Comparison of flow regime transitions with interfacial wave transitions

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Flow geometry of interest

Two-fluid stratified flow

We will consider the transition from a stratified state in this talk. Examining (linear) stability of other structures (e.g., slugs) is also a viable way to formulate the flow regime problem.
Points of this talk

• Examine the use of linear stability models for flow regime transition
  – Long-wave stability is most appropriate based on transition data
  – Models from different studies don’t agree

• Explore the importance of nonlinear effects on the transition using a well-defined system
  – Yes, there are some nonlinear effects that do change the “predicted” transition.
  – But -- linear stability might be a good engineering model if done correctly. It is certainly a useful limiting point.
Some regime transition models

Slug transition models
- Kelvin-Helmholtz
- Wallis & Dobson (1973)
- Taitel & Dukler (1976)
- Lin & Hanratty (1986)
- Barnea (1991)
- Crowley et al. (1993)
- Bendiksen & Espedal (1992)

Lin & Hanratty (1987) data

Long wave linear stability
- laminar-laminar differential

 Slug transition models

<table>
<thead>
<tr>
<th>superficial liquid velocity, m/s</th>
<th>superficial gas velocity, m/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0001</td>
<td>0.01</td>
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<tr>
<td>0.001</td>
<td>0.1</td>
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<tr>
<td>0.01</td>
<td>1</td>
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<tr>
<td>1</td>
<td>10</td>
</tr>
</tbody>
</table>

channel=2.54 cm

liquid
- \( \rho_L = 1 \text{ g/cm}^3 \)
- \( \mu_L = 1 \text{ cp} \)

gas
- \( \rho_G = 0.0112 \text{ g/cm}^3 \)
- \( \mu_G = 0.018 \text{ cp} \)
Oil-Gas at 200 Atm.

channel=20 cm

liquid  gas
ρᵢₗ=0.9 g/cm³  ρᵢ₉=.232 g/cm³
μᵢₗ=50 cp  μᵢ₉=.0371 cp

Barnea (1991)  Kelvin-Helmholtz
Differential, laminar  Bendiksen & Espedal (1992)
Ruder & Hanratty (1989)

superficial liquid velocity, m/s
superficial gas velocity, m/s

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The key issue is the **growing wave peak at low frequency** which can lead to roll waves or slugs.

- When does it occur?

Data of Bruno and McCready, 1988
Wave transition as gas increases

Data of Minato 1996

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Wave transition as gas increases (more)

Data of Minato 1996
The transition at increasing liquid flow rate

Data of Kuru, 1995
Experiments

*laminar, oil-water channel flow*

- Oil phase
- Water phase

- Tracings
- PSD
- CSD - Wave speeds
- Bicoherence

- 2.44 m
- 1 cm
- .2 mm platinum wire probes
- .5-1 cm
- 30 kHz Signal

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Measured wave transitions

$\omega$ (1/s)

$\phi_x$ (cm$^2$/s)

$f$ (Hz)

Linear growth:
- interfacial mode
- "shear" mode
- wave spectrum

Re$\_\text{oil} = 3$
Re$\_\text{water} = 650$

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Measured wave transitions

Linear growth:
- interfacial mode
- "shear" mode

measured spectrum

Re_{oil}=3
Re_{water}=1200
Long wave stability -- THE criterion?

• We might be tempted to conclude that long wave stability, if we could get it correct -- would tell us everything.

• However, there are two problems.
  – 1. "Long" wave stability does not mean all long waves are unstable.
  – 2. We have strong experimental evidence of a range of existence of unstable long waves where there are no visible waves.
Theoretical analysis  (linear laminar theory)

The complete differential linear problem can be formulated as

\[
U = \Phi'(y) \exp[i \ k \ (x-c \ t)] , \quad u = \phi'(y) \exp[i \ k \ (x-c \ t)] ,
\]

\[
V = -i \ k \ \Phi(y) \exp[i \ k \ (x-c \ t)] , \quad v = -i \ k \ \phi(y) \exp[i \ k \ (x-c \ t)]
\]

where \( \Phi(y) \) and \( \phi(y) \) are the disturbance stream functions

\[
\Phi = \Phi = 0 \quad @y=1, \quad [1a]
\]

\[
\phi = \Phi, \quad @y=0, \quad [1b]
\]

\[
\phi' - \frac{u_b \phi}{u_b(0)-c} = \Phi' - \frac{U_b \phi}{U_b(0)-c} \quad @y=0, \quad [1c]
\]

\[
\phi'' + k^2 \phi = \mu (\Phi'' + k^2 \Phi), \quad @y=0, \quad [1d]
\]

\[
\frac{1}{\nu R} (\phi'' - 3 k^2 \phi') + i k (\phi \ u_b' - \phi' (u_b(0) - c)) + \frac{\ i \ k \ \phi}{(u_b(0) - c)} \frac{(F + k^2 \ T)}{R^2} =
\]

\[
\rho \ \frac{R}{R} (\Phi'' - 3 k^2 \Phi') + \rho i k (\Phi \ U_b' - \Phi' \sigma) + \frac{\ i \ \rho \ \phi}{(u_b(0) - c)} \frac{F}{R^2} ,
\quad @y=0, \quad [1e]
\]
Theoretical analysis (linear laminar theory)

