ChEg 355
Fall 1999
Test #2
November 16, 1999

1. Drag force on non spherical particles. (25 points)

Recall that the Stokes' law drag relation for a sphere in creeping flow is:

$F_D = 6 \pi \mu R U$,

where μ is the fluid viscosity, R is the radius of the sphere, and U is the terminal velocity. It was mentioned in class that if the particle is not a sphere, the coefficient (i.e., 6) will change, but not by a lot.

a. Use the following experimental data for terminal velocities of settling particles to find the coefficient for a drag law analogous to the equation above for "ellipsoids". The "nominal radius" gives the correct particle volume with the formula $V = 4/3 \pi r^3$. Recall that the gravitation constant is 980 g/s^2.

Nominal radius	Liquid viscosity	terminal velocity	solid density	fluid density
.1 cm	0.5 g/(cm-s)	24 cm/s	12 g/cm^3	1 g/cm^3
.1 cm	1 g/(cm-s)	12 cm/s	8 g/cm^3	1 g/cm^3
.3 cm	3 g/(cm-s)	36 cm/s	12 g/cm^3	1 g/cm^3
.05 cm	.5 g/(cm-s)	9 cm/s	12 g/cm^3	1 g/cm^3

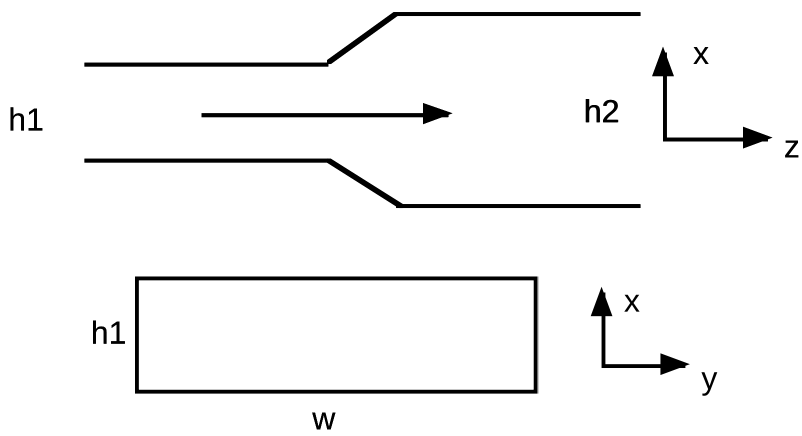
b. The velocity field for the flow past a stationary sphere is:

$$u_r = U \text{Cos}(\theta) (1-3/2\ R/r + R^3/2r^3)$$
$$u_\theta = - U \text{Sin}(\theta) (-1+3/4\ R/r + R^3/4r^3)$$

Quantitatively explain the considerations necessary to obtain accurate data from a falling object experiment.

2. Flow in an expanding channel. (40 points)

Consider pressure driven flow in a rectangular channel with an expansion. The fluid viscosity is μ, the fluid density is ρ. The width of the channel, w, does not change but the height goes from h1 to h2. The average velocity is U in the "h1" section. Gravity is oriented in the x direction.



a. Write down the all of the equations, including boundary conditions, that you need to solve for this flow far upstream of the expansion for the case where the flow is steady in time and there is no turbulence.

b. Now consider the expansion region; write down all of the equations (except skip the BC's) that you need in this region. Consider the case where the expansion is gradual enough that no "jetting" or swirling flows are created. You can assume no time dependence.

c. Find a relation for the approximate distance downstream of the expansion that it is likely to take before the flow is "fully developed" (i.e., parabolic profile) laminar flow in terms of the variables given in the problem.

d. Provide a relation that quantifies how "gradual" or "sudden" the expansion is?

e. Suppose now that the Reynolds number is much less than 1 and the expansion gradual (as you have defined it). (for simplicity, ignore changes in the transverse, y, direction), what equations apply now?

