Coherent Joint Transmission in Downlink Heterogeneous Cellular Networks

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***Abstract*—The analysis of the success probability or signal-to-interference ratio (SIR) distribution for coherent joint transmission (JT) based on stochastic geometry is an open issue. In this letter, we study coherent JT in downlink heterogeneous cellular networks and provide an upper bound of the success probability for the general user and the worst-case user (cell-corner user), and an approximation for the general user. Simulation results show that the derived upper bound and approximation are quite accurate and thus provide an analytical approach to quantify the SIR performance of coherent JT.**

***Index Terms*—Coherent joint transmission, CoMP, success probability, HetNets, stochastic geometry.**

I. Introduction

According to Cisco's forecast, global mobile data traffic will increase sevenfold between 2016 and 2021 [1]. The growing demand for mobile data traffic drives the densification and heterogeneity of cellular networks, which leads to additional inter-cell interference (ICI) [2]. Coordinated multipoint transmission/reception (CoMP) is a key technology in 3GPP LTE to manage the ICI and and enhance the cell edge coverage in cellular networks [3], especially joint transmission (JT) as one of the downlink CoMP transmission technologies. In the framework of 3GPP LTE, JT is categorized into coherent and non-coherent JT [4]. For coherent JT, it is assumed that the BSs in the cooperation set have detailed channel state information (CSI) of the serving links from the BSs in the cooperation set, which is a subset of all BSs, to the same single user [5]. Based on the CSI shared among all cooperating BSs, all the BSs in a cooperation set jointly transmit the same message to the target user on the same time-frequency resource, and the transmitted signals from different BSs are jointly precoded with prior phase alignment and tight synchronization across BSs to achieve coherent combining at the served user, exploiting the phase and potential amplitude relations between channels associated with different serving BSs [6].

Some prior works about JT have used stochastic geometry for modeling and analysis. Due to the simple form of its probability generating functional (PGFL) [7], the Poisson point process (PPP) is the most tractable point process model for the analysis of wireless networks including cellular networks. The success probability, which is the complementary cumulative distribution function (CCDF) of the signal-to-interference ratio (SIR), is focused on as a key performance metric in most prior works. [8] used stochastic geometry to analyze the benefit of JT for the typical general user and the typical user located at the cell-corner (the worst-case user). The success probability was derived under the assumption of no CSI. However, the case of coherent JT was evaluated only with different (coarser) performance metrics (diversity gain and power gain). [9] analyzed non-coherent JT in heterogeneous cellular networks (HCNs), where the cooperation set is determined by the cooperation activation thresholds (i.e., the channel fading including Rayleigh fading and path loss) of each BS tier. Currently, the analysis of the success probability for coherent JT is an open issue.

In this letter, we study coherent JT in downlink HCNs and derive an upper bound on the success probability for the typical general user and the typical worst-case user (cell-corner user). Besides, an approximation of the success probability for the general user is also obtained.

II. System Model

A. HCN Model

We consider a K-tier independent PPP HCN model where the BSs of tier i are distributed in $\mathbb{R}^2$ according to a homogeneous PPP Φ_i with intensity λ_i and transmit power P_i, $i = 1, \ldots, K$. The typical user receives a message that is transmitted by the cooperation set $\mathcal{C} \subset \Phi$, where $\Phi \triangleq \bigcup_{i=1}^{K} \Phi_i$, and n denotes the size of $\mathcal{C}$.

