Nearest-Vehicle Communication in Regular Street Systems

Jeya Pradha Jeyaraj and Martin Haenggi

***Abstract*—We consider a vehicular network with an underlying regular street system formed by horizontally and vertically oriented streets. We model vehicle locations on each street using Poisson point processes, and each vehicle transmits with some probability following ALOHA. Each receiving vehicle connects to its partner-transmitting vehicle based on the following schemes: 1) Nearest-transmitter reception (NTR), 2) Nearest-transmitter reception with selective thinning (NTR-II), and 3) Nearest-receiver transmission (NRT). In NTR and NTR-II, each receiver receives from its nearest transmitter, whereas, in NRT, each transmitter transmits to its nearest receiver. NTR-II is a modified version of NTR, where the transmitters not closest to the receivers are idle. Under each scheme, we calculate the probabilities of successful reception for the general and intersection users, *i.e.,* vehicles not at intersections and at intersections. We study how the nearest-transmitter/receiver distance properties in each scheme differ with general and intersection users, and in turn, how these properties affect the probability of successful reception. Also, we show that we can obtain higher data rates in NTR and NTR-II compared to NRT in the high-reliability regime.**

***Index Terms*—Poisson point process, success probability, stochastic geometry, vehicle-to-vehicle communication.**

I. Introduction

Vehicle-to-vehicle (V2V) communication enables the vehicles to exchange safety messages or warnings to each other to improve the overall traffic safety and efficiency. The reliability of such a communication depends on the channel between vehicles, and locations of vehicles and their interferers. Furthermore, the vehicle locations are subject to considerable uncertainty. Hence, studying a specific instance of the network is insufficient to design and analyze the vehicular network. Stochastic geometry provides a mathematical tool set, specifically, point processes, that allows us to model uncertain, *i.e.,* random vehicle locations. It yields statistical features of the ensembles of vehicular network instances and analytical results that can be evaluated quickly to obtain useful design insights.

It is essential to consider the street geometry and not model only the vehicles using point processes [1]. Some street systems are more regular, while some are less regular. In this work, we focus on the reliability of nearest-vehicle communication in a regular street system. Reliability or success probability is defined as the probability that the vehicle can receive the message successfully. It is reported that almost 50% of accidents occurs at intersections [2]. Moreover, an intersection vehicle sees a different neighborhood than a vehicle not at an intersection. Taking this into account, we consider two kinds of users (vehicles)—the general user and the intersection user. The two-dimensional street system will help us understand the overall network behavior better than one-dimensional street models without intersections. We consider different schemes on how each vehicle chooses its partner vehicle for communication and derive the success probabilities of the general and intersection users under each scheme. We study how the nearest-vehicle distance properties in each scheme affect the success probabilities of the general and intersection users. Finally, we compare the performance of the considered partner-vehicle selection schemes.

Only a few works have focused on the reliability analysis in the two-dimensional street sytems. The reliability of V2V communication in regular street system is studied in [1] with each vehicle connecting to an another vehicle on the same street at a fixed distance. Authors in [3] study the connectivity properties of the vehicles in the irregular street system modeled by Poisson line processes (PLPs). The vehicles on each street are modeled using Poisson point processes (PPPs). In the same setting, [4] investigates the success probability of the general user that receives from its nearest transmitter. The works [1], [4] assume ALOHA as the MAC protocol. Reference [5] analyzes the statistical properties of the PLP such as the nearest distance distribution and the convergence of the typical Voronoi cell as the vehicle density increases.

The authors are with Department of Electrical Engineering, University of Notre Dame, Notre Dame, IN 46556, USA. E-mail : {jjeyaraj, mhaenggi}@nd.edu.

The work is partially supported by the U.S. National Science Foundation through grant CCF 15259204.

II. Network Model

Consider a regular street system, with horizontally and vertically oriented streets forming a square grid. The vehicles on each street form independent homogeneous 1-D PPPs of intensity λ. We refer to the model as *PPP on a square grid* (PPPSG) [1].

