Notes

• This week: Office Hours today, normal Help Sessions
  – I will be away tomorrow-Fri.
    • Prof. Ruggiero will conduct class Wed

• Please pick up exam regrades, old problem sets, new problem set for next week
Beat Intensity (Clarification)

\( \cos \left( \left( \frac{\omega_2 - \omega_1}{2} \right) t \right) \)

You hear \( (\text{Amplitude})^2 = \text{Intensity} \propto \cos^2 \left( \left( \frac{\omega_2 - \omega_1}{2} \right) t \right) \)

\( |\sim|^2 = \sim \sim \)
A tone of 300Hz is played along with another source of frequency $f$. You hear beats in the amplitude that are 2 seconds apart. What are the possible values of $f$?

\[
\frac{\omega_2 - \omega_1}{2} = \frac{2\pi (f_2 - f_1)}{2} = \pi (f_2 - f_1) = 2\pi (0.25)
\]

\[
\left[ f_{\text{beat}} > 0.5 \text{ Hz} \rightarrow \Delta f = 0.25 \text{ Hz} \right] \rightarrow \Delta \omega = 2\pi (0.25 \text{ Hz})
\]

\[
f_2 - f_1 = 0.5 \text{ Hz}
\]

\[
f_2 = 299.5, 300.5
\]
Review: **Standing Waves**

\[ y_1 = A \sin (kx - wt), \quad y_2 = A \sin (kx + wt) \]

\[ \Rightarrow \text{sum gives } \quad y_3 = 2A \cos (wt) \sin (kx) \]

- The envelope varies with time.
- The shape is fixed in space.
Standing Waves on Strings

\[ f \lambda = v = \text{const.} \]

\[ \lambda_n = \frac{2L}{n} \]

\[ f_n = \frac{nv}{2L} \]

<table>
<thead>
<tr>
<th>( n )</th>
<th>( \lambda_n )</th>
<th>( f_n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \frac{2L}{1} )</td>
<td>( \frac{v}{2L} )</td>
</tr>
<tr>
<td>2</td>
<td>( \frac{2L}{2} )</td>
<td>( 2 \frac{v}{2L} )</td>
</tr>
<tr>
<td>3</td>
<td>( \frac{2L}{3} )</td>
<td>( 3 \frac{v}{2L} )</td>
</tr>
<tr>
<td>4</td>
<td>( \frac{2L}{4} )</td>
<td>( 4 \frac{v}{2L} )</td>
</tr>
<tr>
<td>5</td>
<td>( \frac{2L}{5} )</td>
<td>( 5 \frac{v}{2L} )</td>
</tr>
</tbody>
</table>
In the arrangement shown in the figure, an object can be hung from a string with linear mass density $\mu = 0.002$ kg/m that passes over a light pulley. The length of the string between point P and the pulley is 2.0 m. When the mass $m$ is either 16 or 25 kg, standing waves are observed.

a) What is the frequency of the vibrating source?

b) What is the largest mass for which standing waves could be observed?

\[ f = \frac{n\nu}{2L} = \frac{n\sqrt{T}}{2L\mu} \]

\[ f = \frac{n}{2L} \sqrt{\frac{T_{25}}{\mu}} = \frac{n+1}{2L} \sqrt{\frac{T_{16}}{\mu}} \]

\[ \frac{n+1}{n} = \sqrt{\frac{T_{25}}{T_{16}}} = \sqrt{\frac{25}{16}} = \frac{5}{4} \quad n = 4 \implies f = \frac{4\left(\frac{25 \cdot g}{0.002}\right)^{\frac{1}{2}}}{2(2)(0.002)} = 350 \text{ Hz} \]

b) Lowest mode $n=1$ (fundamental)

\[ f = \frac{1}{2L} \sqrt{\frac{Mg}{\mu}} \implies M = 400 \text{ kg} \]
Sound Waves

\[ \frac{dP}{dx} = 0 \] (Constant pressure, no net Force)

Direction of Force changes

\[ F = 0 \Rightarrow \text{no motion} \]

Maximal motion
Standing Sound Waves in Pipes

Both ends open

- Pressure waves
- \( \lambda = 2L, f = \frac{v}{2L} \)
- \( \lambda = L, f = \frac{v}{L} \)
- \( \lambda = \frac{2}{3}L, f = \frac{3v}{2L} \)
- \( \lambda = \frac{1}{2}L, f = \frac{2v}{L} \)

Close both ends

- Pressure wave
- \( \text{disp. = 0 at walls} \)
Standing Sound Waves in Pipes

One end closed

\[ f = \frac{n \nu}{4L} \]
\[ n \text{ is odd only} \]

\[ \lambda = 4L, \quad f = \frac{\nu}{4L} \]

\[ \lambda = \frac{4}{3} L, \quad f = \frac{3\nu}{4L} \]

\[ \lambda = \frac{4}{5} L, \quad f = \frac{5\nu}{4L} \]

\[ \lambda = \frac{4}{7} L, \quad f = \frac{7\nu}{4L} \]
Three successive resonant frequencies in an organ pipe are 1310, 1834, and 2358 Hz.

