Asymptotics and Meta Distribution of the Signal-to-Interference Ratio in Wireless Networks
Part I

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Part I

Overview

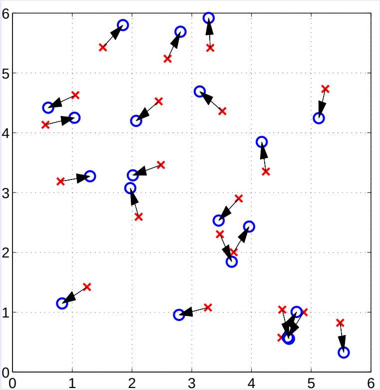
- The Poisson bipolar model
- Joint success probabilities
- The meta distribution of the SIR
- Cellular models
- Homework

Poisson bipolar network with ALOHA random access

- Sources form a Poisson point process (PPP) Φ of intensity λ.
- Each source has a destination at distance r and transmits with probability p in each time slot.
- The SIR at receiver y is

$$\mathrm{SIR}_y \triangleq \frac{S(y)}{I(y)} = \frac{h_{zy}\ell(z-y)}{\sum_{x\in\Phi_{\mathrm{int}}} h_{xy}\ell(x-y)}.$$

- (h_{xy}) are the fading random variables, and ℓ is the path loss function.
- Transmissions succeed if $\mathrm{SIR} > \theta$.



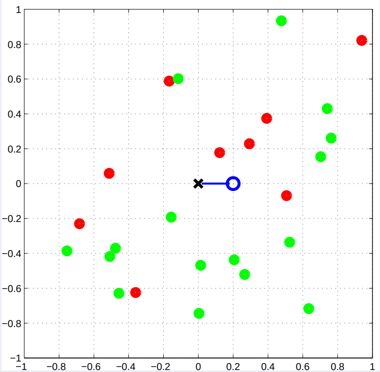
Poisson bipolar network with $\lambda = 1,\ r = 1/2$

What is the SIR distribution (or reliability) $\mathbb{P}(\mathrm{SIR} > \theta)$ of a representative link?

The typical link

- To the PPP, add a (desired) transmitter at location z and a receiver at the origin o. The link $z \to o$ is the typical link.
- Letting $\Phi_{\rm int} = \{x_1, x_2, \ldots\} \subset \Phi$ denote the locations of the interferers in a given time slot, the SIR at the typical receiver is

$$\mathrm{SIR} = \frac{h\ell(z)}{\sum_{x \in \Phi_{\rm int}} h_x \ell(x)}.$$



$\Phi_{\rm int} \subseteq \Phi$ and the typical link

We are interested in the SIR distribution (ccdf) $\mathbb{P}(\mathrm{SIR} > \theta)$.
For each realization of Φ, it is the spatial average of the link success probabilities $\mathbb{P}(\mathrm{SIR}_y > \theta \mid \Phi)$.

SIR distribution for Rayleigh fading

Laplace transform of the interference

Let $\delta \triangleq 2/\alpha$ and $\operatorname{sinc}\delta \triangleq \sin(\pi\delta)/(\pi\delta)$.
For $\ell(x) = \|x\|^{-\alpha} = \|x\|^{-2/\delta}$ and Rayleigh fading ($h \sim \exp(1)$),

$$\mathcal{L}_I(s) = \exp\left(-\lambda p \pi \Gamma(1+\delta)\Gamma(1-\delta)s^\delta\right) = \exp\left(-\frac{\lambda p \pi s^\delta}{\operatorname{sinc}\delta}\right), \quad \delta < 1.$$

Success probability/SIR ccdf

For Rayleigh fading, $p_s(\theta) \equiv \mathcal{L}_I(\theta r^\alpha)$ since[a]

$$p_s(\theta) = \mathbb{P}(hr^{-\alpha} > I\theta) = \mathbb{E}(e^{-\theta r^\alpha I}) = \exp\left(-\frac{\lambda p \pi r^2 \theta^\delta}{\operatorname{sinc}\delta}\right).$$

[a] Baccelli, Blaszczyszyn, and Mühlethaler, "An ALOHA Protocol for Multihop Mobile Wireless Networks". 2006.

