12A.1 Unsteady-state heat conduction in an iron sphere

a. The thermal diffusivity of the sphere is given by Eq. 9.1-8:

\[
\alpha = \frac{k}{\rho \hat{C}_p} = \frac{30}{(436)(0.12)} = 0.573 \text{ ft}^2 / \text{hr}
\]

b. The center temperature is to be 128°F; hence

\[
\frac{T_{\text{ctr}} - T_0}{T_1 - T_0} = \frac{128 - 70}{270 - 70} = 0.29
\]

Then, from Fig. 12.1-3, \(\alpha t/R^2 = 0.1\), and

\[
t = 0.1 \left( \frac{R^2}{\alpha} \right) = 0.1 \left( \frac{1/24}{0.573} \right) = 3.03 \times 10^{-4} \text{ hrs} = 1.1 \text{ s}
\]

c. By equating the dimensionless times, we get

\[
\frac{\alpha_1 t_1}{R_1^2} = \frac{\alpha_2 t_2}{R_2^2}
\]

or

\[
\alpha_2 = \alpha_1 \left( \frac{t_1}{t_2} \right) = 0.573 \left( \frac{1}{2} \right) = 0.287
\]

d. The partial differential equation from which Fig. 12.1-3 was constructed is

\[
\frac{\partial T}{\partial t} = \alpha \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial T}{\partial r} \right)
\]
12A.2 Comparison of the two slab solutions for short times

According to Figure 12.1-1, at $\alpha t/b^2 = 0.01$ and $y/b = 0.9$

$$\frac{T - T_0}{T_1 - T_0} \approx 0.46$$

where $y$ is the distance from the mid-plane of the slab.

Next we use Fig. 4.1-1, which can be interpreted as a plot of $(T - T_0)/(T_1 - T_0)$ vs $y'/\sqrt{4\alpha t}$, where $y' = b - y$ is the distance from the wall. We then get

$$\frac{y'}{\sqrt{4\alpha t}} = \frac{1}{2} \frac{(1 - 0.9)}{\sqrt{\alpha t/b^2}} = \frac{1}{2}$$

Then from Fig. 4.1-1 we find

$$\frac{T - T_0}{T_1 - T_0} \approx 0.48$$

Hence the use of the combination of variables solution introduces an error of about 4%. Smaller errors occur at smaller values of the dimensionless time $\alpha t/b^2$. 
12B.4 Heat transfer from a wall to a falling film (short contact time limit)

a. From Eq. 2.2-18, we get

\[ v_z = v_{z,\text{max}} \left[ 1 - \left( \frac{x}{\delta} \right)^2 \right] = v_{z,\text{max}} \left[ 1 - \left( 1 - \left( \frac{y}{\delta} \right) \right)^2 \right] \]

\[ = v_{z,\text{max}} \left[ 1 - 1 + 2 \left( \frac{y}{\delta} \right) + \left( \frac{y}{\delta} \right)^2 \right] \rightarrow 2v_{z,\text{max}} \left( \frac{y}{\delta} \right) \]

this last expression is good in the vicinity of the wall, where the quadratic term can be neglected.

b. Equation 12B.4-2 presupposes that the heat conduction in the \( z \) direction can be neglected relative to the heat convection in the \( z \) direction. In addition, laminar, nonrippling flow is assumed.

c. The fictitious boundary condition at an infinite distance from the wall may be used instead of the boundary condition at a distance \( \delta \) from the wall, since for short contact times the fluid is heated over a very short distance \( y \). Therefore the infinite boundary condition can be expected to be adequate.

d. Equation 12B.4-3 can be written as \( y \left( \frac{\partial \Theta}{\partial z} \right) = \beta \left( \frac{\partial^2 \Theta}{\partial y^2} \right) \).

Next we have to convert the derivatives to derivatives with respect to the dimensionless variable \( \eta \):

\[ \frac{\partial \Theta}{\partial z} = \frac{d\Theta}{d\eta} \frac{\partial \eta}{\partial z} = \frac{d\Theta}{d\eta} \frac{y}{\sqrt[3]{9\beta z}} \left( -\frac{1}{3z} \right) \]

\[ \frac{\partial \Theta}{\partial y} = \frac{d\Theta}{d\eta} \frac{\partial \eta}{\partial y} = \frac{d\Theta}{d\eta} \frac{1}{\sqrt[3]{9\beta z}} \frac{\partial \eta}{\partial y} = \frac{d^2\Theta}{d\eta^2} \left( \frac{1}{\sqrt[3]{9\beta z}} \right)^2 \]

When these relations are substituted into the partial differential equation and use is made of the defining equation for \( \eta \) we get Eq. 12B.4-7.

e. When we set \( d\Theta/d\eta = p \), we get \( dp/d\eta + 3\eta^2 p = 0 \), which is first-order and separable, and the solution is given in the book. The next integration gives

\[ \Theta = C_1 \int_0^\eta e^{-\eta^3} \, d\eta + C_2 \]