Definition 1 (PPPSG)**.** *Let*

$$L_\ell^\varphi \triangleq \{(x,y) \in \mathbb{R}^2 \colon x\cos\varphi + y\sin\varphi = \ell\}$$

denote a line in $\mathbb{R}^2$, where $\ell \in \mathbb{Z}$ is the location and $\varphi \in [0,\pi)$ is the direction of the line. For example, the x axis $(\mathbb{R},0)$ is $L_0^{\pi/2}$, and the y axis $(0,\mathbb{R})$ is L_0^0.

Let $\mathcal{P}_\ell^\varphi$ denote the 1-D PPP of intensity λ on the line L_ℓ^φ. For different ℓ or φ, the processes are independent. Then

$$\mathcal{P}_\mathbb{Z} \triangleq \bigcup_{k\in\mathbb{Z}} \mathcal{P}_k^0 \cup \mathcal{P}_k^{\pi/2}$$

is a system of horizontally and vertically oriented 1-D PPPs, where exactly one of the coordinates of each point is an integer.

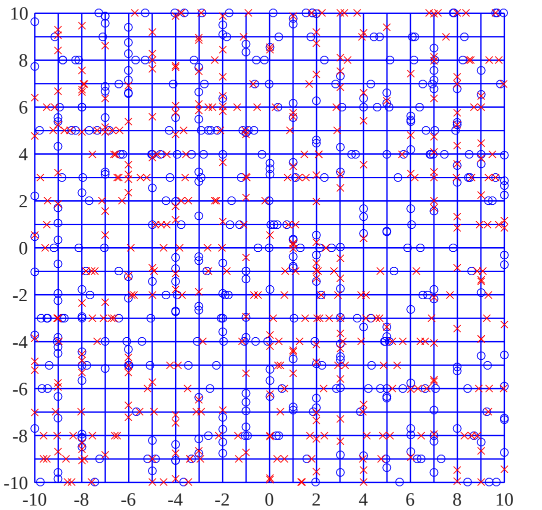

Figure 1. A realization of PPPSG. Transmitters and receivers on each street are denoted by 'o' and '×', respectively. $\lambda_{\rm t} = \lambda_{\rm r} = 0.5$, and $s = 1$.

To make the model stationary with variable two-dimensional intensity, the PPPSG is defined as

$$\mathcal{V} \triangleq s(\mathcal{P}_{\mathbb{Z}} + U),$$

where $s > 0$ *is the inter-street spacing and* U *is uniform on* $[0, 1)^2$*. The corresponding street system is defined as*

$$\mathcal{S} \triangleq s\left(\bigcup_{\ell \in \mathbb{Z}} L_\ell^0 \cup L_\ell^{\pi/2} + U\right).$$

1) Partner-Vehicle Selection Schemes: Each vehicle transmits with probability p at a time instant following the slotted ALOHA protocol unless otherwise stated. Then the intensity of transmitters and receivers on each street are $\lambda_{\rm t} = \lambda p$, and $\lambda_{\rm r} = \lambda(1 - p)$, respectively. Fig. 1 depicts the model. Each vehicle chooses its partner vehicle for communication based on the following schemes:

- *Nearest-transmitter reception (NTR):* Each receiver receives from its nearest transmitter.
- *Nearest-transmitter reception with selective thinning (NTR-II):* Each receiver receives from its nearest transmitter as in NTR. Here, we selectively thin the non-receiving vehicles of intensity λp to obtain the set of transmitters of intensity λpq, *i.e.,* we choose only the non-receiving vehicles closest to the receivers as transmitters. The non-chosen vehicles of intensity $\lambda p(1 - q)$ remain idle.
- *Nearest-receiver transmission (NRT):* Each transmitter transmits to its nearest receiver.

2) Types of Users: Our performance metric of interest is the success probability (reliability). It is defined as the probability that the signal-to-interference ratio (SIR) measured at the receiver exceeds a threshold θ that parametrizes the rate of transmission. For each transmission and each vehicle, the SIR is different. To define a meaningful network-wide metric, we focus on the SIR of a representative vehicle whose performance corresponds to the average of that of all vehicles. In point process theory, this representative vehicle is called 'the typical point' [6]. In our context, it is 'the typical vehicle' or 'the typical user.' We condition the typical user to be at the origin o.[1] In each realization, the user at the origin sees interferers at different locations. The reliability is evaluated by averaging over the point process of vehicle locations.[2]

We consider two types of users—(i) *The typical general user (TGU):* The origin is not an intersection almost surely, *i.e.,* we condition on the random translation U such that $U = [U_1\ 0]$. U_1 is uniform on [0,1) denoted by $U_1 \sim \mathcal{U}[0, 1)$. Then the location of the streets are dependent on $A \triangleq sU_1 \sim \mathcal{U}[0, s)$ (see Definition 1). (ii) *The typical intersection user (TIU):* The origin is an intersection with $U = [0\ 0]$. The term 'typical user' refers to both the TGU and the TIU unless otherwise stated.