Power control and full-duplex operation

ALOHA performs optimum power control

Assumptions:

- A Poisson bipolar network with Rayleigh fading
- No information about the fading at the transmitter (no CSIT)
- There is a peak and an average power constraint.
- In each time slot, the transmitter chooses a transmit power randomly and independently from a distribution that satisfies both constraints.

In turns out that on/off power control is the optimum (memoryless) random power control strategy[a].

So while ALOHA is suboptimum as a MAC scheme, it can be optimum as a power control scheme.

[a] Zhang and Haenggi, "Random Power Control in Poisson Networks". 2012.

Throughput gain with full-duplex links

The bipolar model is a natural model to explore the impact of full-duplex (FD) communication[a]. It turns out that:

- If the links can be used bi-directionally, the throughput

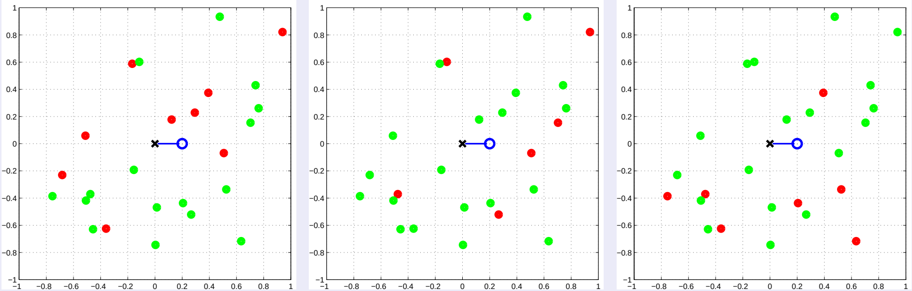
$$\text{(density of active links)} \times \text{(success probability)}$$

  cannot be doubled due to the extra interference.
- Only if the links are not too long and self-interference can be cancelled almost perfectly, FD operation is beneficial. There is a threshold behavior—either all links should be operated in half-duplex or all should use FD.
- Even with perfect self-interference cancellation, the throughput gain does not exceed $2\alpha/(\alpha+2)$, which is $4/3$ for $\alpha \leq 4$.

[a] Tong and Haenggi, "Throughput Analysis for Full-Duplex Wireless Networks with Imperfect Self-interference Cancellation". 2015, Subm.

Joint success probability

SIR in three time slots



The network is static (nodes do not move).

SIR at the receiver in time slot k:

$$\mathrm{SIR}_k = \frac{h_k r^{-\alpha}}{\sum_{x \in \Phi_k} h_{x,k} \|x\|^{-\alpha}}$$

Success event in slot k: $S_k \triangleq \{\mathrm{SIR}_k > \theta\}$.

Interference powers are correlated

The interference power levels in different time slots are correlated (despite ALOHA and iid fading), since the interferers are chosen from the static set of nodes Φ.

For Nakagami-m fading, where the fading coefficients are gamma distributed with pdf

$$f_h(x) = \frac{m^m x^{m-1} \exp(-mx)}{\Gamma(m)},$$

the temporal correlation coefficient of the interference is[a]

$$\zeta_t = p\frac{m}{m+1}\,.$$

As a consequence of the interference correlation, the success events S_k are also dependent.

[a] Ganti and Haenggi, "Spatial and Temporal Correlation of the Interference in ALOHA Ad Hoc Networks". 2009.

The joint success probability

Let $S_k \triangleq \{\mathrm{SIR}_k > \theta\}$ be the event that the transmission succeeds in time slot k. We would like to calculate $\mathbb{P}(S_1 \cap S_2)$.
Letting $\theta' = \theta r^{\alpha}$,

$$\begin{aligned}
\mathbb{P}(S_1 \cap S_2) &= \mathbb{P}(h_1 > \theta' I_1,\ h_2 > \theta' I_2) \\
&= \mathbb{E}(e^{-\theta' I_1} e^{-\theta' I_2}) \\
&= \mathbb{E}\left[\exp\left(-\theta' \sum_{x\in\Phi} \|x\|^{-\alpha}\big(\mathbf{1}(x\in\Phi_1)h_{x,1} + \mathbf{1}(x\in\Phi_2)h_{x,2}\big)\right)\right] \\
&= \mathbb{E}\left[\prod_{x\in\Phi}\left(\frac{p}{1+\theta'\|x\|^{-\alpha}} + 1 - p\right)^2\right] \\
&= \exp\left(-\lambda\int_{\mathbb{R}^2}\left[1-\left(\frac{p}{1+\theta'\|x\|^{-\alpha}} + 1 - p\right)^2\right]\mathrm{d}x\right).
\end{aligned}$$

