separate slot, and receivers do not have to deal with interference. Since all hops have the same distance (attenuation), the same noise level, and no interference, the uniform power allocation is optimal. The bandwidth efficiency–power efficiency characteristic (10) consequently simplifies to

$$R = N^{-1}\log_2\left(1 + \frac{RE_b N^\alpha}{N_0}\right) \tag{16}$$

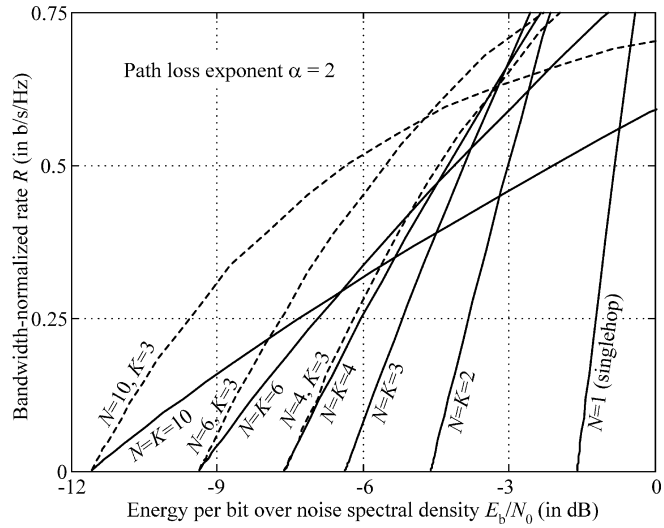

Fig. 2. Performance of the multihop scheme in the power-limited regime for $\alpha = 2$ and selected values of N and K.

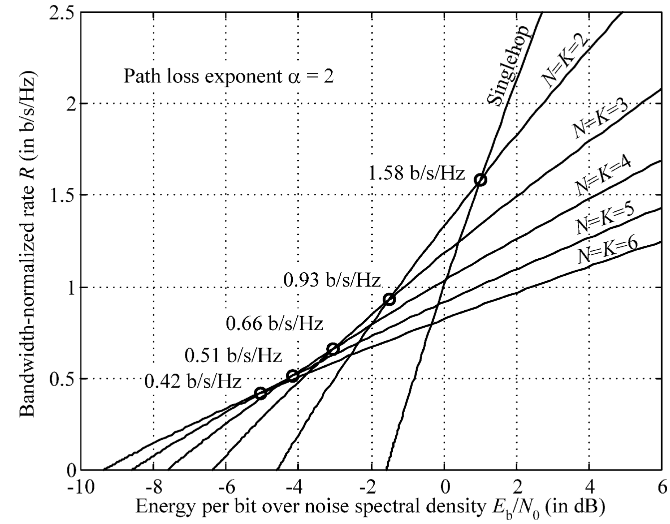

Fig. 3. Performance of the multihop scheme with $K = N$ in the bandwidth-limited regime for $\alpha = 2$.

and

$$\frac{E_b^{\mathrm{MH},K=N}(R)}{N_0} = \frac{2^{RN}-1}{RN^{\alpha}}. \tag{17}$$

The preceding formulas suggest a tradeoff involved in the choice of N—increasing the number of nodes N leads to a shorter inter-node distance and a larger effective received power, at the cost of decreasing the slot duration.

For R sufficiently large, the expression $N^{-\alpha}2^{RN}$ is always increasing in N and so is $E_b(R)$ in (17). Following Definition 4 exactly, we conclude that multihop with $K = N$ is more bandwidth efficient for lower values of N, with the single-hop scheme being the most bandwidth efficient. However, there are still some values of R for which it might be beneficial to choose $N > 1$. Fig. 3, in which we plot the characteristic (16) for several values of N, suggests that there is an interval of rates R for which each given N is optimal. Indeed, we have shown in [22] that, for a given rate R, the number of nodes yielding the least required power can be approximately computed as

$$N_{\mathrm{opt}} \approx \left[R^{-1}\frac{\alpha + \mathcal{W}(-\alpha e^{-\alpha})}{\ln 2}\right]_+ \tag{18}$$

