1. Comparison of CFSTR and Batch Reactors. (60 points)

A 50-Liter cylindrical vessel is considered for both batch-wise and continuous operation in the reaction of $A \rightarrow B$. For batch operation, the initial volume of solution is 23.4 Liters, with an initial concentration, $C_{ao}$, of 2.39 M (moles/liter). In continuous operation, the inlet concentration is also 2.39 M. The reaction is first-order in $A$, and the rate constant $k$ is 0.1 sec$^{-1}$.

a. Assuming Batch Reactor Operation, derive an expression for conversion ($X$) as a function of time. [NOTE: $X = 1 - C_{a}/C_{ao}$]

b. Assuming Continuous Operation (CFSTR), derive an expression for conversion as a function of residence time, $V/q = \square$.

c. If the desired conversion is 90%, calculate the necessary reaction time (for batch) and residence time (for CFSTR) to achieve this target conversion.

d. Briefly explain why the reaction time for batch operation is greater or less than the residence time for a CFSTR.
2. Mr. Whiskers' Recuperation (30 points)

Your stay in New Orleans was mercifully brief (your company went into bankruptcy because the *A*d*rs*n accountants could not disguise the debt) and you have found an exciting new career opportunity: This has lead you to relocate to friendlier surroundings (and better Football, Baseball, Hockey and Lacrosse) in beautiful North Philadelphia. You and your cat, Mr. Whiskers, now live in a spacious, pollution-free third-floor apartment roughly 2 blocks from the University of Pennsylvania. However, thanks to your previously inhospitable surroundings, Mr. Whiskers has lost most of his hair; worse yet, winter just set in Friday afternoon, and even with the heat on, the temperature in your apartment is only 15°C. The rate at which your cat loses body heat is:

\[ Q = - h A (T - T_{air}), \]

where \( T = \) body temperature, \( A = \) Surface area, and \( h = \) heat transfer coefficient.

a. Luckily for Mr. Whiskers (and numerous other mammals), his metabolic system is capable of maintaining a constant body temperature of 38.6°C by converting food into body heat. How many calories of cat food does Mr. Whiskers need to stay warm enough to survive the winter?

b. By next winter, Mr. Whiskers has fully recovered from his haunting stay in New Orleans, and has grown back all his hair. You, in the meantime, have finally purchased that 42" television you always wanted, and watch it incessantly (as it is college basketball season, after all…). Of Course, (i) any good 42" television emits 200 kcal/day of heat, and (ii) any good cat will sleep all day on top of the television to harness *all* of that heat. Calculate how many calories worth of cat food Mr. Whiskers needs to eat to stay warm this winter.

Additional Information:

- Surface Area of Mr. Whiskers: 2 ft\(^2\)
- Heat Transfer Coefficient, Cat: 25 kcal/ft\(^2\)°C.day
- Heat Transfer Coefficient, Cat, Hairless: 42 kcal/ft\(^2\)°C.day
- Heat Capacity, Cat: 0.86 kg/kcal °C
3. Designing an Indoor Football Practice Field (50 points)

The University has decided to construct an all-weather indoor football practice field, as part of an aggressive campaign to recruit another coach. You have been appointed Overseer of Climate-Acclimated Inhalation Nominalization (O.'C.A.I.N.) As the official O.C.A.I.N. (or, as your friends say, 'Coach O'Cain',) you must determine the following design specifications.

a. If the indoor dimensions of the new field are 100x50x15 meters and must be designed to accommodate 250 athletes at any time, at what rate should fresh air be supplied (a.k.a. ventilation rate, in kg air / min) to maintain an oxygen concentration of at least 0.20 weight %?

b. The mean minimum outside air temperature per year is -18°C. How much heat flow is needed to maintain the empty facility at a comfortable 22°C on the coldest day of the year before practice starts (i.e. when there are no athletes)? Calculate Q in KJ/min. Ignore heat losses through the walls. Use the same ventilation rate as in part a.

c. If the heaters stay on during full team practice (100 athletes exercising), and practice lasts three hours, how warm will it get on-field? (HINT: Begin by deriving the Temperature as a function of time). Use the same ventilation rate as in part a.

Suppose that you are asked to determine if adding (more) insulation to the walls and ceiling of the building would be worthwhile.

d. How would you model heat losses through the walls of the building in your equation? (Include the entire new energy balance equation will all of the necessary terms.)

e. What key questions would you ask to determine if the additional insulation was worth the cost?

Additional Information:

Density of Air: 1.00 Kg / m³
Heat Capacity of Air: 1 KJ / Kg. K
O₂ Concentration in Air: 0.21 Kg O₂ / Kg Air
O₂ Consumption per Athlete: 1 g O₂ / min
Heat Generated per Athlete: 80 KJ/min
4. Compression of an ideal gas (60 points)

One mole of an ideal gas is trapped by a moveable piston. The initial pressure is $P_1$, the initial temperature is $T_1$ and the initial volume is $V_1$.