Joint success probability

This generalizes easily to

$$\mathbb{P}(S_1 \cap \ldots \cap S_n) = \exp\left(-\lambda \int_{\mathbb{R}^2} \left[1 - \left(\frac{p}{1+\theta' \|x\|^{-\alpha}} + 1 - p\right)^n\right] \mathrm{d}x\right).$$

Theorem (Joint success probability)

Let $\delta = 2/\alpha$. The probability that a transmission over distance r succeeds n times in a row is[a]

$$p_s^{(n)}(\theta) = e^{-\Delta D_n(p,\delta)},$$

where $\Delta = \lambda \pi r^2 \theta^\delta \Gamma(1+\delta)\Gamma(1-\delta)$ and

$$D_n(p,\delta) = \sum_{k=1}^{n} \binom{n}{k} \binom{\delta-1}{k-1} p^k$$

is the diversity polynomial. It has order n in p and order $n-1$ in δ.

[a] Haenggi and Smarandache, "Diversity Polynomials for the Analysis of Temporal Correlations in Wireless Networks". 2013.

The diversity polynomial

Joint success probability: $p_s^{(n)}(\theta) = e^{-\Delta D_n(p,\delta)}$.

$D_1(p,\delta) = p$
$D_2(p,\delta) = 2p - (1-\delta)p^2$
$D_3(p,\delta) = 3p - 3(1-\delta)p^2 + \frac{1}{2}(1-\delta)(2-\delta)p^3$

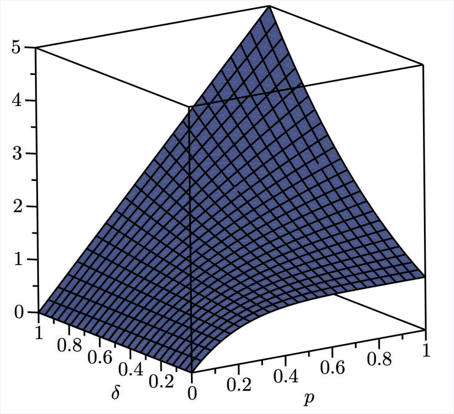
Note that $\delta \in (0,1)$.

The term in p is np, which would be the result in the independent case. So $p \to 0$ restores independence:

$D_n(p,\delta) \sim np, \; p \to 0$.

The same holds for $\delta \to 1$ ($\alpha \downarrow 2$).

For $n = 5$:



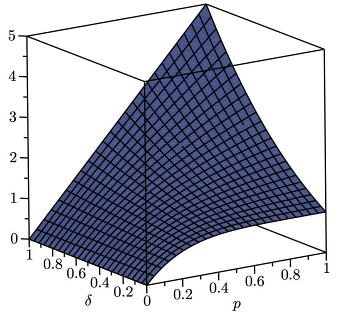
The diversity polynomial

- For small p, the first term dominates, and the transmission success is only weakly correlated.
- If $\delta \uparrow 1$, the success events become independent, but $\Delta \uparrow \infty$.
- If $\delta \downarrow 0$, the correlation is largest, but $\Delta \downarrow \lambda\pi r^2$.
- If $\delta \downarrow 0$ and $p = 1$, $D_n(1,0) = 1$ for all n, so the success events are fully correlated, i.e.,

$$p_s^{(1)} = p_s^{(2)} = \ldots = e^{-\Delta} = e^{-\lambda\pi r^2}.$$

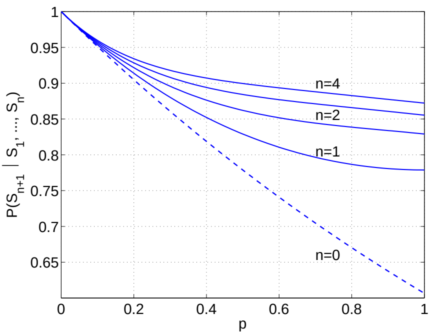
This is just the void probability.