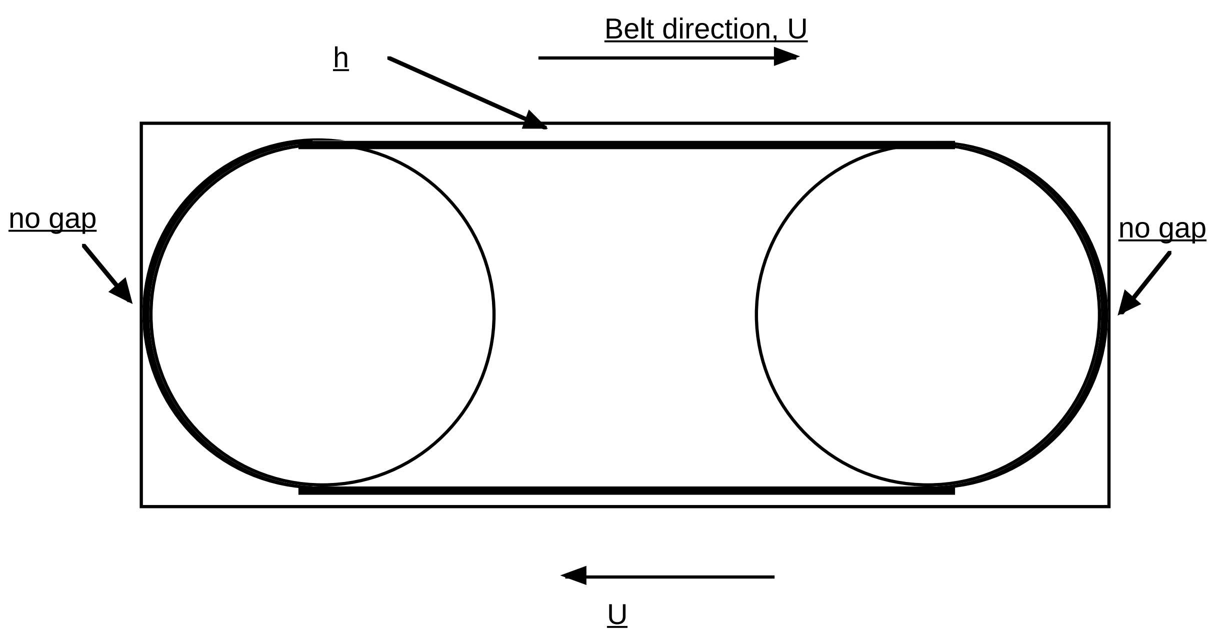
f. Find the velocity profile, $\mathbf{u}(x,z)$ in the expansion region. Use some intuition to get a "pretty good" answer and don't spend too much time trying to get an exact answer-- I am not looking for an exact answer.

3. "Conveyor belt" flow. (35 points)

About a decade ago, Professor Leighton did an elegant experiment to measure the structure of a concentrated suspension of particles. The device consisted of a rectangular box filled with a "secret mix" of fluids that matched the refractive index of Plexiglas®, the material that the spherical particles and the box were made out of. The flow was caused by a gear mechanism that drives uncoated 70 mm Movie Film (to give the holes that the gears fit into to drive it). The particles were visualized using a sheet of Ar-Ion Laser light shining in parallel to the page; individual particles showed up on video as circles. The distribution of particles around a chosen particle was found to be different on the "approach side" than the recession side even at effectively 0 Reynolds number.

For this problem, assume that the fluid is Newtonian. However, note that a real fluid that contains a large volume fraction will not be Newtonian! Thus your answer is only approximate.

a. Find an expression for the base-state flow in the region between the belt and the wall, far away from the ends. The belt velocity is U and the gap is h. The fluid viscosity is μ and the density is ρ. You can assume that the direction perpendicular to the page is much longer than h and that there is no flow in the direction perpendicular to the page. Note the end regions where the fluid does not fit through.



b. Calculate the normal and shear stress profiles in the region between the belt and the wall.

c. Find the value of the force/area that is necessary to "drive" the belt based on the flow that you calculate?