where $[x]_+$ denotes the positive integer closest to x and $\mathcal{W}(x)$ is the principal branch of the Lambert W function. The approximation (18) is obtained by replacing the discrete variable N by the continuous variable N^* in (17) and minimizing E_b by setting the derivative $\partial E_b/\partial N^*$ to zero. This yields

$$2^{RN^*}(RN^*\ln 2 - \alpha) + \alpha = 0 \tag{19}$$

which can be solved for RN^*, giving

$$RN^* = \frac{\alpha + \mathcal{W}(-\alpha e^{-\alpha})}{\ln 2}. \tag{20}$$

The result (18) is obtained from (20) by using the approximation $N_{\mathrm{opt}} \approx [N^*]_+$.

Additionally, we can compute the exact value of R above which single-hop achieves a lower E_b and below which multihop with $N = 2$ performs better. This R must give the same value of E_b in (17) for $N = 1$ and $N = 2$, i.e., it must satisfy

$$\frac{2^R-1}{R} = \frac{2^{2R}-1}{2^{\alpha}R}. \tag{21}$$

After assuming $R > 0$ and rearranging the terms of (21) we obtain

$$(2^R)^2 - 2^{\alpha}2^R + 2^{\alpha} - 1 = 0 \tag{22}$$

which has an admissible solution $R = \log_2(2^{\alpha}-1)$. Hence, for rates $R > \log_2(2^{\alpha}-1)$, single-hop transmission outperforms multihop with $K = N$, for any $N \geq 2$. A yet simpler sufficient condition is $R > \alpha$.

3) Performance of Multihop, $K < N$, in the Bandwidth-Limited Regime: The shape of the power efficiency–bandwidth efficiency characteristic for multihop transmission changes dramatically if we choose $K < N$. In this setting, at least one node will be receiving the usable signal corrupted not only by thermal noise, but also by interference from another node. If our goal is to attain high rates R using high signal power, the hops involving interfering transmitters will become the bottlenecks of the network. In fact, if $K < N$, rates above some $R_\infty(N,K)$ can never be achieved by multihop, even if infinitely large transmit power is available. Hence, according to Definition 4, multihop with $K < N$ is always less bandwidth efficient than multihop with $K = N$ and single-hop.

If the desired communication rate falls below $R_\infty(N,K)$, multihop with $K < N$ may still provide considerable savings in E_b. It is interesting to study what choice of K results in the largest $R_\infty(N,K)$. Suppose we parametrize the power allocation as $P_i = \beta_i K P_T$, $i = 0,\ldots,N-1$, with the new constraint $\sum \beta_i = 1$. We can obtain $R_\infty(N,K)$ by substituting these P_i into (10), taking the limit $P_T \to \infty$, and maximizing this rate over all fractional power allocations β_i. We performed this maximization using a numerical search for several values of N, and the results are plotted in Fig. 4.

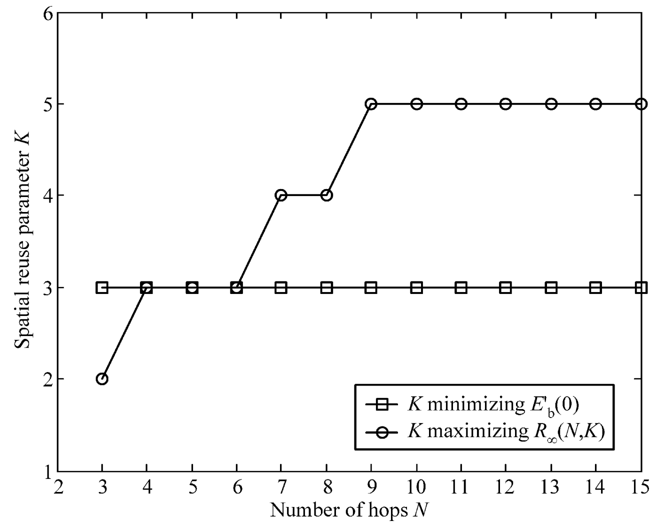

Fig. 4. Choices of K minimizing $E_b'(0)$ and maximizing R_∞. (Note: The same values of K turned out to be optimal for $2 \leq \alpha \leq 4$.