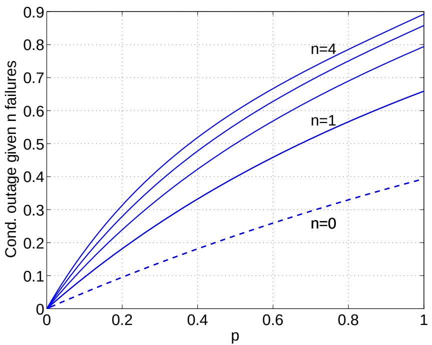
For $n = 5$:



Conditional success and outage probabilities

Using the joint success probabilities, the conditional probabilities of success after n successes and after n failures immediately follow.





$\delta = 1/2$, $\Delta = 1/2$.

Previous success or failures greatly affect future success probabilities.

Asymptotic probability of success in n transmissions

With some work it can be shown that the probability of succeeding at least once in n attempts is

$$p_s^{1|n} = 1 - \Delta p^n \frac{\Gamma(n-\delta)}{\Gamma(n)\Gamma(1-\delta)} + O(\Delta^2), \quad \Delta \to 0.$$

It follows that $p_o^{(n)} = 1 - p_s^{1|n} = \Delta C + O(\Delta^2)$ for some $C > 0$ that does not depend on Δ. Hence the diversity gain is

$$d = \lim_{\Delta \to 0} \frac{\log(\Delta(C + O(\Delta)))}{\log \Delta} = 1.$$

So there is no temporal diversity gain—no matter how many attempts are made and no matter how small p is!

Local delay

Local delay (first approach)

Since after a failure, the probability of success decreases, an interesting question is how long it takes to succeed.

$$\text{Local delay:} \quad M \triangleq \min_{k\in\mathbb{N}}\{S_k \text{ occurs}\}.$$

We have $\mathbb{P}(M > n) = p_{\mathrm{o}}^{(n)} = 1 - p_{\mathrm{s}}^{1|n}$, and the mean local delay can be expressed as

$$\mathbb{E}M = \sum_{k=0}^{\infty} \mathbb{P}(M > k) = \sum_{k=0}^{\infty} p_o^{(k)}.$$

$$= \exp\left(\Delta \frac{p}{(1-p)^{1-\delta}}\right) \gg \exp(\Delta p).$$

So for a deterministic link distance, the mean delay is finite for all $p < 1$, but much larger than in the independent case.

Local delay (second approach)

Success events are conditionally independent given Φ.

Hence, conditioned on Φ, the local delay is geometric with parameter

$$p_s(\Phi) = \mathcal{L}_I(\theta r^\alpha \mid \Phi) = \mathbb{E}(\exp(-\theta r^\alpha I \mid \Phi))\,.$$

It follows that the mean local delay is[a]

$$D = \mathbb{E}_\Phi\left(\frac{1}{\mathcal{L}_I(\theta r^\alpha \mid \Phi)}\right) = \exp\left(\frac{\Delta p}{(1-p)^{1-\delta}}\right),$$

as before.

[a] Baccelli and Blaszczyszyn, "A New Phase Transition for Local Delays in MANETs". 2010.

Mean local delay for nearest-neighbor communication

With random link distance,

$$D = \mathbb{E} \exp\left(\frac{p\lambda\pi R^2\theta^\delta}{\operatorname{sinc}(\delta)(1-p)^{1-\delta}}\right)$$

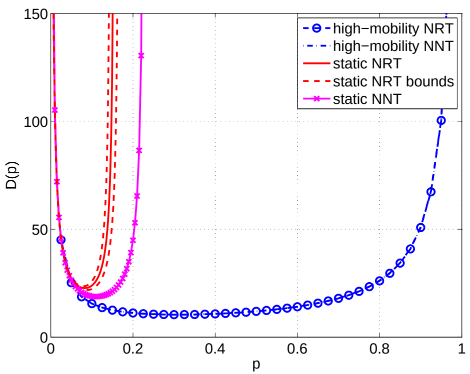
In nearest-neighbor transmission, R is Rayleigh distributed, and there is "tension" between the decay of the Rayleigh tail and the $\exp(cR^2)$ shape of the mean local delay given R:

$$D = c\int_0^\infty r e^{-\xi_1 r^2} e^{\xi_2 r^2} \mathrm{d}r = \frac{c}{2}\frac{1}{\xi_1-\xi_2}, \quad \text{if} \quad \xi_1 > \xi_2.$$