In coherent JT, based on the CSI shared among all cooperating BSs, all cooperating BSs transmit signals that are jointly precoded with prior phase alignment and tight synchronization to the target user. Under the assumption of an interference-limited scenario, noise is neglected. For the purposes of analysis, we assume that the transmitted signals are independent across the interfering base stations (see [8] for a detailed discussion of the impact of this simplification). Conditioning on the typical user at the origin, the SIR at the typical user can be expressed as

$$\mathrm{SIR} = \frac{\left(\sum_{x \in \mathcal{C}} |h_x| P_{\nu(x)}^{1/2} \|x\|^{-\alpha/2}\right)^2}{\sum_{x \in \mathcal{C}^c} g_x P_{\nu(x)} \|x\|^{-\alpha}}, \tag{1}$$

where $g_x = |h_x|^2$, the numerator is the combined desired signal power from the cooperating BSs, and the denominator is the interference power from the non-cooperating BSs; $\nu(x)$ denotes the index of the network tier of the BS located at x, i.e., $\nu(x) = i$ if and only if $x \in \Phi_i$; h_x denotes the Rayleigh fading between the typical user at the origin $(0, 0) \in \mathbb{R}^2$ and the BS at x, $h_x \sim \mathcal{N}_{\mathbb{C}}(0, 1)$ and h_x is i.i.d; $\alpha > 2$ is the path

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loss exponent; $\mathcal{C}^c = \Phi \backslash \mathcal{C}$ denotes the BSs that are not in the cooperation set.

B. General and Worst-case Users

We consider two types of typical users, namely the typical general user and the typical worst-case user. For the general user, we focus on the typical user located at the origin in a K-tier heterogeneous cellular network, and the cooperation set $\mathcal{C}$ consists of the n BSs with the strongest average received power, i.e.,

$$\mathcal{C} = \arg \max_{\{x_1,\ldots,x_n\}\subset\Phi} \sum_{i=1}^{n} \frac{P_{\nu(x_i)}}{\|x_i\|^{\alpha}}. \tag{2}$$

In order to study the cell-edge performance, we consider another type of typical user named the worst-case user as in [8], which is located at a Voronoi vertex in a single-tier network in $\mathbb{R}^2$ modeled by a homogeneous PPP Φ with intensity λ and transmit power P. The Voronoi vertex is a location that has equal distance to the three nearest BSs. In this case, we restrict the size of $\mathcal{C}$ to $n \in \{1, 2, 3\}$. Without loss of generality, we condition on Φ having a Voronoi vertex at $(0, 0)$. Hence the cooperation set $\mathcal{C}$ is a subset of these three BSs which are all closest to the origin. Denoting the location of the i-th closest BS to the origin by x_i, the cooperation set is

$$\mathcal{C} \subseteq \{x_1, x_2, x_3\}, \tag{3}$$

with $\|x_1\| = \|x_2\| = \|x_3\| = D$.

III. Success Probability Bound Analysis

The success probability of downlink HCNs with coherent JT can be expressed as

$$\mathbb{P}(\mathrm{SIR} > \theta) = \mathbb{E}\Big(\mathbb{P}\Big(\Big(\sum_{x\in\mathcal{C}} |h_x| P_{\nu(x)}^{1/2} \|x\|^{-\frac{\alpha}{2}}\Big)^2 > \theta I \mid \boldsymbol{g}, \Phi\Big)\Big),$$

where $I = \sum_{x\in\mathcal{C}^c} g_x P_{\nu(x)} \|x\|^{-\alpha}$, $\boldsymbol{g} = \{g_x \mid x \in \mathcal{C}^c\}$. $\mathbb{P}((\sum_{x\in\mathcal{C}} |h_x| P_{\nu(x)}^{1/2} \|x\|^{-\alpha/2})^2 > \theta I \mid \boldsymbol{g}, \Phi)$ is the CCDF of the square of the weighted sum of Rayleigh random variables (RVs) $|h_x|$. However, determining the probability distribution of a weighted sum of Rayleigh RVs is a long standing open issue, which complicates the analysis of the success probability for coherent JT.

A. Two Inequalities

To derive an upper bound of $\mathbb{P}(\mathrm{SIR} > \theta)$, two known inequalities are needed. Throughout this letter, $\mathrm{Gamma}(k, \theta)$ denotes the gamma distribution with shape parameter k and scale parameter θ. Its probability density function using the shape-scale parametrization is

$$f(x; k, \theta) = \frac{x^{k-1} e^{-x/\theta}}{\theta^k \Gamma(k)}, \quad x > 0,\ k, \theta > 0,$$

where $\Gamma(k)$ is the complete gamma function.

