Notes

• **Reminder:** First Exam Friday, 1:55-3:50pm, Jordan Hall lab room
  – Will cover Gravity, Fluids (Ch.13, 14)
  – Review Session: Tomorrow, 7:30-9:30pm, NSH 118

• **Office Hours:**
  – By appointment the rest of the week

• **Colloquium Today:**
  – “The 2011 Nobel Prize: A personal view”
SHM and Rotation

$$\Theta = \omega t + \phi$$

$$x = A \cos \Theta = A \cos(\omega t + \phi)$$

$$v_x = -|v| \sin \Theta = -|v| \sin(\omega t + \phi)$$

$$|v| = rw = Aw$$

$$v_x = -\omega A \sin (\omega t + \phi)$$
A mass attached to a spring oscillates back and forth as indicated in the position vs. time plot below. At point P, the mass has:

1. positive velocity and positive acceleration.
2. positive velocity and negative acceleration.
3. positive velocity and zero acceleration.
4. negative velocity and positive acceleration.
5. negative velocity and negative acceleration.
6. negative velocity and zero acceleration.
7. zero velocity but is accelerating (positively or negatively).
8. zero velocity and zero acceleration.
A mass suspended from a spring is oscillating up and down. Consider two possibilities:

(i) at some point during the oscillation the mass has zero velocity but is accelerating (positively or negatively);
(ii) at some point during the oscillation the mass has zero velocity and zero acceleration.

1. Both occur sometime during the oscillation.
2. Neither occurs during the oscillation.
3. Only (i) occurs.
4. Only (ii) occurs.
Energy in SHM

\[ F = -kx \quad , \quad U = \frac{1}{2} k x^2 \quad , \quad U(x) = \frac{1}{2} k A^2 \cos^2 (\omega t + \phi) \]

\[ KE = \frac{1}{2} m v^2 \Rightarrow KE(t) = \frac{1}{2} m \omega^2 A^2 \sin^2 (\omega t + \phi) \]

\[ \omega^2 = \frac{k}{m} \quad \Rightarrow \quad KE = \frac{1}{2} k A^2 \sin^2 (\omega t + \phi) \]

\[ E_{tot} = KE + U = \frac{1}{2} k A^2 \cos^2 (\omega t + \phi) + \frac{1}{2} k A^2 \sin^2 (\omega t + \phi) \]

\[ = \frac{1}{2} k A^2 \]

\[ E \propto A^2 \]

Virial Theorem \rightarrow \quad \langle \cos^2 \theta \rangle = \langle \sin^2 \theta \rangle = \frac{1}{2} \]

\[ U_{ave} = KE_{ave} = \frac{1}{2} E_{tot} \]
\[ E_{\text{tot}}(x_1) = \frac{1}{2} k x_1^2 \]

\[ KE(x_1) = \frac{1}{2} m v^2 \]

\[ U(x_1) = \frac{1}{2} k x_1^2 \]
A mass is attached to a spring of spring constant 8 N/m. The angular frequency $\omega$ of its motion is 2 rad/s, the amplitude of its motion is 1 m. At time $t = 0$, the phase of the oscillatory motion is $+53^\circ$ away from zero.

Find:

(a) the position of the mass at $t = 0$

(b) the velocity of the mass at $t = 0$

(c) the total energy of the system

(d) the maximum velocity

(e) the mass

(f) the time at which the maximum kinetic energy is first reached.

\[ x = A \cos(\omega t + \phi) = (1) \cos(53^\circ) = 0.6 \text{ m} \]

\[ v = -\omega A \sin(\omega t + \phi) = -2 \sin(53^\circ) = -1.6 \text{ m/s} \]

\[ E_{tot} = \frac{1}{2} kA^2 = 4 \text{ J} \]

\[ v_{max} = \omega A \sin(\pi/2) = \omega A = 2 \text{ m/s} \]

\[ \omega = \sqrt{\frac{k}{m}}, \quad \omega^2 = \frac{k}{m}, \quad m = \frac{k}{\omega^2} = \frac{8}{4} = 2 \text{ kg} \]

\[ \max \text{ vel.} \rightarrow \sin(\omega t + \phi) = 1, \quad \omega t + \phi = \pi/2, \quad \phi = 53^\circ = 0.925 \text{ rad} \]

\[ wt + 0.925 = \pi/2, \quad wt = \pi/2 - 0.925, \quad t = \frac{\pi/2 - 0.925}{2} = 0.32 \text{ sec} \]
A simple harmonic oscillator consists of a block of mass \( m = 2.00 \text{ kg} \) attached to a spring of constant \( k = 100 \text{ N/m} \). When \( t=1.00 \text{ sec} \), the position and velocity of the block are \( x = 0.129 \text{ meters} \) and \( v = 3.415 \text{ m/s} \), respectively.

a) What is the amplitude of the oscillations?