As a result, there is a phase transition. The mean delay is infinite if p or θ are too large.[a]

[a] Baccelli and Blaszczyszyn, "A New Phase Transition for Local Delays in MANETs". 2010; Haenggi, "The Local Delay in Poisson Networks". 2013.

Mean local delay example ($\alpha = 4$)



- Static networks suffer from increased delay and sensitivity to p.
- Random frequency-hopping multiple access drastically reduces the delay variance compared to ALOHA.[a]

[a] Zhong, Zhang, and Haenggi, "Managing Interference Correlation through Random Medium Access". 2014.

Meta distributions

Back to link reliabilities

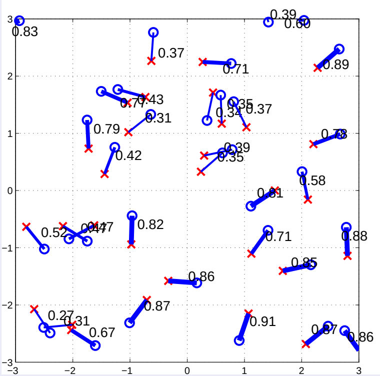
The success probability of the typical link is an average that provides limited information on the performance of an individual link.

For a realization of a Poisson bipolar network, attach to each link the probability

$$P_{\mathrm{s}}(\theta) \triangleq P_{\mathrm{s}}(\mathrm{SIR}_x > \theta \mid \Phi, \mathrm{tx}),$$

which is taken is over the fading and ALOHA and conditioned on Φ and on the partner node transmitting.

$P_{\mathrm{s}}(\theta)$ is a random variable, and its distribution is the meta distribution of the SIR.



What can we say about the random variable P_{s}?

Definition (Meta distribution)

We define the SIR meta distribution (ccdf) as[a]

$$\bar{F}_{P_s}(x) \triangleq \mathbb{P}^{!t}(P_s(\theta) > x), \quad x \in [0,1]$$

Due to the ergodicity of the PPP, the ccdf of P_s can be alternatively written as the limit

$$\bar{F}_{P_s}(x) = \lim_{r\to\infty} \frac{1}{\lambda p \pi r^2} \sum_{\substack{y\in\Phi \\ \|y\|<r}} \mathbf{1}(\mathbb{P}(\mathrm{SIR}_{\tilde{y}} > \theta \mid \Phi) > x),$$

where $\tilde{y}$ is the receiver of transmitter y.

Hence $\bar{F}_{P_s}(x)$ denotes the fraction of links in the network (in each realization of the point process) that, when scheduled to transmit, exceeds an SIR of θ with probability at least x.

[a] Haenggi, "The Meta Distribution of the SIR in Poisson Bipolar and Cellular Networks". 2015, arXiv.

Moments of $P_s(\theta)$

A direct calculation of $\bar{F}_{P_s}$ seems unfeasible, so let us focus on the moments

$$M_b(\theta) \triangleq \mathbb{E}^{!t}(P_s(\theta)^b) = \int_0^1 bx^{b-1}\bar{F}_{P_s}(x)\mathrm{d}x.$$

M_1 is just the "standard" success probability $p_s(\theta)$.

For $b \in \mathbb{N}$,

$$M_b(\theta) = \mathbb{E}^{!t}(P_s(\mathrm{SIR} > \theta \mid \Phi)^b)$$

is a quantity that we have already calculated...