The PPPSG is the Cox process [6, Sec. 3.3] with driving random measure $\Psi(B) = \lambda|B \cap \mathcal{S}|$, where $|\cdot|$ denotes the total line length. Let $\Psi_a^o(B)$ and $\Psi^+(B)$ denote the driving random measures of the TGU conditioned on $A = a$, and the TIU. $\Psi^+(B)$ is deterministic as $U = [0\ 0]$ for the TIU.

3) SIR: Without loss of generality, we order the streets by their shortest (perpendicular) distances to the origin and denote those distances as $r_0 \leq r_1 \leq \ldots$. For the intersection user, $r_0 = r_1 = 0$ as two streets pass through the user, whereas, for the general user, $r_0 = 0$. Let I_j denote the interference from the transmitting vehicles on the jth street. Then the total interference I at the typical user is $I = \sum_{j \in \mathbb{N}_0} I_j$, where $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$. The SIR at the typical user is

$$\mathsf{SIR} = \frac{S}{\sum\limits_{j \in \mathbb{N}_0} I_j} = \frac{h_x \|x\|^{-\alpha}}{\sum\limits_{j \in \mathbb{N}_0} \sum\limits_{z \in \mathcal{V}_j} h_z \|z\|^{-\alpha}}, \tag{1}$$

where x is the location of the intended transmitter, $h_{[.]}$ is the channel power gain exponentially distributed with mean 1, α is the path-loss exponent, and $\mathcal{V}_j$ is the set of transmitters on the jth street $\mathcal{S}_j$.

III. Success Probability

The success probability of the TIU is defined as

$$\begin{aligned} p_{\rm s} &\triangleq \mathbb{P}(\mathsf{SIR} > \theta) = \mathbb{P}(S > I\theta) \\ &= \mathbb{E}(\mathbb{P}(h_x > \theta R^\alpha I \mid R, I)) \\ &= \mathbb{E}(\exp(-\theta R^\alpha I)) \\ &\overset{(a)}{=} \mathbb{E}_R\left(\prod_{j \in \mathbb{N}_0} \mathbb{E}_{I_j}(\exp(-\theta R^\alpha I_j))\right) \\ &\overset{(b)}{=} \int_0^\infty \left(\prod_{j \in \mathbb{N}_0} \mathcal{L}_{I_j}(\theta R^\alpha)\right) f_R(r) \mathrm{d}r, \end{aligned} \tag{2}$$

where (a) follows from the independence of the 1-D PPPs, (b) follows from the definition of Laplace transform, and $f_R(r)$ is the probability density function corresponding to the the nearest-vehicle distance distribution $F_R(r)$. The nearest

[1] The user can be conditioned to be at any location as the street system is stationary (translation-invariant).

[2] We do not consider the mobility as the vehicle locations barely change in a single time slot.

vehicle refers to the nearest transmitter in NTR and NTR-II, and the nearest receiver in NRT. In case of the general user, the shortest distances of the streets to the origin are dependent on $A \sim \mathcal{U}[0, s)$. For example, $r_0 = 0$, $r_1 = A$, and $r_2 = s - A$. Then the success probability of the TGU is given by

$$p_{\mathrm{s}} = \int_0^s \int_0^\infty \left(\prod_{j \in \mathbb{N}_0} \mathcal{L}_{I_j}(\theta R^\alpha \mid A) \right) f_R(r \mid A) f_A(a) \mathrm{d}r \mathrm{d}a. \quad (3)$$

We obtain (2) from (3) by setting $f_A(a) = \delta(a)$, as $U = [0\ 0]$ for the intersection user. $\delta(a)$ denotes the Dirac delta function.

