Multiple Choice Questions:

1. (c) The work done by friction will always be negative. If the block is pushed up the incline by a force that doesn't change the normal force (and hence the magnitude of the friction force), then the friction will do the same negative work on the way back up.

2. (d) Draw free-body diagrams: (ignore y-component here)

$$\sum F_x = m_a = T_1$$
$$\sum F_x = m_a = T_2 - T_1$$

Accelerations must be equal -> use this to solve + substitute:

$$\frac{T_1}{m_1} = \frac{T_2 - T_1}{m_2} \Rightarrow \frac{T_1}{m_1} + \frac{T_1}{m_2} = \frac{T_2}{m_2}$$

$$T_1 \left(\frac{m_1 + m_2}{m_1 m_2}\right) = \frac{T_1}{m_1} T_2$$

$$\frac{T_1}{T_2} = \frac{m_1}{m_1 + m_2}$$

3. (e) Draw a free-body diagram:

$$\sum F_y = m_a = 0 = N + T_y - mg, \ N + T_y = mg$$

$$\sum F_x = m_a = 0 = T_x - F_k, \ T_x = F_k$$

So, since $T > T_x, |T| > F_k$.

4. (e) (a) isn't circular motion; (b) and (c) are wrong because gravity is a conservative force; That leaves (d) or (e):

At bottom:

$$\sum F_y = mv^2 \Rightarrow N - mg = \frac{mv^2}{R}$$

$$N = \frac{mv^2}{R} + mg$$

(e)

5. (e) $W_{fr} = \Delta KE = \frac{1}{2} k x_1^2 = \frac{1}{2} m v_1^2$ ($W_{fr}$ is positive as spring expands)

Second case $\frac{1}{2} k x_2^2 = \frac{1}{2} (3m)(2v)^2 = 12 \left(\frac{1}{2} m v_1^2 \right)$

$x_2^2 = 12 x_1^2 \Rightarrow x_2 = 4x_1$
Problems

II. Start with a free-body diagram:

\[ \Sigma F_y = 0 = N - mg \cos \theta \]

\[ F_{max} = UsN = Us mg \cos \theta = 11 \, N \]

b.) The force exerted by the spring will just balance the force of friction and the horizontal component of the weight.

\[ \Sigma F_x = 0 = F - F_s - mgsin\theta \]

\[ \Rightarrow F = F_s + mgsin\theta = 21.1 \, N \]

This is equal to the force provided by the spring: \( F = kx \); \( 200 \, N/m \cdot x = 21.1 \, N \)

(no sign is needed, since we only want the change in length.)

\[ x = 0.1 \, m \]

c.) The force and the stretching are in the same direction, so the work done in stretching will be opposite in sign to and equal to the work done by the spring.

\[ W_p = \frac{1}{2} kx^2 = 1 \, J \]

III. a.) free-body diagram:

b.) \( \Sigma F_y = 0 = F \cos \theta - mg \Rightarrow F = \frac{mg}{\cos \theta} \)

\[ \Sigma F_x = max = \frac{mv^2}{r} = \frac{mv^2}{L_0 \sin \theta} = F \sin \theta \]

\[ v = \frac{2\pi r}{T} = \frac{2\pi L_0 \sin \theta}{T} \Rightarrow v^2 = \frac{4\pi^2 L_0^2 \sin^2 \theta}{T^2} \]

Substitute into force equation:

\[ \frac{4\pi^2 m L_0^2 \sin^2 \theta}{T^2 L_0 \sin \theta} = F \sin \theta \]

\[ \frac{4\pi^2 m L_0}{T^2} = F = \frac{mg}{\cos \theta} \quad (\text{from } F \cos \theta) \]

\[ \cos \theta = \frac{gT^2}{4\pi^2 L_0} \]

\[ \theta = \cos^{-1} \left( \frac{gT^2}{4\pi^2 L_0} \right) \]
Even though this looks like circular motion, it's really just linear kinematics and work.