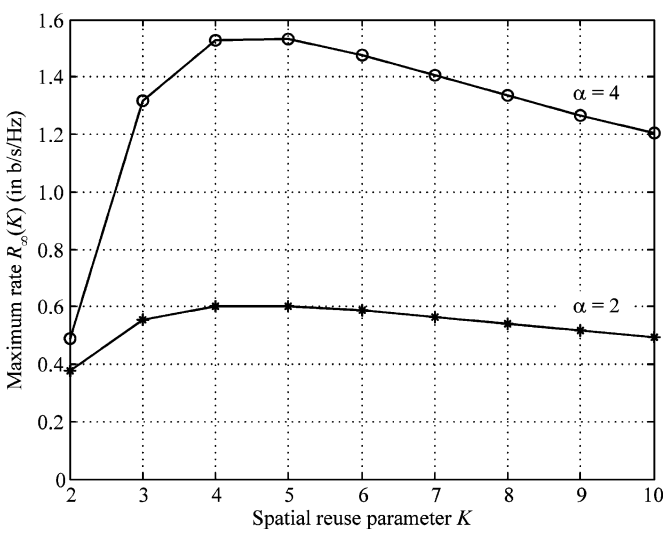

Fig. 5. Maximum rate $R_\infty(K)$ as a function of the spatial reuse parameter K for very large N.

For very large N ($N \gg K$), the uniform power allocation becomes asymptotically optimal, and $R_\infty(K, N)$ can be accurately approximated as

$$R_\infty(K, N) \approx R_\infty(K) \triangleq \frac{1}{K} \log_2 \left(1 + \frac{1}{Z(K, \alpha)}\right) \tag{23}$$

which does not depend on N. The values of $R_\infty(K)$ versus K are plotted in Fig. 5 for $\alpha = 2$ and $\alpha = 4$. In both cases, the highest value of $R_\infty(K)$ is achieved for $K = 5$.

It is interesting to observe that there are three distinct rate regimes, each of which requires a different multihop strategy. If the rate is above $\log_2(2^\alpha - 1)$, single-hop transmission offers the smallest E_b. If the rate is less than $\log_2(2^\alpha - 1)$ but above $R_\infty(5)$, some combination of N and K yields the smallest E_b. However, if the desired rate is below $R_\infty(5)$, transmission can be performed with arbitrarily small E_b, provided that $K = 5$ and N is sufficiently large.

V. Multihop With Interference Cancellation

Interference cancellation (IC) is a powerful technique that can significantly improve the performance of schemes aiming at high spectral efficiency. Schemes performing full IC can be costly in terms of memory, complexity, and delay, and they are much more susceptible to error propagation. Even though in this paper we regard them as less practical than schemes without IC, they are of interest from an information-theoretic perspective for establishing the range of rates and energies per bit that are achievable.

Successful IC is only possible if the interfering signal and the channel between the interferer and the receiver are known with very high reliability to the IC module in the receiver. In our system model, the receivers know the channel state for the incoming signals by assumption, but if the interfering signal can be determined, the channel state can be reliably estimated with a practical receiver by using this signal as a pilot sequence.

In this section, we incorporate the IC technique into the multihop scheme with $K < N$. Among the interfering signals at node $\boldsymbol{F}_i$ (i.e., all signals arriving at $\boldsymbol{F}_i$ from nodes different than $\boldsymbol{F}_{i-1}$), it is convenient to distinguish between *downstream interference* (from $\boldsymbol{F}_k$, $k > i$) and *upstream interference* (from $\boldsymbol{F}_k$, $k < i - 1$, and, if $i > 1$, from $\boldsymbol{S}$). Node $\boldsymbol{D}$ receives only upstream interference. Since the downstream nodes are always transmitting messages that were already decoded at the current node, the downstream interfering signals are known exactly (if no transmission errors occurred), and hence they can be canceled without additional processing. All that is required at each node is to keep track of the last N decoded messages. Removal of the upstream interference turns out to be more challenging and is addressed next.

