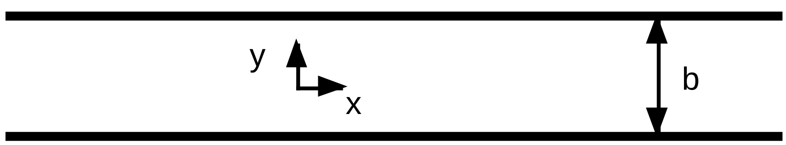
ChEg 355
Fall 1999
Final Exam

1. Flow between parallel plates (75 points)

Consider a pair of infinitely wide parallel plates that have a liquid trapped between them, which will be flowing somewhere at sometime during this problem. The flow direction is x, the direction normal to the plates is y and z is the transverse direction that is not important for this problem.



a. First consider that the flow is caused by a pressure differential. Of course this is a final exam and the pressure differential may not be constant and can vary as
$\Delta P(t) = \Delta P_0 + P_1 \cos(\omega t)$, where ω is the frequency of oscillation.

i. For the oscillatory pressure driven flow, write down the non zero terms in the Navier - Stokes equation and give reasons for those that are kept and those which are 0.

b. Now consider the case where the pressure gradient is steady (does not change in time.), $P_1=0$.

i. Write down the boundary conditions and the relevant differential equations.
ii. Solve these to get the velocity profile.
iii. Also calculate the average velocity.

c. It is probably a bad time to tell you, but many fluids are not Newtonian. The viscosity is not constant but can be a very complex function of the stress. Whipped cream, whipped egg whites, Mayonnaise and Ketchup are examples of fluids that (roughly) follow a model called a Bingham Plastic. An obvious physical characteristic of these fluids is that they can sustain a non-zero shear stress without flow (the "peaks" in the whipped cream for example) but if they are flowing, the stress-strain rate relation is linear. Such fluids do not follow the Navier-Stokes equations because the consitutive relation for the shear stress and velocity gradients is more complicated. In terms of the current geometry, the relation for the shear stress, *T*, in terms of the velocity gradients is:

$$\frac{du_x}{dy} = 0, if \left|T_{xy}\right| \le T_0,$$

$$T_{xy} = \mu \frac{du_x}{dy} + T_0, if \left|T_{xy}\right| > T_0$$

i. Find a relation for the minimum pressure drop necessary cause flow of a Bingham Plastic between your parallel plates.
ii. Find an equation for the velocity profile.
iii. Sketch the velocity profile and explain your answer.

2. Boundary layer flows (60 points)

a. Write the defining physical characteristic of a boundary-layer flow that leads to the form of the boundary-layer equations.
b. Write down the primary mathematical characteristic of a boundary - layer flow
c. Write down the main geometric characteristic of a boundary-layer flow, explain how is related to the need for a large Reynolds number for a boundary-layer to form.

The boundary - layer equations are:

$$u_x \frac{du_x}{dx} + u_y \frac{du_x}{dy} = -\frac{1}{\rho}\frac{dp}{dx} + \frac{\mu}{\rho}\frac{d^2u_x}{dy^2}$$

$$\frac{du_x}{dx} + \frac{du_y}{dy} = 0$$

$$\frac{dp}{dy} = 0$$

While this would never happen in real life, suppose that at a televised debate, one of your political rivals (for a major government position) is touting a new transportation vehicle that will save the environment by using much less energy and produce much less CO_2, but still has performance characteristics of at least a gasoline powered Honda Accord. The other candidate claims that the secret is that the power needed to for the car to cruise at 70 MPH is only 20% more than needed at 50 MPH.

Please ignore all non-fluidic issues such as rolling friction and the efficiencies of the engine and transmission (which should be good assumptions)

d. Assuming that you can communicate the science to a non-literate public, score a really big debating point by explaining why this is nonsense. (Use equations for this exam, but think how you could do it without equations for nonengineers.)

I expect that air drag will be a big part of your argument and use what you have learned about low and high Reynolds number flows to make your case. It would be useful to give the change in the power needed for cruising as a function of velocity to make it clear for the exam.

3. **Surge and storage tanks**. **(85 points)**

While you have accepted the job with XYZ chemical in anticipation of soon being promoted to CEO, you realize that at least your first job could actually require using engineering -- that hopefully you have acquired (somewhere) before taking the job. Your first assignment involves a review of the storage and surge tanks in your process; some capital has been allocated to be spent on this process and you would like to be one who gets to spend it. You are in competition with a new engineer from Rice who apparently has a better position since the Rice Engineer is involved a more glamorous part of the process: reactor technology. Fortunately, a representative from ACME storage tanks (newly promoted to *Senior* Sales Engineer after getting a *renewal* of the Loony Tunes Contract), is available to help you.

Consider the simplest problem first.

a. Find the volumetric rate of fluid discharge from a tank, as a function of the liquid height, (shown below) that vented to the atmosphere. You can assume that the tank volume is very large and thus this is a pseudo steady state (in terms of liquid height) problem.

b. Suppose that the tank is not vented, what is the steady state exit flow rate? What assumptions are you making to get this answer?