The gas is first compressed to a final state, $P_2$, $T_2$, and $V_2$.

a. Write down the energy balance that describes the gas inside the piston for this compression. Note that no information has been given about how fast this occurs and if any heat exchange has occurred (so be general).

b. If you know both initial and final temperatures, pressures and volumes, find an expression for the change in internal energy.

c. Suppose that the gas is compressed isothermally (at $T_1$) from $P_1$ to $P_2$. Find an expression for the work and heat for this step.

d. Now suppose that the pressure is held constant at $P_2$, and the gas is heated from $T_1$ to $T_2$. Find the heat and work for this step.

e. For all of these steps, is the work done on the atmosphere always the same as the work done by the gas? Explain if/when differences can arise with equations.

For the second part of this problem, the compression is being done by a "compressor" operating at steady state. Again the initial values are $P_1$, $T_1$ and $V_1$ and the final values are $P_2$, $T_2$, and $V_2$. The gas flow rate is $N$.

f. Write down the energy balance that describes this compression. Note that no information is given about any heat exchange that may be occurring so please be general.

g. Find an expression for the rate of work if there is no heat exchange. Note that $P_1$ and $P_2$ are given and the gas is known to follow a path where $PV^{\gamma}$ is constant.

h. Find the change in internal energy (per mole) for the compression that is occurring.
5. **Salt dissolving in water (35 points)**

A quantity of salt tablets of mass M and total area, \(a\) have been placed in a tank filled with a volume, \(V\) of water that has density, \(\rho\). The concentration of salt in the water is \(C_A\) and the equilibrium solubility of salt in water at this temperature is \(C_{A^e}\). The mass transfer coefficient is \(K_m\) and the greatest engineering equation of them all governs the rate of mass transfer.

a. Write down the equation that describes the rate at which salt is accumulating in the water phase.

b. Write down the equation that describes the rate at which salt is leaving the salt phase.

c. Find an algebraic expression for the concentration of the salt in the water as a function of time. You can assume that the area, \(a\) remains constant, but there is no special limiting values of the concentration compared to the equilibrium concentration. The initial salt concentration is 0.

d. Explain why allowing the area to remain constant simplifies the problem for you. (Please discuss what changes with this problem if the area is not constant.)

e. Explain the effect of changing the initial amount of water on the rate at which salt is dissolving. Please use equations.

f. Find a group of variables in your solution that is dimensionless and explain the physical relevance of it. Note that this group should include \(K_m\).
Potentially useful formulas

Reacting systems:
\[ \frac{d (c_A V)}{dt} = -r_{A - V} \]
\[ \frac{d (c_B V)}{dt} = -r_{B - V} \]
\[ \frac{d (c_D V)}{dt} = r_{D + V} \]
\[ \frac{d V}{dt} = q f_A + q f_B - q \]
\[ \frac{d (c_A V)}{dt} = q f_A c_A - q c_A - r_{A - V} \]
\[ \frac{d (c_B V)}{dt} = q f_B c_B - q c_B - r_{B - V} \]
\[ \frac{d (c_D V)}{dt} = -q c_D + r_{D - V} \]

Non reacting systems:
\[ \frac{d (\Box V)}{dt} = q_1 - q_i \]
\[ \frac{d (c_i V)}{dt} = q_1 c_i - q_i c_i + A - D \]

Multiphase systems
\[ \frac{d \Box V^I}{dt} = a r_A \]
\[ \frac{d \Box V^{II}}{dt} = a r_A \]
\[ \frac{d c_A V^I}{dt} = a r_A \]
\[ \frac{d c_A V^{II}}{dt} = a r_A \]
\[ r_A = K_m [c_A^I \Box f(c_A^{II})] \]

\[ \ln[15] := \int_{x_0}^{x_1} \frac{1}{y} \, dy \]
\[ \text{Out}[15] = -\log(x_0) + \log(x_1) \]

\[ \ln[8] := \int_{x_0}^{x} \frac{1}{x^2} \, dx \]
\[ \text{Out}[8] = -\frac{1}{y} + \frac{1}{y^2} \]
\[ \text{In}[9] := \int_{x_0}^{x} \frac{1}{x^n} \, dx \]

\[ \text{Out}[9] = -\frac{x^{1-n}}{-1+n} + \frac{x_0^{1-n}}{-1+n} \]

\[ \text{In}[11] := a \, x^2 + b \, x + c = 0 \]

\[ \text{Out}[11] = a x^2 + b x + c = 0 \]

\[ \text{In}[13] := \text{Solve}[x^2 + b x + c = 0] \]

\[ \text{Out}[13] = \left\{ \left\{ x \rightarrow \frac{-b - \sqrt{b^2 - 4ac}}{2a} \right\}, \left\{ x \rightarrow \frac{\sqrt{b^2 - 4ac} - b}{2a} \right\} \right\} \]