First, we consider the scenario where the vehicle chooses its partner vehicle on the same street.[3] Then, we extend the results to the general setup where the partner vehicle can be on the same or on a different street. For each case, we derive the the nearest-vehicle distance distribution, and the Laplace transform of the interference under the considered partner-vehicle selection schemes, with which we can evaluate the success probabilities of the TIU and the TGU given by (2) and (3), respectively.

A. Partner Vehicle on the Same Street

The probability $F_R(r)$ that the typical user has the nearest-vehicle within distance r is

$$\begin{aligned} F_R(r) &= \mathbb{P}(R \le r) = 1 - \mathbb{P}(R \ge r) \\ &= 1 - \mathbb{P}(\text{no vehicle within distance } r) \\ &= 1 - \exp(-2\mu r), \end{aligned} \quad (4)$$

where $\mu = \lambda_{\mathrm{t}} m$ for NTR, and NTR-II, with $m = 1$ for the general user and $m = 2$ for the intersection user as two streets pass through the user. For NRT, $\mu = \lambda_{\mathrm{r}} = \lambda(1-p)$. $\lambda_{\mathrm{t}} = \lambda p$ for NTR, and λ_{t} for NTR-II is given in Lemma 1.

Lemma 1. *The intensity of the transmitters in NRT-II is*

$$\lambda_{\mathrm{t}} = \lambda p \left(1 - \left(\frac{2p}{1+p} \right)^2 \right). \quad (5)$$

Proof: See Appendix A. ∎

In this scenario, $f_R(r) = 2\mu \exp(-2\mu r)$ is independent of A. Next, we will derive the Laplace transform of the interference from the same and different streets as that of the typical user.

Let $\mathcal{H}_j$ denote the region containing no interferers on the jth street based on our ordering of the shortest distances of the streets to the typical user. We condition on the nearest-transmitter distance R from the typical user. In NTR and NTR-II, since the partner-vehicle is the nearest transmitter on the same street(s), there are no interferers within distance R. Then $\mathcal{H}_0 = [-R, R]$, and $\mathcal{H}_{j \neq 0} = \emptyset$ for the TGU, and for the TIU, as two streets pass through the user, $\mathcal{H}_0 = \mathcal{H}_1 = [-R, R]$, and $\mathcal{H}_{j>1} = \emptyset$. On the other hand, in NRT, as each transmitter transmits to its nearest receiver, there may be interferers closer than the intended transmitter at R, *i.e.*, $\mathcal{H}_j = \emptyset\ \forall j$.

[3] 1-D highway model can be seen as a subset of the regular street system with only a vertical street, and each vehicle connecting to its partner-vehicle on the same street.

Proposition 1. *The Laplace transform of the interference from the same street(s) as that of the typical user in NTR and NTR-II is given by*

$$\prod_{j=0}^{m-1} \mathcal{L}_{I_j}(\theta R^\alpha) = \exp\left(-2m\lambda_{\mathrm{t}} R \left(g(\delta, \theta) - 1\right)\right), \quad (6)$$

and in NRT, it is given by

$$\prod_{j=0}^{m-1} \mathcal{L}_{I_j}(\theta R^\alpha) = \exp(-2m\lambda_{\mathrm{t}} R \theta^{\delta/2} f(\delta)), \quad (7)$$

where $\delta \triangleq 2/\alpha$, $g(\delta, \theta) = {}_2F_1(1, -\delta/2; 1-\delta/2; -\theta)$ *is the hypergeometric function,* $f(\delta) = \Gamma(1+\delta/2)\Gamma(1-\delta/2)$, *and* $m = 1$ *and* 2 *for the TGU and the TIU, respectively.*