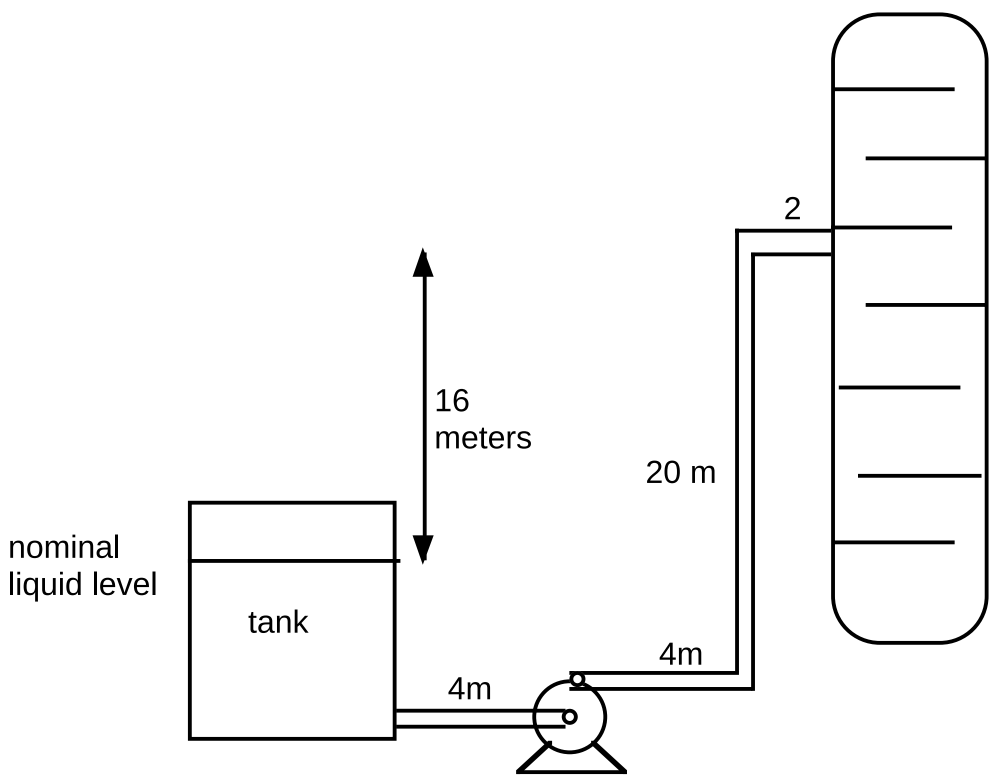
c. If you solved part *a* using the Bernoulli equation, now get an answer using the momentum balance. Likewise, if you used the momentum balance, get an answer using the Bernoulli equation. Comment on the differences.

Now on to the main issue, winning one for ND. The storage tank you identify as possibly needing improvement is one that feeds a slurry (small solid particles in a viscous liquid) into the (Rice Engineer's) reactor. The problem is the excessive energy cost in the form of stirring needed to keep the solid particles suspended. Note that this tank holds (in this and other reasonable cases for difficult or dangerous processes), 24 hours of feed. You believe that the aspect ratio, height to diameter, of the tank does not allow for most efficient mixing.

d. Use one of your favorite macroscopic equations applied to the tank with your perceived need to have an organized circulation velocity in the tank of at least 1 m/s (which is much greater than the settling velocity of the particles) to explain how the aspect ratio of the tank is important in this problem. Don't be unduly influenced by the ACME engineer who in typical cartoon fashion is suggesting short and fat for a tank shape. (don't worry, this is worth just 20 points).

4. **Flow in a pipe network. (50 points)**

You need to pump 10,000, Kg/hour of a mixture of light hydrocarbons from a storage tank to the "debutanizer" distillation column to remove butanes to be used for (say, portable lighters, solvent for WD -40 , etc.) and to make the rest of the mixture legal to be added to gasoline for the summer months. A reasonably sharp separation is needed to meet the required spec and the distillation column has 40 trays -- which is more than enough so that energy costs for this separation will be low. The column has multiple feed points and the best location appears to be the 12th tray that is 16 meters above the liquid level. The column is operated at approximately atmospheric pressure and the tank is vented though a carbon adsorption trap to the atmosphere.



The density of the liquid is 888 kg/m^3, the viscosity in the summer is about .0015 Kg/m/s, (it would be substantially more/less(?) in the winter if you did it?) The piping is 4 cm in diameter.

a. What is the power requirement for a pump that is 65% efficient? Be sure to include some fittings so that I know you know how to do this.

b. Is the pipe size reasonable for this purpose? Give some justification for your answer.

5. Growth of a bubble in a liquid. (30 points)

If a liquid is rapidly depressurized, dissolved gases can form bubbles that grow with time. In this problem it is desired to obtain an equation for the flow field and the pressure field for a bubble that is growing at a constant volumetric rate. The liquid density is ρ_L, the gas density is ρ_G, the liquid viscosity is μ_L, the gas viscosity is μ_G. The bubble volumetric growth rate is Q, (length^3/time). Interfacial tension between the gas and liquid is σ.

- **a. Find the velocity field inside the bubble as a function of time in terms of the bubble radius, R(t). As an idealization, you can assume that the gas is fed into the bubble from the center of the bubble.**

- **b. Find the pressure field inside the bubble as a function of the bubble radius, R(t).**

Be sure to state any restrictions on the range of validity of your solution.