TEST #2
ANSWERS

1) a

\[ \text{eq.} \quad - \frac{2 U_0^2}{a} = - \frac{\partial \varphi}{\partial n} \]

10 points

\[ \text{eq.} \quad 0 = M \left( \frac{2}{a} \left[ \frac{1}{n} \frac{\partial (\varphi)}{\partial n} \right] \right) \]

b) \quad U_0 = R U_0 \quad \text{at} \quad n = R \quad 5 \text{ points}
\quad U_0 = 0 \quad \text{as} \quad n \to \infty

C) \quad M \left( \frac{2}{a} \left[ \frac{1}{n} \frac{\partial (\varphi)}{\partial n} \right] \right) = 0
\quad \frac{d}{dn} \left( \frac{1}{n} \frac{\partial (\varphi)}{\partial n} \right) = 0
\quad \frac{1}{n} \frac{\partial (\varphi U_0)}{\partial n} = C_1
\quad d (\varphi U_0) = C_1 n \, dn
\quad n U_0 = C_1 \frac{n^2}{2} + C_2 \quad 5 \text{ points}

\[ U_0 = \frac{C_1 n}{2} + \frac{C_2}{n} \]

(\text{diagram})


**APPLY BCS**

\[ n \to \infty \quad U_0 \to 0 \quad c_1 = 0 \quad n = \varphi \]

\[ \frac{Rw_2}{\varphi} = \frac{C_2}{R} \quad \therefore C_2 = R^2w_2 \]

\[ U_0 = \frac{R^2w_2}{\varphi} \quad 5 \text{ POINTS} \]

**d.** Look at answer, which dies off as \( \frac{1}{n} \). (This is slow)

Choose \( n \sim 20 \ R \) to get a 5% error.

**Q.** Pressure can be obtained from

\[ \frac{dp}{dn} = \frac{8U_0^2}{\varphi} \]

\[ \frac{dp}{dn} = \frac{8R^4w_2}{\varphi R^3} \]
\[ p = \frac{-8 \gamma \omega z}{\eta \omega^2} + c, \]

As \( \omega \to \infty \), \( p \to p_0 \) (Background pressure)

\[ p = p_0 - \frac{8 \gamma \omega z}{2 \eta \omega^2} \]

5 points

f. Minimum pressure is at \( \omega = R \).
This is where outward radial force is largest.

5 points

g. Pressure in a liquid will not drop below vapor pressure.

5 points

h. \[ \tau_{\omega \theta} = -\mu \left( \frac{\partial}{\partial \theta} \left( \frac{\omega z}{\eta \omega^2} \right) + \frac{\partial}{\partial \theta} \right) \]

\[ \tau_{\omega \theta} \bigg|_R = -\mu \frac{\partial}{\partial \theta} \left( \frac{R^2 \omega z}{\eta^2} \right) = 2\mu \omega z \]

\[ \Pi = \frac{2\mu \omega z (2\pi R)}{\eta} \]

= \frac{4\pi \mu \omega z R^2 L}{\eta}

5 points
i. TIME SCALE FOR MOMENTUM DIFFUSION IS
\[ t \sim \frac{L^2}{V} \quad L = \text{LENGTH SCALE} \]
\[ L \sim 10^{-202} \quad SD \]
\[ t \sim \frac{(202)^2}{V} \]

2) \( \alpha \)

i. ALL TERMS ARE DIVIDED BY APPROPRIATE LENGTH OR VELOCITY SCALES SO THAT THE VARY ONLY BETWEEN 0 TO 1.

5 POINTS

ii. FLOW FIELD ADJUSTS ON A LENGTH SCALES OF SOME SMALL MULTIPLE OF, SAY, SPHERE RADIUS SO DENOMINATORS ARE NOT MUCH LARGER THAN 1.

5 POINTS

5) \( 8 \left( \frac{\partial u_x}{\partial t} + \bar{u} \cdot \frac{\partial u_x}{\partial x} \right) + \frac{\partial p}{\partial x} = 0 \)

5 POINTS

5) \( 8 \left( \frac{\partial u_y}{\partial t} + \bar{u} \cdot \frac{\partial u_y}{\partial y} \right) + \frac{\partial p}{\partial y} = 0 \)
c) Should keep all right side terms in \( 1 + \beta \) Eqs. 5 points

Ignore left side terms.

d) There is no "no-slip" condition. This makes sense because there is no viscous force to transmit a tangential stress.

e) For the inviscid flow there is a large change in pressure that varies as the local \(-\frac{\partial U^2}{\partial x}\).

For creeping flow there is a gradual decrease in pressure and no areas of increase.

3) a) Just need continuity.

\[ \nabla \cdot \mathbf{u} = 0 \]

Look at Table

\[ \frac{1}{n^2} \left( \frac{\partial^2 u_n}{\partial x^2} \right) = 0 \]

5 points

\[ U_0 = U_p = 0 \]
b. NO RE RESTRICTION OTHER THAN NOT TURBULENT. 5 POINTS

c. NOW USE R-DIRECTION N-S EQUATION. PLUG IN $u_1(x)$. SOLVE FOR $p$.

5 POINTS

$$
\frac{3u_1 du_1}{dz} = - \frac{\partial P}{\partial z} + \nu \left[ \frac{1}{R^2} \frac{d^2 (u_1^2 u_1)}{dz^2} \right]
$$

COULD INCLUDE $\frac{\partial u_1}{\partial z}$ IF YOU NEEDED TO.