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is a quantity that we have already calculated:

$$\begin{aligned} M_b(\theta) &= \mathbb{P}(S_1 \cap \ldots \cap S_b) \\ &= \exp\left(-\lambda \int_{\mathbb{R}^2} \left[1 - \left(\frac{p}{1+\theta'\|x\|^{-\alpha}} + 1 - p\right)^b\right] \mathrm{d}x\right). \end{aligned}$$

Moments of $P_s(\theta)$

So for $b \in \mathbb{N}$,

$$M_b(\theta) = \mathbb{E}^{!t}(P_s(\mathsf{SIR} > \theta \mid \Phi)^b)$$

is the probability that the transmission succeeds b times in a row.
For arbitrary $b \in \mathbb{C}$, generalizing the diversity polynomial to

$$D_b(p, \delta) \triangleq \sum_{k=1}^{\infty} \binom{b}{k} \binom{\delta - 1}{k - 1} p^k, \quad b \in \mathbb{C} \text{ and } p, \delta \in [0, 1],$$

we have, with $C \triangleq \lambda \pi r^2 \theta^\delta \Gamma(1 - \delta)$,

$$M_b(\theta) = \exp\left(-C\Gamma(1 + \delta) D_b(p, \delta)\right), \quad b \in \mathbb{C}.$$

D_b can be expressed using the Gaussian hypergeometric function ${}_2F_1$ as

$$D_b(p, \delta) = pb\, {}_2F_1(1 - b, 1 - \delta; 2; p).$$

Variance of $P_s(\theta)$

The variance of $P_s(\theta)$ follows as[a]

$$\operatorname{var} P_s(\theta) = M_1^2 (M_1^{p(\delta-1)} - 1).$$

Remarkably, fixing the transmitter density to $\tau \triangleq \lambda p$ (and thus fixing M_1) and letting $p \to 0$, we have $\operatorname{var} P_s \to 0$ and thus

$$\lim_{\substack{p\to 0\\ \lambda p=\tau}} P_s(\theta) = p_s(\theta)$$

in mean square (and probability and distribution).

So in an ultra-dense network with very small transmit probability, the success probability of each link is identical.

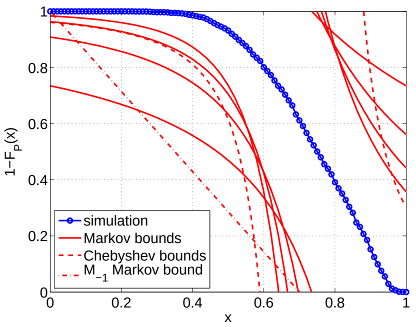
[a] Haenggi, "The Meta Distribution of the SIR in Poisson Bipolar and Cellular Networks". 2015, arXiv.

Bounds

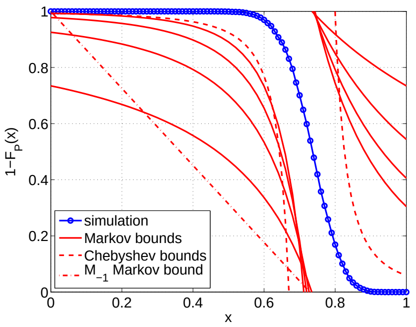
For $x \in [0,1]$, the ccdf $\bar{F}_{P_s}$ is bounded as

$$1 - \frac{\mathbb{E}^{!t}((1 - P_s(\theta))^b)}{(1-x)^b} < \bar{F}_{P_s}(x) \le \frac{M_b}{x^b}, \quad b > 0.$$

Illustrations for $p_s = M_1 = 0.735$:



$\lambda = 1,\ p = 1/4,\ \mathrm{var}(P_s) = 0.0212$



$\lambda = 5,\ p = 1/20,\ \mathrm{var}(P_s) = 0.00418$

Exact expression

Since we know the moments for $b \in \mathbb{C}$, we can use the Gil-Pelaez theorem to obtain an exact expression for the meta distribution:

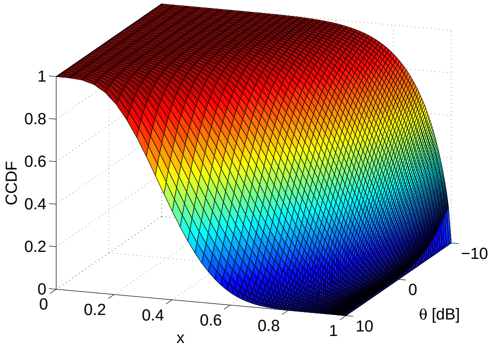
$$\bar{F}_{P_s}(x) = \frac{1}{2} - \frac{1}{\pi}\int_0^\infty \frac{e^{-C\Gamma(1+\delta)\Re(D_{jt})}\sin(t\log x + C\Gamma(1+\delta)\Im(D_{jt}))}{t}\mathrm{d}t\,.$$

This can be evaluated quite efficiently.