What were the (b) position and (c) velocity at time \( t=0? \)

(a) \( E_{tot} = \frac{1}{2} kA^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 \Rightarrow A = 0.5 \text{ m} \)

(b) \( @ t=1, \quad x = 0.129 = A \cos(\omega(1) + \phi) = (0.5) \cos(\sqrt{50} + \phi) \)

\[
\begin{align*}
0.258 &= \cos\left(\sqrt{50} + \phi\right) \Rightarrow \\
\cos(0.258) &= \pm 1.31 \\
&= \pm 5.76 \text{ rad} \\
&= \pm 8.38 \text{ rad} \\
&= \pm 2.097
\end{align*}
\]
Solving $x = x_0 \cos t$ leaves 2 solutions -> need to look at $v$ to resolve.

\[ v = -wA \sin(\omega t + \phi) = -\sqrt{50} (0.5) \sin (\sqrt{50} t + \phi) \] @ $t=1$

\[ v = \begin{cases} 3.415 @ t=1 & \text{if } \phi = -2.097 \checkmark \\ -3.415 @ t=1 & \text{if } \phi = -5.76 \end{cases} \]

So, \[ x = (0.5) \cos (\sqrt{50} t - 2.097), \] @ $t=0$ \[ x = 0.5 \cos (-2.097) = -0.25 \text{ m} \]

\[ v = -\sqrt{50} (0.5) \sin (\sqrt{50} t - 2.097), @ t=0 \]

\[ v = +3.06 \text{ m/s} \]
Oscillating Systems

**Pendulum**

\[ g, \ L, \ m, \ \theta \]

\[ \downarrow \ \downarrow \ \downarrow \ \downarrow \ \downarrow \]

\[ m/\text{s}^2, \ m, \ \text{kg} \] (no units)

\[ \frac{4}{g} = \frac{m}{m/L^2} = s^2, \ \sqrt{\frac{L}{g}} \rightarrow \text{sec.} = T? \]

\[ \Sigma F_{\parallel} = ma_{\parallel} = -mgsin\theta = m\frac{d^2s}{dt^2}, \ s = \text{arc length} \]

\[ s = L\theta \Rightarrow -mgsin\theta = mL\frac{d^2\theta}{dt^2} \]

If \ \[ \theta \ll 1, \ \sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} + \cdots \Rightarrow \sin\theta \approx \theta \quad \text{small} \ \theta \]

\[ -mgh = mL\frac{d^2\theta}{dt^2}, \quad \frac{d^2\theta}{dt^2} = -\frac{g}{L}\theta \quad \text{SHM!} \]

\[ \theta = A\cos(\omega t + \phi) \]

\[ w = \sqrt{\frac{g}{L}}, \ T = \frac{2\pi}{w} = 2\pi\sqrt{\frac{L}{g}} \]
Physical Pendulum

\[ r \times F \]

\[ \ddot{\phi} = \ddot{\phi} = \frac{d^2 \phi}{dt^2} \quad \text{and} \quad \ddot{\phi} = -Mg \sin \phi \]

\[ I \frac{d^2 \phi}{dt^2} = -Mg \sin \phi \quad \text{similar to \quad \ddot{x} + \omega^2 x = 0} \]

\[ \frac{d^2 \phi}{dt^2} = -\frac{Mg}{I} \phi \quad ? \text{SHM!} \]

\[ \phi = A \cos(\omega t + \phi), \quad \omega = \sqrt{\frac{Mg}{I}} \quad \tau = \frac{2\pi}{w} = 2\pi \sqrt{\frac{I}{Mg}} \]

\[ \omega_1 = \sqrt{\frac{Mg \frac{1}{3} L}{\frac{1}{3} mL^2}} = \sqrt{\frac{3}{2}} \frac{g}{L}, \quad \omega_2 = \sqrt{\frac{Mg L}{mL^2}} = \sqrt{\frac{g}{L}} \]

\[ d = \frac{1}{2} L \quad \text{and} \quad \ell = mL^2 \]

\[ I = \frac{1}{3} mL^2 \]
Ultimate bungee jumping: a straight tunnel is dug all of the way through the earth. You are dropped into the frictionless tube that lines the tunnel and appear on the other side, only to fall back into the hole. Show that your motion is simple harmonic motion, and find the period.

\[ F_g = ma \implies m \frac{d^2r}{dt^2} = F_g \]

\[ F(r) = -\frac{G M_{\text{enc}} m}{r^2} = -\frac{G M_E m}{R_E^3} r \]

\[ M_{\text{enc}} = \frac{4/3 \pi R^3}{4/3 \pi R_E^3} M_E = \frac{r^3}{R_E^3} M_E \]

\[ m \frac{d^2r}{dt^2} = -\frac{G M_E m}{R_E^3} r \implies \text{SHM!} \]

\[ \omega = \sqrt{\frac{G M_E}{R_E^3}} \quad T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{R_E^3}{G M_E}} = 5000 \text{ sec} \]
A pendulum is made of a rod of length $L = 1 \text{ m}$ and mass $m_{\text{rod}} = 3 \text{ kg}$, attached to a solid sphere of radius $R = 0.2 \text{ m}$ and mass $m_{\text{sphere}} = 4.5 \text{ kg}$. The axis of rotation is at the end of the rod.

a) What is the moment of inertia of the system about the rotation axis?

b) Where is the center-of-mass of the pendulum relative to the axis of rotation? 

$$r_{\text{cm}} = \frac{m_r (\frac{L}{2}) + M_s (L + R)}{m_r + M_s} = 0.92 \text{ m}$$

c) Write down Newton’s 2nd Law (for rotational motion) for the system configuration shown. Assume the angular displacement $\phi$ is small.

$$I = I_{\phi} = I \frac{d^2 \phi}{dt^2} = -M_{\text{tot}} g r_{\text{cm}} \sin \phi$$

d) Find the period of the pendulum for small angular displacement $\phi$.

$$\omega = \sqrt{\frac{M_{\text{tot}} g r_{\text{cm}}}{I_{\text{tot}}}} = 3.0 \text{ rad/s}, \quad T = \frac{2\pi}{\omega} = 2.1 \text{ sec}.$$