Proof: The Laplace transform of the interference from the same street as that of the TGU is

$$\begin{aligned} \mathcal{L}_{I_0}(\theta R^\alpha) &= \mathbb{E}(\exp(-\theta R^\alpha I_0)) \\ &= \mathbb{E}\left[\prod_{z \in \mathcal{V}_0 \setminus \mathcal{H}_0} \mathbb{E}_h \left(\exp\left(-\theta R^\alpha h_z \|z\|^{-\alpha}\right) \right) \right] \\ &= \mathbb{E}\left[\prod_{z \in \mathcal{V}_0 \setminus \mathcal{H}_0} \frac{1}{1 + \theta R^\alpha |z|^{-\alpha}} \right] && (8) \\ &\overset{(a)}{=} \exp\left(-2\lambda_{\mathrm{t}} \int_R^\infty \frac{1}{1 + \left(\frac{u^2}{R^2\theta^\delta}\right)^{1/\delta}} \mathrm{d}u \right) && (9) \\ &\overset{(b)}{=} \exp\left(-2\lambda_{\mathrm{t}} R \theta^{\delta/2} \int_{\theta^{-\delta/2}}^\infty \frac{1}{\left(1 + v^{2/\delta}\right)} \mathrm{d}v \right) \\ &= \exp\left(-2\lambda_{\mathrm{t}} R \left({}_2F_1(1, -\delta/2; 1-\delta/2; -\theta) - 1\right)\right), && (10) \end{aligned}$$

where (a) applies the probability generating functional (PGFL) of the PPP, and (b) results from the change of variable $v = \frac{u}{R\theta^{\frac{\delta}{2}}}$. We observe that (10) is independent of the orientation of the street φ. Then, $I_0 = I_1$ in distribution for the intersection user as $r_0 = r_1 = 0$. Hence, we can express the corresponding Laplace transform of the interference from the same streets as

$$\begin{aligned} &\mathcal{L}_{I_0}(\theta R^\alpha) \mathcal{L}_{I_1}(\theta R^\alpha) = \mathcal{L}^2_{I_0}(\theta R^\alpha) && (11) \\ &= \exp\left(-4\lambda_{\mathrm{t}} R \left({}_2F_1(1, -\delta/2; 1-\delta/2; -\theta) - 1\right)\right). && (12) \end{aligned}$$

Summarizing (10) and (12), we obtain (6). For NRT, $\mathcal{H}_j = \emptyset$ $\forall j$. Based on (8) and (9), we write the Laplace transform of the interference from the same street for the TGU as

$$\begin{aligned} \mathcal{L}_{I_0}(\theta R^\alpha) &= \exp\left(-2\lambda_{\mathrm{t}} \int_0^\infty \frac{1}{1 + \left(\frac{u^2}{R^2\theta^\delta}\right)^{1/\delta}} \mathrm{d}u \right) \\ &= \exp(-2\lambda_{\mathrm{t}} R \theta^{\delta/2} \Gamma(1+\delta/2)\Gamma(1-\delta/2)), \end{aligned}$$

and using the same approach as in (11) for the TIU, we obtain (7). ∎

Proposition 2. *The Laplace transform of the interference from a different street than that of the typical user in all the schemes is expressed as*

$$\mathcal{L}_{I_j}(\theta R^\alpha) = \exp\left(-\lambda_t R\theta^{\delta/2} \int_{v_0}^{\infty} \frac{1}{\left(1+v^{1/\delta}\right)\sqrt{v-v_0}} \mathrm{d}v \right), \quad (13)$$

where $\delta \triangleq 2/\alpha$, *and* $v_0 = r_j^2 R^{-2}\theta^{-\delta}$.

Proof: See Appendix B. ■

For $\alpha = 2$ and 4, (13) simplifies to

$$\mathcal{L}_{I_j}(\theta R^2) = \exp\left(\frac{-\pi\lambda_t R^2\theta}{\sqrt{r_j^2 + R^2\theta}}\right),$$

$$\mathcal{L}_{I_j}(\theta R^4) = \exp\left(\frac{-\pi\lambda_t R^2\sqrt{\theta}\sin\left(\frac{1}{2}\arctan\left(\frac{R^2\sqrt{\theta}}{r_j^2}\right)\right)}{\left(r_j^4 + R^4\theta\right)^{\frac{1}{4}}}\right).$$

B. Partner Vehicle on Any Street

In this scenario, the partner vehicle can be either on the same or on a different street. Then $\mathcal{H}_j = \mathcal{S}_j \cap b(o,R)$ for NTR and NTR-II, where $b(o,R)$ denotes the ball of radius R centered at the origin o. In NRT, $\mathcal{H}_j = 0\ \forall j$ as in III-A.