Approximation with beta distribution

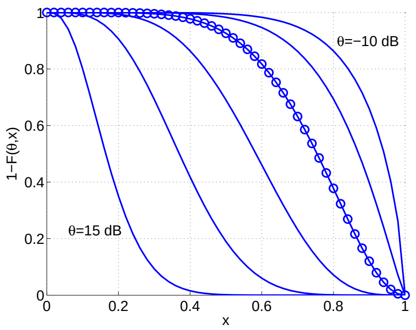
The beta distribution with moments M_1 and M_2 provides an excellent approximation.

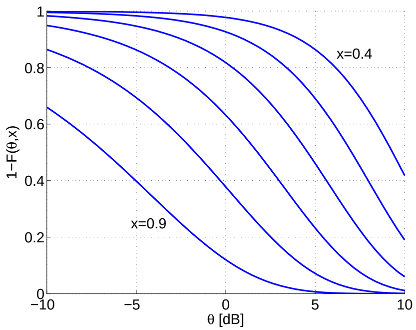
Example of meta distribution



$\lambda = 1$, $p = 1/4$, $\alpha = 4$, and $r = 1/2$

Cross-sections





$\theta = -10, -5, 0, 5, 10, 15$ dB.

$x = 0.4, 0.5, 0.6, 0.7, 0.8, 0.9$

Left: For a transmission at a certain rate, what fraction of links achieve reliability x? At $\theta = 0$ dB, 80% of the links succeed 60% of the time.

Right: For a given fraction of links x, what θ can be sustained?
A target reliability of 90% is only achieved by 40% of the links for $\theta = -5$ dB.

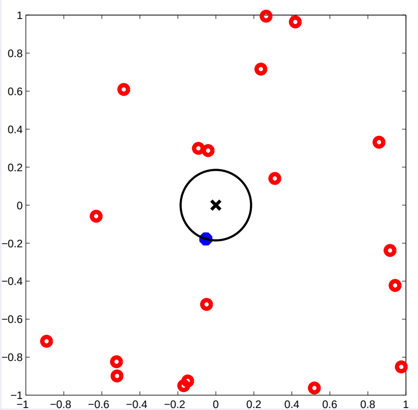
Discussion

- The meta distribution provides much more fine-grained information than the success probability of the typical link.
- It shows that stochastic geometry is not restricted to spatial averaging but can provide spatial distributions.
- In cellular networks, operators may be more interested in the performance of the "5% user" than the typical user. Using the meta distribution, we can answer questions such as "what spectral efficiency can be guaranteed with 90% probability for 95% of the users?"
- As in bipolar networks, the moments of the conditional success probability can be calculated in Poisson cellular networks.

From bipolar to cellular networks

A generic cellular network (downlink)

- Base stations form a stationary point process and all transmit at equal power.
- Assume a user is located at o. Its serving base station is the nearest one (strongest on average).
- The other base stations are interferers (frequency reuse 1).



Differences to bipolar model: (1) No ALOHA; (2) random link distance; (3) no interferers within black disk.

Basic result for Poisson cellular networks

If the BS form a PPP and fading is Rayleigh,[a]

$$p_{\rm s}(\theta) \triangleq \bar{F}_{\rm SIR}(\theta) = \frac{1}{{}_2F_1(1,-\delta;1-\delta;-\theta)}, \quad \delta \triangleq 2/\alpha.$$

For $\alpha = 4$ $(\delta = 1/2)$, $p_{\rm s}(\theta) = \left(1 + \sqrt{\theta}\arctan\sqrt{\theta}\right)^{-1}$.

This is obtained by conditioning on the link distance R, noting that the point process of interferers is a PPP on $b(o,R)^{\rm c}$, and taking the expectation w.r.t. R, which is Rayleigh distributed.

The density and the transmit power do not matter.