A. Multihop With Genie-Aided Interference Cancellation

Suppose that each receiving node could employ a genie to determine all upstream interference signals and cancel them. Multihop transmission would then consist of N hops with identical attenuation and noise power, making the uniform power distribution optimal. After removing the interference terms from (10), the achievable performance is described by

$$R = \frac{1}{K} \log_2 \left(1 + \frac{P_T}{W N_0} K N^{\alpha - 1}\right) \tag{24}$$

and

$$\frac{E_b^{IC}(R)}{N_0} = \frac{2^{KR} - 1}{R K N^{\alpha - 1}}. \tag{25}$$

The above equations show that, with perfect IC, multihop benefits from increasing N at all rates (not just below some R_∞).[1] Also, not surprisingly, since the rate (24) is strictly greater than (10) for a fixed N, K, and E_b, multihop with perfect IC is both more power efficient and more bandwidth efficient than multihop without IC.

For the power-limited regime we can compute

$$\frac{E_b^{IC}(0)}{N_0} = N^{1-\alpha} \ln 2 \tag{26}$$

[1]Note that (24) still relies on point-to-point coding. Higher rates can be achieved with synchronous cooperation and/or sliding window decoding.

and

$$\frac{E_b'^{IC}(0)}{N_0} = \frac{(\ln 2)^2}{2} K N^{1-\alpha}. \tag{27}$$

Expression (27) shows that decreasing K improves power efficiency, and hence the most power-efficient choice is $K = 2$ ($K = 1$ is not allowed since it would require simultaneous transmission and reception at relay nodes). The same choice of K turns out to be optimal for the bandwidth-limited regime, since, for fixed P_T and N, the rate in (24) is strictly decreasing in K for $K \geq 2$. Surprisingly, according to Definition 4, single-hop is still more bandwidth efficient than multihop without IC for any fixed N, i.e., there exists a rate $R_0(N)$ above which single-hop requires a lower E_b. For $K = 2$, this $R_0(N)$ satisfies

$$\frac{2^{2R_0(N)} - 1}{N^{\alpha-1}} = 2^{R_0(N)} - 1 \tag{28}$$

which then gives

$$R_0(N) = \log_2(2N^{\alpha-1} - 1). \tag{29}$$

For larger N we have $R_0(N) \approx 1 + (\alpha - 1)\log_2 N$, which can be relatively large, especially for higher α. Thus, practical scenarios in which single-hop could actually outperform multihop with IC are rare.

B. Multihop With Recursive Backward Interference Cancellation

In this subsection, we present a new IC scheme based on backward decoding [15]–[17] that allows for cancellation of the upstream interference at an arbitrarily small rate loss and does not require the aid of a genie to achieve (24). The scheme is constructed recursively, with each recursion allowing interference-free communication over an additional hop. The scheme is then complete after N such recursions. The possibility of extending backward decoding to multiple hops using nested blocks was first suggested in [7] in the context of the multiple-relay channel and block Markov encoding, but the authors focused on sliding-window decoding and did not develop the idea further. The main advantage of recursive backward IC in our scenario is the fact that it relies exclusively on buffering, IC, and point-to-point decoding.

For the sake of a clearer presentation, we first describe this process for a linear network in which full-duplex communication (simultaneous transmission and reception at the relay nodes) is allowed, along with a spatial reuse parameter $K = 1$, even though our system model explicitly prohibits such a setting. We will later argue that the same principle applies to half-duplex networks.

Let us fix N and assume a uniform power allocation of P_T/N per node. The basic building block for our scheme will be a point-to-point coding scheme that achieves rate $R = \log_2(1 + P_T N^{\alpha-1}/WN_0)$ over a hop between two neighboring nodes when no interference is present. We assume a finite block length, which will later be made arbitrarily large to drive the probability of error to zero. Suppose that we are first interested only in delivering the messages from the source node $\boldsymbol{S}$ to the first relay $\boldsymbol{F}_1$ using the coding scheme mentioned above. Since $\boldsymbol{F}_1$ receives only downstream interference, it can cancel the interference and decode the message from $\boldsymbol{S}$ upon reception.