Lemma 1 (A lower bound of the CDF of the square of the weighted sum of Rayleigh RVs [10]) *If X_i, $i = 1, \ldots, n$ are the i.i.d. Rayleigh RVs with scale parameter σ, the CDF of the square of their weighted sum is bounded as*

$$\mathbb{P}\bigg(\Big(\sum_{i=1}^{n} w_i X_i\Big)^2 \le x\bigg) \ge \mathbb{P}\bigg(\sum_{i=1}^{n} X_i^2 \le \frac{x}{s}\bigg), \tag{4}$$

where $w_i \in \mathbb{R}^+$, $\sum_{i=1}^n X_i^2 \sim \mathrm{Gamma}(n, 2\sigma^2)$, and $s \triangleq \sum_{i=1}^n w_i^2$.

A RV X with $\mathrm{Gamma}(n, 1)$ distribution has a CDF that is given by the normalized lower incomplete gamma function, i.e., $F_X(x; n, 1) = \frac{\gamma(n,x)}{\Gamma(n)} \triangleq \int_0^x (t^{n-1} e^{-t}/((n-1)!))\mathrm{d}t$, $n \in \mathbb{Z}^+$. The next lemma gives a bound on $F_X(x; n, 1)$.

Lemma 2 (Alzer's Inequality [11]) *If RV X has a gamma distribution $\mathrm{Gamma}(n, 1)$, $n \in \mathbb{Z}^+$, the CDF $F_X(x; n, 1)$ is bounded as*

$$(1 - e^{-\beta x})^n \le F_X(x; n, 1) \le (1 - e^{-x})^n, \tag{5}$$

where $\beta \triangleq (n!)^{-1/n}$. If and only if $n = 1$, (5) holds with equality, i.e., X is an exponential RV with unit mean.

B. General User

Letting $\Xi_i = \{\|x\|^{\alpha}/P_i, x \in \Phi_i\}$, by the mapping theorem and the superposition property [7] of PPP, $\Xi = \bigcup_{i=1}^{K} \Xi_i$ is a non-homogeneous PPP on $\mathbb{R}^+$ with intensity function

$$\lambda(x) = \pi\delta x^{\delta-1} \sum\nolimits_{i=1}^{K} \lambda_i P_i^{\delta}, \quad x \in \mathbb{R}^+, \tag{6}$$

where $\delta = 2/\alpha$. We sort the elements of Ξ in ascending order, define $\gamma_k = \|x_k\|^{\alpha}/P_{\nu(x_k)}$ as the k-th element in the ordered set. The SIR of the general user can be expressed as

$$\mathrm{SIR_g} = \frac{\Big(\sum_{k\le n} |h_{x_k}| \gamma_k^{-1/2}\Big)^2}{\sum_{k>n} g_k \gamma_k^{-1}}, \tag{7}$$

where $g_k = |h_{x_k}|^2$.

Before Theorem 1 and Cor. 1, we define a function

$$G(\theta, \beta) = \sum_{i=1}^{n} (-1)^{i+1} \binom{n}{i} \int\limits_{\substack{0<u_1<\cdots\\ \cdots<u_n<\infty}} \exp\Bigg(-u_n\ {}_2F_1\bigg(1, -\delta; 1-\delta; -\frac{i\beta\theta}{\sum\limits_{k=1}^{n}\left(\frac{u_n}{u_k}\right)^{1/\delta}}\bigg)\Bigg)\mathrm{d}\mathbf{u}, \tag{8}$$

where $\delta = 2/\alpha$, and ${}_2F_1(a, b; c; z)$ is the Gaussian hypergeometric function.

