Notes

• Please look over exams, compare with posted solutions, turn in re-grades
• Please turn in old Problem Set, pick up new one
• Colloquium Today: “Order, Fluctuations, And New Frontiers In Quantum Materials”
Reminder: Harmonic (Sinusoidal) Waves

Snapshot at an instant in time \( t = \text{const} \):

Motion at one point in space \( x = \text{const} \):

\[
y = A \sin(kx - \omega t + \phi)
\]
Types of Waves

• Transverse
  – Local motion is perpendicular to wave propagation

• Longitudinal
  – Local motion is along/against direction of wave propagation
  – Examples: sound, slinky compression wave

• Toroidal
  – Sort of a transverse wave with angular displacement
Wave Speed for a String

move w/ wave, string moves left with $v$

$\Delta s \rightarrow \delta m = \Delta s \cdot \mu$

$\Sigma F_r = 2 |\mathbf{T}| \sin \theta \approx 2T \theta = 2 \theta T = \frac{\Delta s \cdot T}{R}$

$2 \theta = \frac{\Delta s}{R}, R(2\theta) = \Delta s$

$T \frac{\Delta s}{R} = \delta m a_r = \delta m \frac{v^2}{R} = \Delta s \mu \frac{v^2}{R}, v^2 = \frac{1}{\mu}$

$v = \sqrt{\frac{T}{\mu}}$

$v = f \lambda$
Boundaries

\[ f_1 = f_2 \]

Reflection

Transmission

different \( \mu \), different medium

\[ f = \frac{\nu}{\lambda} \quad \rightarrow \quad \frac{\nu_1}{\lambda_1} = \frac{\nu_2}{\lambda_2} \]
The Wave Equation

\[ \sin \theta \approx \tan \theta \approx \theta \]

\[ \Delta F_y = F \sin \theta_2 - F \sin \theta_1 \approx F \tan \theta_2 - F \tan \theta_1 \]

\[ = F \Delta \delta (\tan \theta) \quad \Delta (\tan \theta) = \tan \theta_2 - \tan \theta_1 \]

\[ \Delta F_y = \delta m \Delta y = F \delta (\tan \theta) = \mu \Delta x \Delta y \]

\[ \Delta \cos \theta = \Delta x \approx \Delta x = \frac{\Delta y}{\Delta x} \quad \Rightarrow \quad \Delta (\tan \theta) \quad \frac{\Delta (\tan \theta)}{\Delta x} = \frac{\mu}{F} \Delta y \]

\[ \longrightarrow y \text{ motion:} \quad A_y = \frac{d^2 y}{dt^2} = \frac{\partial^2 y}{\partial x^2}, \quad \tan \theta = \frac{\partial y}{\partial x} \quad \text{local slope not time dep} \]

\[ \frac{\delta \left( \frac{\partial y}{\partial x} \right)}{\delta x} = \frac{\mu}{F} \frac{\partial^2 y}{\partial t^2} \quad \text{make } \Delta x \text{ small} \]

\[ \lim_{\Delta x \to 0} \frac{\delta \left( \frac{\partial y}{\partial x} \right)}{\delta x} = \frac{\partial^2 y}{\partial x^2} = \frac{\mu}{F} \frac{\partial^2 y}{\partial t^2} \quad \frac{\mu}{F} = \frac{1}{v^2} \Rightarrow \frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2} \]

\[ y = A \sin (kx - \omega t) \Rightarrow \frac{\partial y}{\partial x} = kA \cos (kx - \omega t), \quad \frac{\partial^2 y}{\partial x^2} = -k^2 A \sin (kx - \omega t) \]

\[ \frac{\partial y}{\partial t} = -\omega A \cos (kx - \omega t), \quad \frac{\partial^2 y}{\partial t^2} = -\omega^2 A \sin (kx - \omega t) \]
\[ \frac{\partial^2 y}{\partial x^2} = -k^2 A \sin (kx - wt), \quad \frac{\partial^2 y}{\partial t^2} = -w^2 A \sin (kx - wt) \]

\[ \frac{\partial^2 y}{\partial x^2} \to \frac{\partial^2 y}{\partial t^2}, \quad k^2 A \sin (kx - wt) \to w^2 A \sin (kx - wt) \]

\[ \frac{\partial^2 y}{\partial x^2} \cdot \frac{w^2}{k^2} = \frac{\partial^2 y}{\partial t^2} \quad \therefore \quad \frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2} \]

\[ y = A \sin (kx - wt) \] is a solution of wave equation.