The Laplace transform of the interference from the same street(s) is the same as in Proposition 1, and that of from a different street that does not intersect with $b(o,R)$, *i.e.,* $\mathcal{H}_j = \emptyset$, is the same as in Proposition 2. For a different street that intersects with $b(o,R)$, we obtain the corresponding Laplace transform of the interference as

$$\mathcal{L}_{I_j}(\theta R^\alpha) = \exp\left(-\lambda_t R\theta^{\delta/2} \int_{\theta^{-\delta}}^{\infty} \frac{1}{\left(1+v^{1/\delta}\right)\sqrt{v-v_0}} \mathrm{d}v \right),$$

where $v_0 = r_j^2 R^{-2}\theta^{-\delta}$. We omit the detailed proof as it is similar to the proof of Proposition 2 given in Appendix B, and $\lambda_t = \lambda p$ in NTR and NRT. The analytical expression for λ_t in NTR-II is intractable in this setup, but we can evaluate it through Monte-Carlo simulations. Next, we will analyze the nearest-vehicle distance distributions in the considered partner-vehicle selection schemes.

1) Nearest-Vehicle Distance Distribution in NTR and NTR-II: The nearest-transmitter distance distribution for the TIU is given by

$$\begin{aligned} F_R(r) &= 1 - \mathbb{P}(\text{no transmitter within } b(o,r)) \\ &= 1 - \exp(-\lambda_t \Psi^+(b(o,r))), \end{aligned} \quad (14)$$

where $\Psi^+(b(o,r))$ is the total length of the streets that intersect $b(o,r)$ (see Section II-2). The total number of segments that intersect $b(o,r)$ is $2+4(\lceil\frac{r}{s}\rceil-1)$. The two street segments that pass through o have a length of $2r$ each. The street segment which is at a distance s from the origin is of length $2\sqrt{r^2-s^2}$. Similarly, we can find the length of the other segments using their shortest distances to the origin and r. Therefore, $\Psi^+(b(o,r))$ is given by

$$\Psi^+(b(o,r)) = 4r + 8\sum_{k=1}^{\lceil\frac{r}{s}\rceil-1}\sqrt{r^2-(ks)^2}. \quad (15)$$

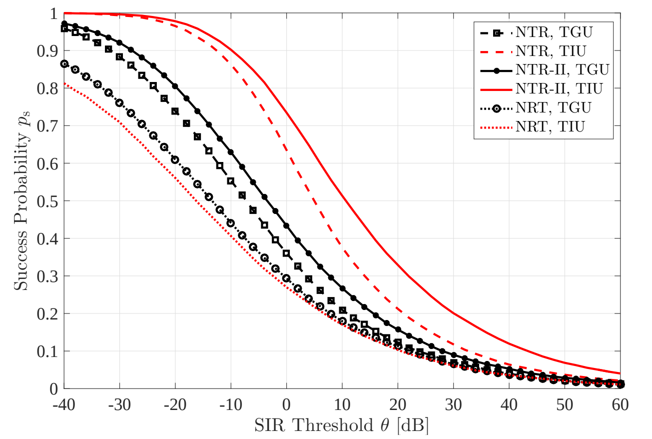

Figure 2. Comparison of the schemes when the partner vehicle is on the same street. $\lambda = 1$, $p = 0.5$, and $s = 1$.

In case of the general user, conditioning on A, we obtain $F_R(r \mid A = a) = 1 - \exp(-\lambda_t \Psi_a^o(b(o,r)))$. We can compute $\Psi_a^o(b(o,r))$ similar to (15) as

$$\Psi_a^o(b(o,r)) = 2r + 2r\mathbb{1}_A(0) + 2\sum_{k=0}^{\lceil\frac{r-a}{s}\rceil-1}\sqrt{r^2-(ks+a)^2} + 2\sum_{k=1}^{\lceil\frac{r+a}{s}\rceil-1}\sqrt{r^2-(ks-a)^2} + 4\sum_{k=1}^{\lceil\frac{r}{s}\rceil-1}\sqrt{r^2-(ks)^2}, \quad (16)$$

where $\mathbb{1}_A(0) = 1$ if $A = 0$, and 0 otherwise. Note that $\Psi_0^o(b(o,r)) = \Psi^+(b(o,r))$.