a.) The net acceleration is $a = 1.5 \, m/s^2$.  
$$v_f = at, \quad 6 m = (1.5 m/s) \cdot t$$
$$t = 4 \, \text{sec.}$$

b.) $W_{tot} = \Delta KE = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}mv_f^2 = \frac{1}{2} (2)(6)^2 = 36 \, J$

We can use this and the magnitude of the friction force to find the work done by the axle, or use the sum of the forces to find the axle's force, then use the distance travelled.

\[ \begin{align*}
\vec{F}_k &= \mu_N = \mu mg, \quad W = -\vec{F} \cdot \Delta = -\mu mg \cdot d = \text{need d!} \\
\text{d} &= \frac{1}{2}at^2 = \frac{1}{2} (1.5)(4)^2 = 12 \, m \Rightarrow W = -(0.3)(2)(9.8)(12) \\
W_{tot} &= W_{ax} + W_{f} = W_{ax} - 70.6 \, J = \Delta KE = 36 \, J \\
W_{ax} &= 106.6 \, J
\end{align*} \]

\[ \begin{align*}
(\text{Or), } \sum F & \parallel = F_{ax} - F_k = m (a_{\parallel}), \quad F_{ax} = F_k + m (a_{\parallel}) = 5.9 \, N + 3 \, N = 8.9 \, N \\
W_{ax} &= F_{ax} \cdot d = (8.9 \, N)(12 \, m) = 106.6 \, J
\end{align*} \]

c.) $W = \Delta KE = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = -36 \, J$  

\[ \begin{align*}
W &= W_f = -\vec{F} \cdot S = -(5.9 \, N) \cdot S = -36 \, J \Rightarrow S = 6.1 \, m \\
(\text{distance traveled})
\end{align*} \]

(Could also use $\vec{F}_k = ma$, $a = \frac{\vec{F}_k}{m} = 2.95 \, m/s^2$)

\[ \begin{align*}
V_f^2 &= V_o^2 - 2a \cdot S, \quad V_f^2 = 2a \cdot S \Rightarrow (6 \, m/s)^2 = 2(2.95) \cdot S \\
S &= 6.1 \, (m)
\end{align*} \]
V. a.) \( W = -mg \Delta h \) (gravity does positive work on the way down, \( \Delta h \) increases upwards)
\[ = -(0.5 \text{ kg})(9.8 \text{ m/s}^2)(-15 \text{ m}) = 73.5 \text{ J} \]

b.) \( W_{\text{tot}} = W_{\text{grav}} - W_f = \Delta KE = \frac{1}{2}m v_f^2 - \frac{1}{2}m v_i^2 \)

\[ 73.5 \text{ J} - 40 \text{ J} = \frac{1}{2}m v_f^2 - \frac{1}{2}m v_i^2 \]
\[ 53.5J = \frac{1}{2}m v_f^2, \quad v_f^2 = 234, \quad v_f = 15.3 \text{ m/s} \]

c.) Final speed would be the same, since gravity will do the same work on the ball, raising its kinetic energy by the same amount. (Assume the energy lost to air resistance is the same in both cases)

d.) Again, \( W_{\text{tot}} = W_{\text{grav}} - W_f = \Delta KE \)

This time, \( \Delta KE = \frac{1}{2}m v_f^2 - \frac{1}{2}m v_0^2 = \frac{1}{2}m v_{xf}^2 + \frac{1}{2}m vy_f^2 - \frac{1}{2}m v_{x0}^2 \)

Since there is no acceleration in the x-direction,
\[ \frac{1}{2}m v_{xf}^2 = \frac{1}{2}m v_{x0}^2, \quad \text{so,} \quad \Delta KE = \frac{1}{2}m vy_f^2 \]

\[ W_{\text{tot}} = (73.5 - 40) \text{ J} = 33.5 \text{ J} = \frac{1}{2}m vy_f^2, \quad vy_f^2 = 134 \]

The total speed is then
\[ v_f = \sqrt{v_{xf}^2 + vy_f^2} = \sqrt{(10)^2 + (11.6)^2} = 15.3 \text{ m/s} \]