Theorem 1 (An upper bound of the success probability for the general user with coherent JT) *The success probability for the general user in downlink cellular networks with coherent JT from n BSs is upper bounded as $\mathbb{P}(\mathrm{SIR_g} > \theta) \le G(\theta, (n!)^{-1/n})$.*

Proof: Given Ξ, the conditional success probability for the general user is

$$\begin{aligned}\mathbb{P}(\mathrm{SIR_g} > \theta \mid \Xi) &= \mathbb{P}\bigg(\Big(\sum_{k\le n} |h_{x_k}| \gamma_k^{-1/2}\Big)^2 > \theta I \mid \Xi\bigg)\\ &= \mathbb{E}_{\boldsymbol{g}}\bigg(\mathbb{P}\bigg(\Big(\sum_{k\le n} |h_{x_k}| \gamma_k^{-1/2}\Big)^2 > \theta I \mid \boldsymbol{g}, \Xi\bigg)\bigg),\end{aligned} \tag{9}$$

where $I = \sum_{k>n} g_k \gamma_k^{-1}$, $\boldsymbol{g} = \{g_k \mid k > n\}$, and $|h_x|$ are Rayleigh RVs with scale parameter $\sigma = \sqrt{2}/2$.

Using Lemma 1, (9) is upper bounded by

$$\mathbb{P}(\mathrm{SIR_g} > \theta \mid \Xi) \le \mathbb{E}_{\boldsymbol{g}}\left(\mathbb{P}\left(\sum_{k\le n} |h_{x_k}|^2 > \frac{\theta I}{s} \mid \boldsymbol{g}, \Xi\right)\right), \tag{10}$$

where $s = \sum_{k\le n} \gamma_k^{-1}$, and $\sum_{k\le n} |h_{x_k}|^2 \sim \mathrm{Gamma}(n,1)$.

The right side of (10) can be expressed as

$$\begin{aligned}
&\mathbb{E}_{\boldsymbol{g}}\left(\mathbb{P}\left(\sum_{k\le n} |h_{x_k}|^2 > \frac{\theta I}{s} \mid \boldsymbol{g}, \Xi\right)\right) \overset{(a)}{=} \mathbb{E}_{\boldsymbol{g}}\left(1 - \frac{\gamma(n, \frac{\theta I}{s})}{\Gamma(n)}\right) \\
&= 1 - \mathbb{E}_{\boldsymbol{g}}\left(\int_0^{\frac{\theta I}{s}} \frac{t^{n-1}e^{-t}}{(n-1)!}\mathrm{d}t\right) \overset{(b)}{\le} 1 - \mathbb{E}_{\boldsymbol{g}}\left((1-e^{-\beta\frac{\theta I}{s}})^n\right) \\
&\overset{(c)}{=} \sum_{i=1}^{n} (-1)^{i+1}\binom{n}{i} \underbrace{\mathbb{E}_{\boldsymbol{g}}\left(\exp\left(-i\beta\frac{\theta I}{s}\right)\right)}_{A},
\end{aligned} \tag{11}$$

where (a) follows since $\sum_{k\le n} |h_{x_k}|^2 \sim \mathrm{Gamma}(n,1)$; (b) follows from the lower bound in Lemma 2 and $\beta = (n!)^{-1/n}$; (c) follows from the binomial theorem $(a+b)^n \equiv \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$.

The term A in (11) can be derived as

$$A = \mathbb{E}_{\boldsymbol{g}}\left(\prod_{k>n} \exp\left(-\frac{i\beta\theta}{s} g_k \gamma_k^{-1}\right)\right) \overset{(a)}{=} \prod_{k>n} \frac{1}{1 + \frac{i\beta\theta}{s}\gamma_k^{-1}},$$

where (a) follows since $g_k = |h_{x_k}|^2$ is independently exponentially distributed with unit mean.