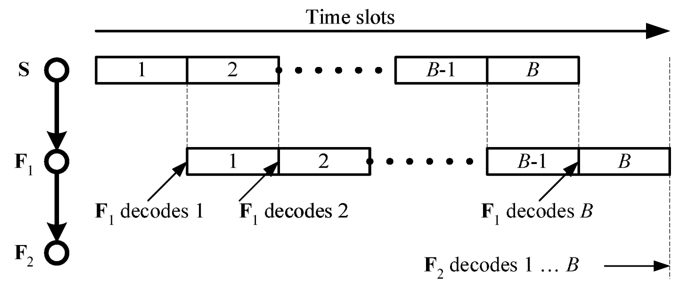

Fig. 6. Multihop with recursive backward IC up to $\boldsymbol{F}_2$.

In order to further deliver the message to relay $\boldsymbol{F}_2$, node $\boldsymbol{F}_1$ retransmits the message immediately after it receives and decodes it. Even though node $\boldsymbol{F}_2$ can cancel all the downstream interference, it will still receive upstream interference from $\boldsymbol{S}$. To deal with this interference, we can use the backward-decoding principle. Suppose that, after transmitting B blocks of data, the source node $\boldsymbol{S}$ does not transmit anything in slot $B + 1$, as illustrated in Fig. 6. During slot $B + 1$, the relay $\boldsymbol{F}_1$ retransmits message B, which is then received by $\boldsymbol{F}_2$ with no upstream interference. It is only at this point in time that $\boldsymbol{F}_2$ attempts to recover messages 1 to B based on previously received signals. First, it decodes message B and uses it to cancel the upstream interference corrupting message $B-1$, then it decodes message $B - 1$ and uses it to cancel the interference corrupting message $B - 2$, and so on. Eventually, all messages 1 to B are decoded by node $\boldsymbol{F}_2$.

The scheme just presented uses $B + 1$ time slots to deliver B data blocks from $\boldsymbol{S}$ to $\boldsymbol{F}_1$ and $\boldsymbol{F}_2$. We will now extend it to deliver B^2 data blocks in $(B + 1)^2$ time slots to nodes $\boldsymbol{F}_1$, $\boldsymbol{F}_2$, and $\boldsymbol{F}_3$. Suppose that node $\boldsymbol{F}_2$, after listening to the channel for $B + 1$ time slots and decoding B blocks of data, starts to retransmit these B blocks to node $\boldsymbol{F}_3$. During the same time nodes $\boldsymbol{S}$ and $\boldsymbol{F}_1$ proceed with the transmission of blocks $B+1$ to $2B$, causing upstream interference at $\boldsymbol{F}_3$. Node $\boldsymbol{F}_3$ just buffers the received signal, but does not attempt decoding. Eventually, after delivering B^2 data blocks to $\boldsymbol{F}_2$, nodes $\boldsymbol{S}$ and $\boldsymbol{F}_1$ stop transmitting for a duration of $B + 1$ time slots, as shown in Fig. 7. During these time slots, node $\boldsymbol{F}_3$ receives no upstream interference, and it can decode the data blocks $(B - 1)B + 1$ to B^2 received from $\boldsymbol{F}_2$. Using the backward decoding principle, it can then cancel these blocks from signals received earlier and continue decoding the initial B^2 blocks.

One more recursion is shown in Fig. 8. These recursive extensions continue until the messages reach the destination node $\boldsymbol{D}$. The final step involves transmitting B^{N-1} data blocks using $(B + 1)^{N-1}$ time slots. Since, for any N, the rate loss incurred by this scheme tends to zero as B goes to infinity, we have shown that the complete removal of all upstream interference is indeed possible at an arbitrarily small rate loss. We emphasize, however, that the delay of this scheme grows exponentially in N, which limits its applicability to extremely delay-insensitive applications.