a) Is the pipe closed at one end or open on both ends?
b) What is the fundamental frequency?
c) What is the effective length of the pipe?

\[
\frac{1310}{262} = 5 \quad \frac{1834}{262} = 7 \quad \frac{2358}{262} = 9 \quad \rightarrow \text{odd multiples of 262} \\
\rightarrow \text{closed at one end}
\]

\[
f_1 = 262 \text{ Hz}
\]

\[
f_1 = \frac{v}{4L} \quad L = \frac{v}{4f_1} \quad \rightarrow \text{speed of sound} \\
343 \text{ m/s} \quad 20^\circ \text{C}
\]

\[
L = 0.37 \text{ m}
\]
The “flaming tube” demo had a second harmonic frequency $f = 545$ Hz. The tube is 0.9 m long. What is the speed of sound in natural gas in this situation?

$$v = f \lambda$$

$$= (545) (0.9) = 490 \text{ m/s}$$

$> 343 \text{ m/s}$
Interference of Waves

\[ r_2 - r_1 = \frac{\Delta r}{\lambda} = \frac{\Phi}{2\pi} \quad \text{e.g.} \quad \Delta r = \frac{\lambda}{2} \]

\[ \frac{\Delta r}{\lambda} = \frac{1}{2} \implies \Delta \phi = \frac{\pi}{180} \]

Two sources: really far.

\[ r \gg d \]

\[ \Delta r = d \sin \theta \]

\[ \max \implies d \sin \theta = m \lambda \]
Diagnostic ultrasound of frequency 4.50 MHz is used to examine tumors in soft tissue.

a) What is the wavelength of the sound in air?

b) Assuming the speed of sound is 1500m/s in the tissue, what is the wavelength inside the body?
Two sound sources radiating in phase at a frequency of 480 Hz interfere such that maxima in the intensity pattern are heard at 0° and 23° from a line perpendicular to that joining the two sources. The listener is a large distance away and hears no other maxima between these. How far apart are the two sources? At which angles will other maxima occur?

\[
d = \frac{d \sin \theta}{\sin \theta} = \frac{0.71}{\sin 23^\circ} = 1.8 \text{ m}
\]

a. \[d \sin \theta = \frac{\lambda}{f} = \frac{343}{480} = 0.71 \text{ m}\]

b. Next, \[d \sin \theta = 2 \lambda, \quad (1.8 \text{ m}) \sin \theta_2 = 2(0.71)\]

\[\lesssim \theta_2 = 52^\circ\]

\[d \sin \theta_3 = 3 \lambda, \quad \sin \theta_3 = \frac{3(0.71)}{1.8} \geq 1\]

\[\Rightarrow \text{no 3rd max.}\]
An astronomical radio telescope consists of two antennas separated by 200m. Both are tuned to the frequency of 20 MHz. The signals from both antennas are fed to a common amplifier, but one signal first passes through a fixed phase delay so that the telescope can “look” in different directions. When the phase delay is zero, the telescope looks straight up (meaning signals coming from that direction will add coherently). What phase delay would be needed to look 10 degrees to the vertical?

\[
\Delta \phi = \frac{2\pi d \sin \theta}{\lambda} = 14.55 \text{ rad}
\]

\[
f \lambda = c, \quad \lambda = \frac{c}{f} = \frac{3 \times 10^8}{20 \times 10^6} = 15 \text{ m}, \quad \Theta = 10^\circ,
\]
The brain determines the direction of a source of sound by sensing the phase difference (i.e., arrival time difference) between sound striking your eardrums. A distant source emits sound of frequency 680 Hz. When you are directly facing the sound source, there is no phase difference. Estimate the phase difference between the sounds received by your ears when you are facing 90 degrees away from the source.

\[
\frac{\Delta r}{\lambda} = \frac{\Delta \phi}{2\pi},
\]

\[
\Delta r = 25\text{cm}, \quad \lambda = \frac{v}{f} = \frac{343}{680} \approx \frac{1}{2} \text{m}
\]

\[
\frac{\Delta r}{\lambda} = \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}, \quad \frac{\Delta \phi}{2\pi} = \frac{1}{2}, \quad \Delta \phi = \pi
\]

\[
343\text{m/s} = 25\text{cm}, \quad \sqrt{\Delta t} = \Delta r
\]

\[
\Delta t \approx 0.7\text{ms}
\]

\[
\text{brain} \leq 0.1\text{ms}
\]