The joint probability density function of $\boldsymbol{\gamma} = (\gamma_1, \gamma_2, \ldots, \gamma_n)$ is given by [8], i.e., for $0 < \gamma_1 < \cdots < \gamma_n$,

$$f_{\boldsymbol{\gamma}}(\boldsymbol{r}) = \left(\pi\delta\sum_{j=1}^{K} \lambda_j P_j^{\delta}\right)^n \exp\left(-\pi\sum_{j=1}^{K} \lambda_j P_j^{\delta} r_n^{\delta}\right) \prod_{j=1}^{n} r_j^{\delta-1}.$$

Using the PGFL of the non-homogeneous PPP Ξ, we have

$$\begin{aligned}
&\mathbb{P}(\mathrm{SIR_g} > \theta) \triangleq \mathbb{E}_{\Xi}(\mathbb{P}(\mathrm{SIR_g} > \theta \mid \Xi)) \\
&\le \sum_{i=1}^{n} (-1)^{i+1}\binom{n}{i} \mathbb{E}_{\Xi}\left(\prod_{k>n} \frac{1}{1 + \frac{i\beta\theta}{s}\gamma_k^{-1}}\right) \\
&= \sum_{i=1}^{n} (-1)^{i+1}\binom{n}{i} \\
&\int\limits_{\substack{0<r_1<\cdots \\ \cdots<r_n<\infty}} \exp\Bigg(\underbrace{-\int_{r_n}^{\infty}\left(1 - \frac{1}{1+\frac{i\beta\theta}{s}x^{-1}}\right)\lambda(x)dx}_{B}\Bigg) f_{\boldsymbol{\gamma}}(\boldsymbol{r})\mathrm{d}\boldsymbol{r},
\end{aligned}$$

where the term B can be derived as

$$B = -\pi r_n^{\delta} \sum_{i=1}^{K} \lambda_i P_i^{\delta} \left({}_2F_1\left(1, -\delta; 1-\delta; \frac{-i\beta\theta}{r_n \sum\limits_{k=1}^{n} r_k^{-1}}\right) - 1\right),$$

using the Gaussian hypergeometric function ${}_2F_1(a,b;c;z)$. By changing the variable of this integration, i.e., letting $u_k = (\pi \sum_{i=1}^{K} \lambda_i P_i^{\delta}) r_k^{\delta}$, the result is obtained. ■

Corollary 1 (An approximation of the success probability for the general user) *An approximation of the success probability for the general user in downlink cellular networks with coherent JT from n BSs is $\mathbb{P}(\mathrm{SIR_g} > \theta) \approx G(\theta, 1)$, where the function G is defined in* (8).

Proof: Same as the proof of Theorem 1, except that we use the upper bound in Lemma 2 to obtain an approximation of the success probability for the general user. The tightness of this approximation is demonstrated by a comparison with simulation results in Sec. IV. ■

C. Worst-case User

Similar to the general user, for $n = 1, 2, 3$, the SIR of the worst-case user is

$$\mathrm{SIR_w} = \frac{\left(\sum\limits_{k\le n} |h_{x_k}| D^{-\alpha/2}\right)^2}{\sum\limits_{k=n+1}^{3} g_k D^{-\alpha} + \sum\limits_{k>3} g_k \|x_k\|^{-\alpha}}. \tag{12}$$

Theorem 2 (An upper bound of the success probability for the worst-case user with coherent JT) *The success probability for the worst-case user in downlink cellular networks with coherent JT from $n \in \{1,2,3\}$ BSs is upper bounded as*

$$\mathbb{P}(\mathrm{SIR_w} > \theta) \le \sum_{i=1}^{n} \frac{(-1)^{i+1}\binom{n}{i}\left(1 + \frac{i\beta\theta}{n}\right)^{n-3}}{\left({}_2F_1\left(1, -\delta; 1-\delta; -\frac{i\beta\theta}{n}\right)\right)^2},$$

where $\beta = (n!)^{-1/n}$ and $\delta = 2/\alpha$.