2) Nearest-Vehicle Distance Distribution in NRT: Here, we are interested in the nearest-receiver distance distribution of a transmitter whose location is varying. Since the street system is stationary, we need to average over only the transmitter locations $Y \subset [0,s)$ to evaluate the nearest-receiver distribution. We can do this either by conditioning $A = 0$ and averaging over Y or conditioning on Y to be at the origin and averaging over A. Then we can express $F_R(r)$ as

$$\begin{aligned} F_R(r) &= 1 - \mathbb{E}(\mathbb{P}(\text{no receiver within } b(Y,r) \mid A = 0, Y)) \\ &= 1 - \mathbb{E}(\exp(-\lambda_r \Psi_0^o(b(Y,r)))), \\ &= 1 - \mathbb{E}(\exp(-\lambda_r \Psi_A^o(b(o,r)))), \end{aligned} \quad (17)$$

where $\Psi_A^o(b(o,r))$ conditioned on $A = a$ is given by (16). We observe from (17) that $F_R(r \mid A) = F_R(r)$ unlike in the other schemes.

IV. Results and Discussion

Figs. 2 and 3 compare the success probabilities of the TGU and the TIU for all the schemes when the partner vehicle is on the same street and on any street, respectively. The success probability of the typical user is higher when the partner vehicle can be on any street for all the schemes. The reason is that the user closer to the intersection has the flexibility to connect with the nearest-vehicle on the streets adjoining the intersection as well than that of only on the same street. In both NTR and NTR-II, the TIU has a higher success probability

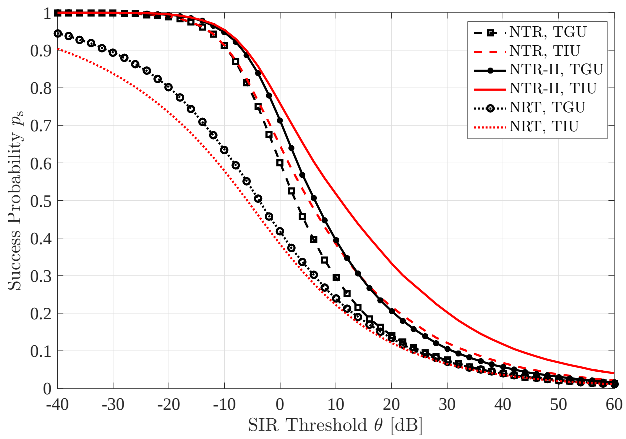

Figure 3. Comparison of the schemes when the partner vehicle can be on any street. $\lambda = 1$, $p = 0.5$, and $s = 1$.

than the TGU. As two streets pass through the intersection user, the probability that it finds the nearest-transmitter closer is higher than that of the general user. In contrast, the TGU has a higher success probability than the TIU in NRT. Since the transmitter transmits to its nearest-receiver, there can be an interferer closer than the intended transmitter. The intersection user has higher chances of having an interfering transmitter closer than the general user, reducing its success probability.

From Figs. 2 and 3, we see that NTR-II performs better than NTR as the transmitters not closest to any of the receivers are idle. In the high-reliability regime ($p_{\mathrm{s}} \to 1$ or $\theta \to 0$), for the SIR to be greater than θ, there should not be any interferers in a small disk around the typical user. When the vehicle has the flexibility to choose its partner from any street, there are no interferers on the different streets closer than the intended transmitter. This explains the diminishing gap between NTR and NTR-II as $\theta \to 0$ in Fig. 3. The flatness as $\theta \to 0$ indicates that we can obtain higher data rates when receiving from the nearest-transmitter than in NRT.

V. Conclusions

We considered a two-dimensional regular street system involving intersections with vehicles on each street forming Poisson point processes. We studied the reliability of the nearest-vehicle communication under different schemes that govern how each vehicle chooses its partner vehicle for communication. Specifically, we derived the success probabilities of the general and intersection users under each scheme. Using the analytical expressions for the success probability, one can determine the fraction of vehicles that satisfies a certain reliability/rate constraint. We showed that the reliability of the inter-vehicle communication on the same street is worse than the communication between arbitrary nearest vehicles. Furthermore, the performance of the ALOHA-based MAC schemes considered here can be viewed as lower bounds to the performance of the CSMA-based MAC schemes used in vehicle-to-vehicle communication protocols.

