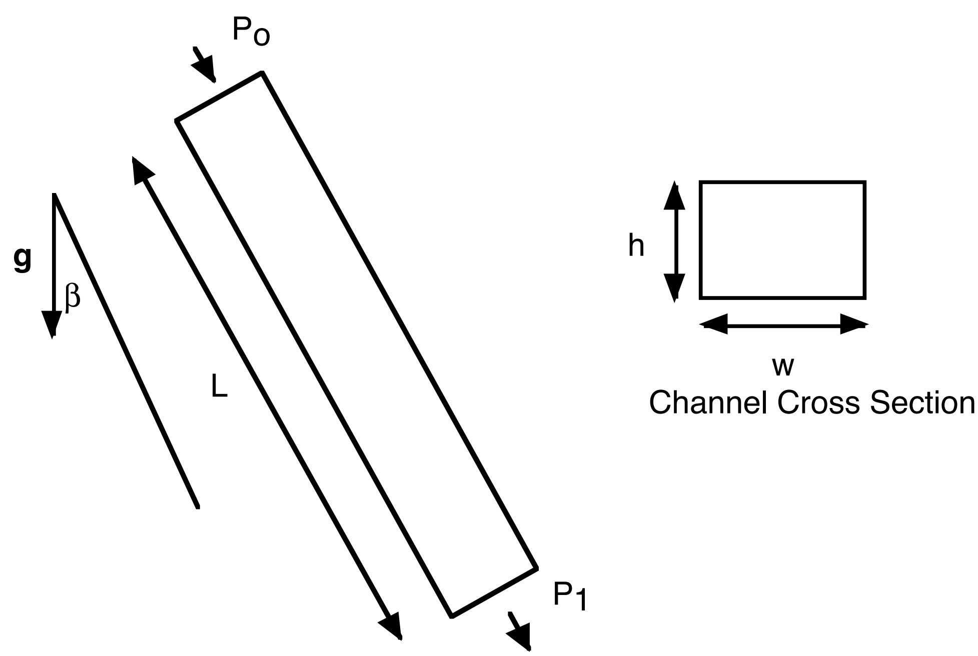
ChEg 355
Fall 1999
Test #1
10/5/99

1. Laminar flow in a channel. (50 points)

Consider a rectangular channel with a width w and thickness, b, where $b \leq w$ filled with a liquid that will, for this problem at least, eventually flow. The channel is oriented at an angle β relative to vertical.



a. Since it is much easier to solve a problem where the velocity gradient is non-zero in only one direction, we would like to do this here. What approximation (i.e. restriction on the domain of validity of your solution) can you make to simplify this problem so that you can obtain an ordinary differential equation for the stress and velocity of flow in this channel?

b. <u>Derive</u> the differential equation for the shear stress in terms of the pressure change and gravity.

c. What boundary conditions are appropriate for solving this problem?

d. Calculate the pressure distribution in a stagnant fluid over the entire length of the channel.

e. What must be true about the pressures, P_0 and P_1, if the fluid is to flow downward?

f. What must be true about the pressures, P_0 and P_1, if the fluid is to flow upward?

g. Starting with the equation for the stress distribution, derive an equation for the velocity profile. Assume that the shear stress and velocity are related by

$$\tau_{zx} = -\mu \, dv_z/dx$$

h. Where is the location and what is the value of the maximum velocity?

i. Calculate the average velocity.

Extra credit. (10 points)

j. Now suppose that the flow is not just due to pressure difference, but could also be caused by one of the walls moving. Find a dimensionless group that would tell how important the two different flow mechanisms would be and explain the physical meaning of the group.

2. Capillary rise (20 points)

A light silicon oil produces the following set of static rise heights for a set of capillary tubes with circular cross sections, that are identical except for the diameter.

Tube diameter (cm)	Observed rise (cm)
.01	2.55
.05	0.51
0.1	0.245
0.5	0.045
1.0	0.019

a. If the fluid has a density of 0.8 g/cm^3 and gravity is 980 cm/s^2, what value is the surface tension of this liquid with air?

b. Do all of the data give this same value?

c. If they do not, explain what is happening with the experiment. Which data point is most likely to give a true value of surface tension, why?

3. Floating objects on liquids using surface tension force (30 points)

We would like to get a general understanding of the problem where an object that is more dense than a liquid, is floated on the surface of the liquid using surface tension forces. That is, we need to find one or more dimensionless groups that give insight into this problem. Note that we are neglecting any buoyancy contribution to holding the solid up, because the density of the solid, ρ_S is much larger than the density of the liquid.

Consider long, (length, l >>diameter, d) cylinders as a generic shape. The liquid can have arbitrary density, ρ_L, and surface tension σ.

a. List all the variables that could be important.

b. For your list of variables, how many dimensionless groups should you get?

c. Find your dimensionless group(s). There is no need to use a formal procedure to find the obvious one(s).

d. Which group, (could be a combination of your groups) gives primary physical understanding for this problem?

e. Explain the physical significance of this group and why it gives this insight.

Potentially useful equations.

$V^*_{drop} = V_{drop}[\, D^a\, \sigma^b\, \rho^c]$

$$p_2 - p_1 = \rho g h$$

$$\dot{\gamma} = \frac{dv_z}{dx}, \tau_{zx} = -\mu \frac{dv_z}{dx}$$

$$-\frac{1}{r}\frac{d}{dr}\left(r\tau_{zr}\right) = C = \frac{dp}{dz} - \rho g$$

$$Q = \frac{\pi R^4}{8\mu}\frac{\Delta \boldsymbol{P}}{L}, \boldsymbol{P} = P_o - P_L + \rho g L$$

$$mg = \rho A h g$$

$$p_i - p_o = \sigma\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$