Proof: Using Lemma 1 and Lemma 2, the conditional success probability for the worst-case user is bounded as

$$\begin{aligned}
&\mathbb{P}(\mathrm{SIR_w} > \theta \mid \Phi) \\
&= \mathbb{E}_{\boldsymbol{g}}\left(\mathbb{P}\left(\left(\sum_{k\le n} |h_{x_k}| D^{-\alpha/2}\right)^2 > \theta I \mid \boldsymbol{g}, \Phi\right)\right) \\
&\le \mathbb{E}_{\boldsymbol{g}}\left(\mathbb{P}\left(\sum_{k\le n} |h_{x_k}|^2 > \frac{\theta I}{nD^{-\alpha}} \mid \boldsymbol{g}, \Phi\right)\right) \\
&\le \sum_{i=1}^{n} (-1)^{i+1}\binom{n}{i} \underbrace{\mathbb{E}_{\boldsymbol{g}}\left(\exp\left(-i\beta\frac{\theta I}{nD^{-\alpha}}\right)\right)}_{A},
\end{aligned} \tag{13}$$

where $I = \sum_{k=n+1}^{3} g_k D^{-\alpha} + \sum_{k>3} g_k \|x_k\|^{-\alpha}$ and $\boldsymbol{g} = \{g_k \mid k > n\}$. The term A in (13) can be derived as

$$\begin{aligned}
A &= \mathbb{E}_{\boldsymbol{g}}\left(\prod_{k=n+1}^{3} \exp\left(-\frac{i\beta\theta g_k}{n}\right) \prod_{k>3} \exp\left(-\frac{i\beta\theta g_k \|x_k\|^{-\alpha}}{nD^{-\alpha}}\right)\right) \\
&\overset{(a)}{=} \left(\frac{1}{1+\frac{i\beta\theta}{n}}\right)^{3-n} \prod_{k>3} \frac{1}{1 + \frac{i\beta\theta}{nD^{-\alpha}}\|x_k\|^{-\alpha}},
\end{aligned}$$

where (a) follows since $g_k = |h_{x_k}|^2$ is independently exponentially distributed with unit mean.

The probability density function of D is [8]

$$f_D(r) = 2\pi^2\lambda^2 r^3 e^{-\lambda\pi r^2}, \quad r \ge 0.$$

An upper bound of the success probability $\mathbb{P}(\mathrm{SIR_w} > \theta)$ can be obtained by

$$\begin{aligned}
&\mathbb{P}(\mathrm{SIR_w} > \theta) \triangleq \mathbb{E}_{\Phi}(\mathbb{P}(\mathrm{SIR_w} > \theta \mid \Phi)) \\
&\le \sum_{i=1}^{n} (-1)^{i+1}\binom{n}{i} \mathbb{E}_{\Phi}(A)
\end{aligned}$$

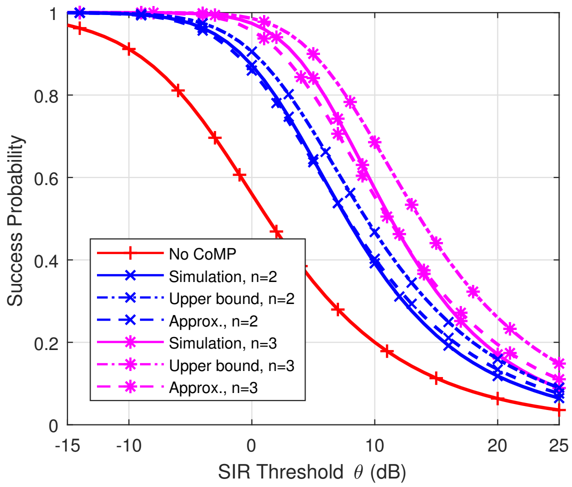

Fig. 1. The upper bounds, the approximations, and the simulation results of success probability for the general user.