If \( y_1 \) and \( y_2 \) are waves, \( y_1 + y_2 \) is also a solution.

\[ ay_1 + by_2 \] is also a solution.
The wave function
\[ y(x,t) = (0.3\text{m}) \times \sin((2.2\text{m}^{-1})x - (3.5\text{s}^{-1})t) \] is for a harmonic wave traveling down a string.

a) In what direction does this wave travel, and what is its speed?
\[ v = \frac{\omega}{k} = \frac{3.5}{2.2} = 1.6 \text{ m/s} \]

b) Find the wavelength, frequency, and period.
\[ k = \frac{2\pi}{\lambda}, \quad \lambda = \frac{2\pi}{k} = 2.9 \text{ m}, \quad f = \frac{v}{\lambda} = \frac{\omega}{2\pi} = 0.56 \text{ Hz} \]
\[ T = \frac{1}{f} = 1.8 \text{ sec} \]

c) What is the maximum displacement of any point on the string? \[ A = 0.3 \text{ m} \]
d) What is the maximum speed of any point on the string?
\[ v_{\text{max}} = v_{y\text{max}} = \frac{\partial y}{\partial t}\big|_{\text{max}} = -\omega A \cos(ke - \omega t) \]
\[ v_{y\text{max}} = \omega A = 0.11 \text{ m/s} \]
A simple harmonic transverse wave is propagating to the left (-x) direction. The figure shows the displacement as a function of x at time t = 0. The string tension is 3.6 N and its mass density is 25 g/m. Find

a) the wavelength

\[
\lambda = 20 \text{ cm}
\]

b) the amplitude

\[
A = 5 \text{ cm}
\]

c) the wave speed

\[
v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{3.6}{0.025}} = 12 \text{ m/s}
\]

d) the period

\[
T = \frac{1}{f} = \frac{1}{f} = T = 3.3 \times 10^{-2} \text{ sec}
\]

e) the maximum speed of a particle in the string

\[
\omega = \frac{2\pi}{T} = \frac{2\pi}{1} = 60\pi \text{ rad/s}
\]

f) an equation representing this wave

\[
y = (0.05 \text{ m}) \sin \left( \left( \frac{5\pi}{x} \right) x + (60\pi \text{ rad/s})t + 0.93 \right)
\]
Energy and Power in Wave Motion

harmonic wave → Power is rate of energy transfer
work done on a segment of a string by the adjacent segment
Rate at which work is done = Power

\[ P = \overrightarrow{F_T} \cdot \overrightarrow{v_y} = F_T \cdot v_y \cos \phi = -F_T \cdot v_y \sin \theta \approx -F_T \cdot v_y \tan \theta \]

\[ P = -F_T \frac{\partial y}{\partial t} \frac{\partial y}{\partial x}, \quad y = A \sin (kx - \omega t) \]

\[ \frac{\partial y}{\partial t} = -\omega A \cos (kx - \omega t), \quad \frac{\partial y}{\partial x} = kA \cos (kx - \omega t) \]

\[ P = F_T k \omega A^2 \cos^2 (kx - \omega t) \rightarrow \langle P \rangle_{\text{ave}} = \frac{1}{2} F_T k \omega A^2, \quad \langle P \rangle \propto A^2 \]

\[ V = \sqrt{\frac{F_T}{\mu}}, \quad V^2 = \frac{F_T}{\mu}, \quad \frac{w^2}{k^2} = \frac{\omega}{k} \cdot V = \frac{F_T}{\mu}, \quad F_T k \omega = \mu V \]

\[ \langle p \rangle = \frac{1}{2} \mu V w^2 A^2, \quad \langle p \rangle_{\text{ave}} = \frac{\Delta E_{\text{ave}}}{\Delta t}, \quad \Delta E_{\text{ave}} = \langle p \rangle_{\text{ave}} \Delta t = \frac{1}{2} \mu V w^2 A^2 \Delta t \]

\[ v \Delta t = \Delta x, \quad \left( \frac{\Delta E_{\text{ave}}}{\Delta x} = \frac{1}{2} \mu V w^2 A^2 \right) \]
Intensität: \[ I = \frac{\text{Power}}{\text{Area}_\perp}, \quad \text{Power} \perp \text{Area} \]

\[ P = P_0 \hat{n}, \quad A_\perp = A \hat{n} \cdot \hat{x} \]

\[ A_\perp = A \cos \theta \]
Power is to be transmitted along a taut string by means of transverse harmonic waves. The wave speed is 10 ms/\(^{-1}\), and the linear mass density of the string is 0.01 kg/m. The power source oscillates with an amplitude of 0.5 mm.

a) What average power is transmitted along the string if the source frequency is 400 Hz?

\[ P_{ave} = \frac{1}{2} \mu v \omega^2 A^2 = \frac{1}{2} \mu v (2\pi f)^2 A^2 = 7.9 \times 10^{-3} \text{W} \]

b) The power transmitted can be increased by increasing independently the tension in the string, the frequency of the source, or the amplitude of the waves. By how much would each of these quantities need to increase to cause a factor of 100 increase in the transmitted power?

\[ A \rightarrow 10 \times A \quad ; \quad f \rightarrow 10 \times f \quad ; \quad v^2 = \frac{F_t}{\mu} \]

\[ P_{ave} = \frac{1}{2} F_t k \omega A^2 \quad ; \quad kw = \frac{w}{\sqrt{\omega}} = \frac{w^2}{\sqrt{\omega}} = \frac{k^2 v}{\sqrt{\omega}} \]

\[ = \frac{1}{2} F_t k^2 v A^2 \quad \rightarrow \quad k = \text{const} = \sqrt{\frac{v}{u \sqrt{F_t}}} \]

\[ P \propto F^{3/2} \quad ; \quad F^{3/2} = 100 \quad ; \quad F = (100)^{2/3} \]