The same principle of operation can be used in multihop transmission with $K > 1$. In fact, increasing K reduces the number of upstream interferers and can only result in a simplification of the algorithm (e.g., with careful scheduling it might be possible to extend the number of hops by K in each recursion).

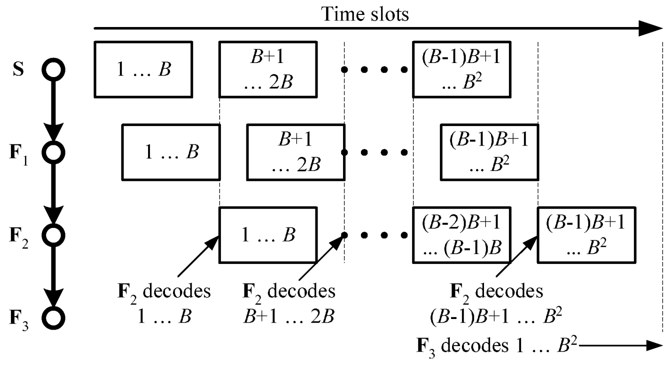

Fig. 7. Multihop with recursive backward IC up to $\boldsymbol{F}_3$.

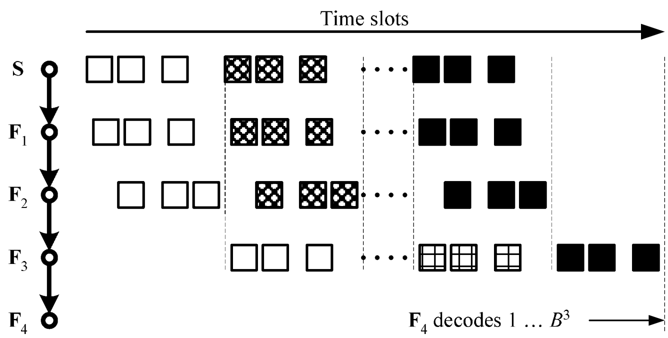

Fig. 8. Multihop with recursive backward IC up to $\boldsymbol{F}_4$.

Also, if $K \geq 2$, when $\boldsymbol{F}_i$ is transmitting, we are guaranteed that $\boldsymbol{F}_{i-1}$ is not attempting to send anything to $\boldsymbol{F}_i$, and any signal that $\boldsymbol{F}_i$ could receive at this time would be useless. Hence, for $K \geq 2$, this scheme works and performs identically in a half-duplex network, and all the results derived in the previous subsection for multihop with genie-aided IC apply without modification to multihop with recursive backward IC.

C. Upper and Lower Bounds on the Capacity of a Linear Wireless Network

The existence of a concrete scheme capable of complete removal of interference shows that the capacity of the considered network is lower-bounded by (24). Since (24) is $O(\log N)$ and the upper bound (6) is also $O(\log N)$, the capacity itself must also be $O(\log N)$. Since (6) is also an upper bound on the capacity of a linear network with synchronous cooperation, we have shown that synchronous cooperation is not a prerequisite for $O(\log N)$ rate growth. (Similar results were reported in [8].) The lower bound $R_{\min} \gtrsim \frac{\alpha-1}{2}\log_2 N$ and the upper bound $R_{\max} \lesssim \alpha \log_2 N$ have different proportionality constants, so at least one of them is not asymptotically tight. Nevertheless, we can conclude that multihop with IC is order-optimal in N.

VI. Conclusion

In this paper, we consider one-way communication between a single source and destination over a regular linear wireless network with $N-1$ relays. We adopt a conservative approach in deciding which communication techniques can be regarded as practical, by allowing only point-to-point coded transmission, conditionally allowing IC, and excluding the possibility of synchronous cooperation between nodes. In the class of networking schemes not employing IC, we analyze the performance of single-hop and multihop with spatial reuse. Additionally, we present a new information-theoretic scheme, called multihop with recursive backward IC, that removes all interference caused by spatial reuse at an arbitrarily small rate loss.