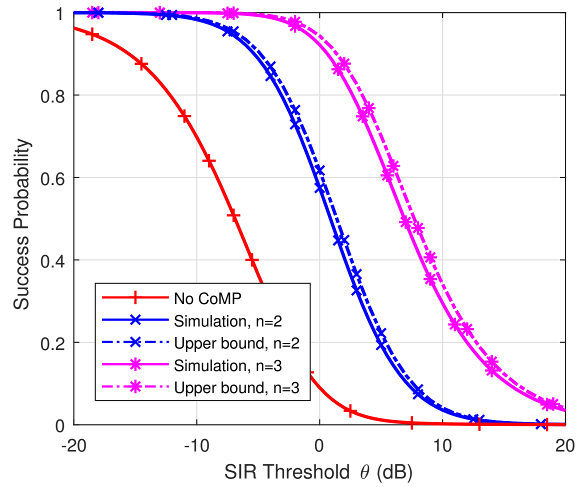

Fig. 2. The upper bounds and the simulation results of the success probability for the worst-case user.

$$\overset{(a)}{=} \sum_{i=1}^{n} (-1)^{i+1} \binom{n}{i} \int_0^\infty \left(1 + \frac{i\beta\theta}{n}\right)^{n-3} \exp\left(-\int_r^\infty \left(1 - \frac{1}{1 + \frac{i\beta\theta}{nr^{-\alpha}} x^{-\alpha}}\right) 2\pi\lambda x dx\right) f_D(r) \mathrm{d}r$$

$$\overset{(b)}{=} \sum_{i=1}^{n} (-1)^{i+1} \binom{n}{i} \int_0^\infty 2\pi^2 \lambda^2 r^3 \left(1 + \frac{i\beta\theta}{n}\right)^{n-3} \exp\left(-\pi\lambda r^2 \, {}_2F_1\left(1, -\delta; 1-\delta; -\frac{i\beta\theta}{n}\right)\right) \mathrm{d}r,$$

where (a) follows from the PGFL of the homogeneous PPP Φ, and (b) follows from the Gaussian hypergeometric function ${}_2F_1(a, b; c; z)$. By using integration by parts, the result is obtained. ■

IV. Tightness of Bounds and Approximations

Since the SIR distribution only depends on the intensity $\lambda(x)$ in (6)[1], the success probability for coherent JT is independent of the number of network tiers and their respective transmit powers and densities. Thus, without loss of generality, we focus on the case of a single-tier network with P and λ set arbitrarily. For the simulations in this letter, the simulation parameters are: $\alpha = 4$, $P = 1$, $\lambda = 1$, the size of cooperation set $n = 1, 2, 3$, and simulation region $[-30, 30]^2$.

In order to verify the tightness of the upper bounds and approximations, we compare them with simulation results for the general and worst-case users, as shown in Fig. 1 and Fig. 2. In Fig. 1, we observe that the approximation for the general user is very accurate. In Fig. 2, it is apparent that the upper bound for the worst-case user is rather tight. Moreover, for the upper bound for both general and worst-case users, the smaller the value of n, the better the accuracy. Comparing Fig. 1 and Fig. 2, it can be observed that the accuracy of the upper bound for the worst-case user is better than that for the general user.

[1]The density of the mapped PPP (6) is of the form $cx^{\delta-1}$, and the number of tiers and their densities and power levels only affect the constant c. This constant, however, has no influence on the SIRs in the network, since it merely corresponds to a scaling of the plane, and the network model is scale-invariant since a scaling of the plane by a reduces both the signal and the interference power by a factor a^α in each realization of the HCN.

For the general user, the horizontal gap between the bound and the simulation result is increasing with increasing θ, while for the worst-case user, it appears to remain constant.

V. Conclusion

In this letter, we derive an upper bound of the success probability for the general user and the worst-case user with coherent JT in downlink HCNs. Furthermore, an approximation for the general user is also derived. Comparing with the simulation results, the upper bounds and approximations are quite accurate and demonstrate that significant SIR gains are possible using coherent JT. Moreover, we clearly show that worst-case users (a special case of edge users) benefit significantly more from coherent JT than general users.

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