TEST #2
ANSWERS

1

1) a

r eq. $$-\rho\frac{u_\theta^2}{r} = -\frac{\partial P}{\partial r}$$

10 POINTS

θ eq. $$0 = \mu\left(\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial(r u_\theta)}{\partial r}\right]\right)$$

b) $u_\theta = R\omega_z$ @ $r = R$ 5 POINTS

$u_\theta = 0$ $r \to \infty$

c) $$\mu\left(\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}(r u_\theta)\right]\right) = 0$$

$$d\left(\frac{1}{r}\frac{d}{dr}(r u_\theta)\right) = 0$$

$$\frac{1}{r}\frac{d}{dr}(r u_\theta) = C_1$$

$$d(r u_\theta) = C_1 r\, dr$$

$$r u_\theta = \frac{C_1 r^2}{2} + C_2$$

5 POINTS

$$\boxed{u_\theta = \frac{C_1 r}{2} + \frac{C_2}{r}}$$

2

APPLY BC'S

$r \to \infty \quad u_\theta \to 0 \therefore$

$C_1 = 0$

$r = R$

$$R\omega_z = \frac{C_2}{R} \quad \therefore C_2 = R^2\omega_z$$

$$\boxed{u_\theta = \frac{R^2\omega_z}{r}}$$

5 POINTS

d. LOOK AT ANSWER, WHICH DIES OFF AS $1/r$. (THIS IS SLOW.)

CHOOSE $r \sim$ 20 R TO GET A 5% ERROR.

5 POINTS

e. PRESSURE CAN BE OBTAINED FROM

$$\frac{dP}{dr} = \rho \frac{u_\theta^2}{r}$$

$$\frac{dP}{dr} = \rho \frac{R^4\omega_z}{r^3}$$

$$P = \frac{-\rho R^4 \omega_z^2}{2r^2} + C_1$$

AS $r \Rightarrow \infty$ $\quad P \Rightarrow P_0$ (BACKGROUND PRESSURE)

$$\boxed{P = P_0 - \frac{\rho R^4 \omega_z^2}{2r^2}}$$

5 POINTS

f. MINIMUM PRESSURE IS AT $r = R$. THIS IS WHERE OUTWARD RADIAL FORCE IS LARGEST. 5 POINTS

g. PRESSURE IN A LIQUID WILL NOT DROP BELOW VAPOR PRESSURE! 5 POINTS.

h.
$$\tau_{r\theta} = -\mu \left(r \frac{\partial}{\partial r}\left(\frac{u_\theta}{r}\right) + \cancel{\frac{1}{r}\frac{\partial u_r}{\partial \theta}}^{\,0} \right)$$

$$\tau_{r\theta}\Big|_R = -\mu\, r \frac{d}{dr}\left(\frac{R^2 \omega_z}{r^2}\right) = 2\mu\omega_z$$

$$\Gamma = 2\mu \omega_z \underbrace{(2\pi R L)}_{\text{AREA}}\, R$$

SHOULD ALSO HAVE THIS

$$= 4\pi\mu\,\omega_z R^2 L$$

5 POINTS

i. TIME SCALE FOR MOMENTUM DIFFUSION IS

$$t \sim \frac{L^2}{\nu} \qquad L = \text{LENGTH SCALE}$$

$L \sim 10-20a$ SO

$$\boxed{t \sim \frac{(20a)^2}{\nu}}$$

10 POINTS

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2) a i ALL TERMS ARE DIVIDED BY APPROPRIATE LENGTH OR VELOCITY SCALES SO THAT THE VARY ONLY BETWEEN 0 + 1.

5 POINTS ii FLOW FIELD ADJUSTS ON A LENGTH SCALE OF SOME SMALL MULTIPLE OF, SAY, SPHERE RADIUS SO DENOMINATORS ARE NOT MUCH LARGER THAN 1.

b) $$\rho\left(\frac{\partial u_x}{\partial t} + \vec{u}\cdot\vec{\nabla}u_x\right) + \frac{\partial P}{\partial x} = 0$$ NEED PRESSURE

5 POINTS $$\rho\left(\frac{\partial u_y}{\partial t} + \vec{u}\cdot\vec{\nabla}u_y\right) + \frac{\partial P}{\partial y} = 0$$

c) SHOULD KEEP ALL ~~B~~ RIGHTSIDE TERMS IN r + θ EQS.
5 POINTS IGNORE LEFT SIDE TERMS.

d) THERE IS NO, "NO-SLIP" CONDITION, THIS MAKES SENSE BECAUSE THERE IS NO VISCOUS FORCE TO TRANSMIT A TANGENTIAL STRESS.

e) FOR THE INVISCID FLOW THERE IS A LARGE CHANGE IN PRESSURE THAT VARIES AS THE LOCAL $\frac{-\Delta U^2}{2}$.

FOR CREEPING FLOW THERE IS A GRADUAL DECREASE IN PRESSURE AN NO AREAS OF INCREASE.

3) a) JUST NEED CONTINUITY.

$$\vec{\nabla} \cdot \vec{u} = 0$$

LOOK AT TABLE

$$\boxed{\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 u_r) = 0}$$

5 POINTS

$$U_\theta = U_\phi = 0.$$

b. NO Re RESTRICTION OTHER THAN NOT TURBULENT. 5 POINTS

C. NOW USE R-DIRECTION N-S EQUATION. PLUG IN $u_r(r)$. SOLVE FOR P.

5 POINTS →

$$\rho u_r \frac{\partial u_r}{\partial r} = -\frac{\partial P}{\partial r} + \mu\left[\frac{1}{r^2}\frac{\partial^2}{\partial r^2}\left(r^2 u_r\right)\right]$$

COULD INCLUDE $\rho \frac{\partial u_r}{\partial t}$ IF YOU NEEDED TO.