Our analysis reveals that multihop transmission performs very well in the power-limited regime but can become inefficient in the bandwidth-limited regime without IC. In the latter case, single-hop provides reliable communication at lower energy per bit than multihop whenever the required spectral efficiency exceeds $\log_2(2^\alpha - 1)$. The removal of interference with recursive backward IC improves the performance of multihop at higher rates and, at a fixed energy per bit, provides asymptotic rate growth of order $O(\log N)$.

The high-level conclusion is that physical layer and medium-access layer resource allocation, half-duplex transmission, and IC dramatically impact the power and bandwidth efficiency of multihop routing schemes. Thus, it is important to take these interdependencies into account when designing routing algorithms for real-world networks intended to operate in different regimes.

Appendix
Derivation of $E_b(0)$ and $E_b'(0)$ for Multihop Without Interference Cancellation

The main problem in the derivation of $E_b(0)$ and $E_b'(0)$ based on (10) is the necessity of finding the optimal power allocation. Let $P_i = \beta_i KRWE_b$, $i = 0, \ldots, N-1$, where $\beta_i(R)$ are all functions of R, satisfying $\sum \beta_i(R) = 1$. We will use the fact that the power allocation maximizing (10) makes all terms under the minimum operator equal. Hence, for all $i = 0, \ldots, N-1$

$$R = \frac{1}{K}\log_2\left(1 + \frac{\beta_i(R)RE_bKN^\alpha}{N_0 + RE_bKN^\alpha \sum_{s\in S_i}\beta_s(R)|i+1-s|^{-\alpha}}\right) \tag{30}$$

and

$$\frac{E_b(R)}{N_0} = \frac{(2^{KR}-1)K^{-1}N^{-\alpha}}{R\beta_i(R) - (2^{KR}-1)R\sum_{s\in S_i}\beta_s(R)|i+1-s|^{-\alpha}} \tag{31}$$

where S_i is the set of transmitters active simultaneously with node i, as defined in (11). By taking the limit as $R \to 0$ and applying l'Hôpital's rule, we obtain

$$\frac{E_b(0)}{N_0} = \beta_i(0)^{-1}N^{-\alpha}\ln 2 \tag{32}$$

where equality must hold for all i. Hence, for $R = 0$, the uniform power allocation is optimal, i.e., $\beta_i(0) = N^{-1}$, and

$$\frac{E_b(0)}{N_0} = N^{1-\alpha}\ln 2. \tag{33}$$

Next we want to find $E_b'(0)$ by first taking the derivative of (30) with respect to R and then taking the limit as $R \to 0$. After lengthy but straightforward calculations involving two applications of l'Hôpital's rule, we arrive at

$$\frac{E_b'(0)}{N_0} = \frac{K(\ln 2)^2}{N^\alpha}\left(\frac{1}{2\beta_i(0)} - \frac{\beta_i'(0)}{\beta_i^2(0)}K\ln 2 + \sum_{s\in S_i}\frac{\beta_s(0)}{\beta_i^2(0)}|i+1-s|^{-\alpha}\right) \quad (34)$$

and after substituting the known values for $\beta_i(0)$

$$\frac{E_b'(0)}{N_0} = \frac{K(\ln 2)^2}{N^{\alpha-1}}\left(\frac{1}{2} - \frac{N\beta_i'(0)}{K\ln 2} + \sum_{s\in S_i}|i+1-s|^{-\alpha}\right). \quad (35)$$

The above expression for $E_b'(0)$ must yield the same value independent of $i = 0, \ldots, N-1$, and the average over i also gives the same value. By taking the average and exploiting the fact that $\sum \beta_i' = 0$, we arrive at

$$\frac{E_b'(0)}{N_0} = \frac{K(\ln 2)^2}{N^{\alpha-1}}\left(\frac{1}{2} + \frac{1}{N}\sum_{i=0}^{N-1}\sum_{s\in S_i}|i+1-s|^{-\alpha}\right). \quad (36)$$

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