Appendix

A. *Proof of Lemma 1*

In NTR-II, only the non-receiving vehicles closest to the receiving vehicles transmit. The Voronoi cell of a non-receiving vehicle Z contains the receiving vehicles whose distances to Z are smaller than their distances to any other non-receiving vehicle. Then the probability that Z is not retained is the same as the probability that its Voronoi cell is empty. Let N denote the number of receiving vehicles in the Voronoi cell of Z. The probability density function corresponding to the size of the Voronoi cell X normalized by $1/\lambda p$ is given by $f_X(x) = 4x\exp(-2x)$ [7]. The probability $1-q$ that Z is not retained is

$$
\begin{aligned}
1-q = \mathbb{P}(N=0) &= \int_0^\infty \mathbb{P}(N=0 \mid X=x) f_X(x)\mathrm{d}x, \\
&= \int_0^\infty \exp\left(-\frac{\lambda_{\mathrm{r}} x}{\lambda p}\right) 4x\exp(-2x)\mathrm{d}x, \\
&= \frac{4}{\left(2+\frac{\lambda_{\mathrm{r}}}{\lambda p}\right)^2} = \left(\frac{2p}{1+p}\right)^2.
\end{aligned}
$$

The intensity of the non-receiving vehicles chosen to transmit is $\lambda_{\mathrm{t}} = \lambda p q$ as shown in (5).

B. *Proof of Proposition 2*

Similar to (8), we can express the Laplace transform of the interference from a different street as

$$
\begin{aligned}
\mathcal{L}_{I_j}(\theta R^\alpha) &= \mathbb{E}\left[\prod_{z\in\mathcal{V}_j\setminus\mathcal{H}_j} \frac{1}{1+\theta R^\alpha \|z\|^{-\alpha}}\right] \\
&\overset{(c)}{=} \exp\left(-2\lambda_{\mathrm{t}} \int_0^\infty \frac{1}{1+\left(\frac{r_j^2+u^2}{R^2\theta^\delta}\right)^{1/\delta}}\mathrm{d}u\right) \\
&\overset{(d)}{=} \exp\left(-\lambda_{\mathrm{t}} R\theta^{\delta/2} \int_{\frac{r_j^2}{R^2\theta^\delta}}^\infty \frac{1}{\left(1+v^{1/\delta}\right)\sqrt{v-\frac{r_j^2}{R^2\theta^\delta}}}\mathrm{d}v\right),
\end{aligned}
$$

where (c) follows from $\mathcal{H}_j = \emptyset$ and the PGFL of the PPP, and (d) is due to the change of variable $v = \frac{r_j^2+u^2}{R^2\theta^\delta}$.

References

[1] J. P. Jeyaraj and M. Haenggi, "Reliability analysis of V2V communications on orthogonal street systems," in *2017 IEEE Global Communications Conference (GLOBECOM), Singapore*, Dec. 2017.

[2] U. S. Dept. of Transportation, "Traffic safety facts 2015," Jan. 2017. [Online]. Available: https://crashstats.nhtsa.dot.gov/Api/Public/Publication/812384.

[3] D. H. Thuen, H. V. Cuu, and H. N. Do, "VANET modelling and statistical properties of connectivity in urban environment," *REV Journal on Electronics and Communications*, vol. 3, no. 1-2, Jun. 2013.

[4] V. V. Chetlur and H. S. Dhillon, "Coverage analysis of a vehicular network modeled as Cox process driven by Poisson line process," 2017. [Online]. Available: http://arxiv.org/abs/1709.08577.

[5] C.-S. Choi and F. Baccelli, "Poisson Cox point processes for vehicular networks." [Online]. Available: https://arxiv.org/abs/1801.04556.

[6] M. Haenggi, *Stochastic Geometry for Wireless Networks*. Cambridge University Press, 2012.

[7] J.-S. Ferenc and Z. Néda, "On the size-distribution of Poisson Voronoi cells," *Physica A: Statistical Mechanics and its Applications*, vol. 385, no. 2, pp. 518–526, 2007.