[a] Andrews, Baccelli, and Ganti, "A Tractable Approach to Coverage and Rate in Cellular Networks". 2011.

Moments of conditional success probability

The moments of the conditional success probability are[a]

$$M_b(\theta) = \frac{1}{{}_2F_1(b, -\delta; 1-\delta; -\theta)}.$$

[a] Zhang and Haenggi, "A Stochastic Geometry Analysis of Inter-cell Interference Coordination and Intra-cell Diversity". 2014.

Beyond the basic Poisson model

- The single-tier Poisson model can be extended to a multi-tier model consisting of independent PPPs.
- Extensions to other types of fading and non-Poisson models are difficult.

More on this topic tomorrow...

Conclusions

- For Poisson bipolar networks, the joint success probability can be expressed using the diversity polynomial.
- Retransmissions do not provide diversity gain.
- The joint success probability is closely related to the local delay and the moments of the conditional success probability given the point process.
- The distribution of the conditional success probablity is termed meta distribution. It can be expressed in integral form and provides fine-grained information about the performance of individual links or users in cellular networks.
- Tomorrow we will focus on cellular networks and discuss an approximate analysis framework of the SIR distribution that is applicable for general base station processes.
- Now it is time for the homework assignment...

Homework problem

One-dimensional Point process with constant pair correlation

Let $W \subseteq \mathbb{R}$, and let $W_{\text{int}} = W \setminus \partial W$. Find a point process on W such that

$$g(x, y) = \mathbf{2} \quad \forall x, y \in W_{\text{int}}.$$

g is the pair correlation function, defined as

$$g(x, y) \triangleq \frac{\rho^{(2)}(x, y)}{\lambda(x)\lambda(y)},$$

and $\rho^{(2)}$ is the second moment density, i.e., the density pertaining to the factorial second moment measure.

For the PPP, $g(x, y) = 1$.

References I

J. G. Andrews, F. Baccelli, and R. K. Ganti, "A Tractable Approach to Coverage and Rate in Cellular Networks". In: *IEEE Transactions on Communications* 59.11 (Nov. 2011), pp. 3122–3134.

F. Baccelli and B. Blaszczyszyn, "A New Phase Transition for Local Delays in MANETs". In: *IEEE INFOCOM'10*. San Diego, CA, Mar. 2010.

F. Baccelli, B. Blaszczyszyn, and P. Mühlethaler, "An ALOHA Protocol for Multihop Mobile Wireless Networks". In: *IEEE Transactions on Information Theory* 52.2 (Feb. 2006), pp. 421–436.

R. K. Ganti and M. Haenggi, "Spatial and Temporal Correlation of the Interference in ALOHA Ad Hoc Networks". In: *IEEE Communications Letters* 13.9 (Sept. 2009), pp. 631–633.

M. Haenggi, "The Local Delay in Poisson Networks". In: *IEEE Transactions on Information Theory* 59.3 (Mar. 2013), pp. 1788–1802.

M. Haenggi, "The Meta Distribution of the SIR in Poisson Bipolar and Cellular Networks". ArXiv, `http://arxiv.org/abs/1506.01644`. June 2015.

References II

M. Haenggi and R. Smarandache, "Diversity Polynomials for the Analysis of Temporal Correlations in Wireless Networks". In: *IEEE Transactions on Wireless Communications* 12.11 (Nov. 2013), pp. 5940–5951.

Z. Tong and M. Haenggi, "Throughput Analysis for Full-Duplex Wireless Networks with Imperfect Self-interference Cancellation". In: *IEEE Transactions on Communications* (2015). Submitted.

X. Zhang and M. Haenggi, "A Stochastic Geometry Analysis of Inter-cell Interference Coordination and Intra-cell Diversity". In: *IEEE Transactions on Wireless Communications* 13.12 (Dec. 2014), pp. 6655–6669.

X. Zhang and M. Haenggi, "Random Power Control in Poisson Networks". In: *IEEE Transactions on Communications* 60.9 (Sept. 2012), pp. 2602–2611.

Y. Zhong, W. Zhang, and M. Haenggi, "Managing Interference Correlation through Random Medium Access". In: *IEEE Transactions on Wireless Communications* 13.2 (Feb. 2014), pp. 928–941.