\[ i k (U_b -c) (\Phi'' - k^2 \Phi) - i k U_b'' \Phi = R^{-1}(\Phi^{iv} - 2 k^2 \Phi'' + k^4 \Phi), \]
for \(0 \leq y \leq 1 \) \[1f\]

\[ i k (u_b -c) (\phi'' - k^2 \phi) - i k u_b'' \phi = (v R)^{-1}(\phi^{iv} - 2 k^2 \phi'' + k^4 \phi), \]
for \(-1/d \leq y \leq 0 \) \[1g\]

\[ \phi = \phi' = 0, \quad @ y = -d^{-1}. \] \[1h\]

viscosity ratio ==> \( \mu = \mu_2/\mu_1 \), density ratio ==> \( \rho = \rho_2/\rho_1 \),

ratio of kinematic viscosities ==> \( \nu, \quad \sigma = u_b(0) -c \)

depth ratio ==> \( d = D_2 \uparrow/D_1 \uparrow \),

wavenumber ==> \( k \)

liquid average velocity profiles ==> \( u_b \),

gas velocity ==> \( U_b \)
Unstable long waves, stable intermediate region

\[ \text{growth rate (1/s)} \]

\[ \text{wavenumber (m}^{-1}) \]

- \( R_G = 20000 \)
- \( \mu_L = 5 \text{ cP} \)
- \( R_L = 35 \)
- \( R_L = 25 \)
- \( R_L = 20 \)
- \( R_L = 15 \)
- \( R_L = 10 \)
Two-(matched density) liquid, rotating Couette device

laser

camera

mirror

mercury

Couette Cell

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Rotating Couette experiment

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Wave map for rotating Couette flow experiment

Regions of "no waves" exist where long waves are unstable

- Stable long waves
- Steady 2-D waves occur in most of this range

Legend:
- No waves
- Steady periodic
- Unsteady waves
- Solitary
- Long wave stability boundary

Will show spectral simulation later at this condition.
Weakly- nonlinear theory

\[ \frac{Du}{Dt} = -\nabla p + \frac{1}{R} \nabla^2 u \]

Spectral reduction of Navier-Stokes equations and boundary conditions.

The interface is represented by, \( \psi = (u, p, h) \)

We make the assumption that the waves can be represented by a series of modes which have a complex amplitude, A, multiplying a linear eigenfunction, \( \zeta \),

\[ \psi = \sum A_i \zeta_i \]

A series of amplitude coupled amplitude equations is integrated.

\[ \frac{\partial A_i}{\partial t} = L(k_i) + \alpha_{ji} A_j A^*_j - i + \beta_{ji} A_i - j A_j + \gamma_{kji} A_i A_j A_k \]

Both the dynamic and steady state behavior are watched.
Comparison of oil-water experiments and simulation

![Graph showing wave amplitude vs. wavenumber for different ReW values.]

- Linear stability
  - ReW=650
  - ReW=1200

- Simulated Spectra
  - ReW=650
  - ReW=1200
Measured wave transitions

![Graph showing wave transitions with parameters and modes labeled]

**Linear growth:**
- interfacial mode
- "shear" mode
- wave spectrum

Re_{oil} = 3
Re_{water} = 650
Measured wave transitions

Linear growth:
- interfacial mode
- "shear" mode
- measured spectrum

$\omega$ (1/s) (linear growth)

$f$ (Hz)

$\phi_{xx}$ (cm$^2$/s) (wave spectrum)

$Re_{oil}=3$
$Re_{water}=1200$

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Nonlinear effect on long wave formation

$Re_w=650$, cubic nonlinear coefficients are more balanced

$Re_w=1200$, large cubic nonlinear coefficients between large and small wavenumbers

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Gas-liquid simulations

wave amplitude

wavenumber (1/m)

Temporal Growth Rate (1/sec)

f (Hz)

Re_G = 9800
F_vich = 3.5 m
Re_1 = 240
Re_1 = 350
Re_1 = 470
Growth
μ_l = 1 cP

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Nonlinear coefficients for gas-liquid flow

Quadratic interactions with mean flow mode are much stronger at the higher liquid Reynolds number

Re\textsubscript{G}=9800, Re\textsubscript{L}=240

Re\textsubscript{G}=9800, Re\textsubscript{L}=470
Couette flow linear growth rate

- Depth ratio = 1.5
- Viscosity ratio = 55
- Density ratio = 1
- Rotation rate = 15 cm/s
Spectral simulation, Couette flow

Weakly-nonlinear spectral simulation

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Two-layer Couette flow

viscosity ratio = 55
density ratio = 1
depth ratio = 1.5
rotation rate = 15 cm/s

Wavenumber
Couette flow simulations

Evolution of wave spectrum with time. No preferred wavenumber exists. Should explain why we see no waves.
Conclusions

1. All evidence is that long wave stability is a necessary condition for the formation of long wave disturbances.

2. For many flow situations, there is a good correspondence between the long wave stability boundary and the formation of growing long waves.

3. However, there is the nagging problem with Couette flow, where a nonlinear cascade appears to saturate waves at an amplitude too small to see.
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Conclusions (cont.)

4. There is also the complication that the entire long wave region may not be unstable, this probably does prevent significant long wave formation, because …

5. The simulations suggest that different kinds of nonlinear interactions (e.g., cubic and or quadratic with various modes) are important in the development of the spectrum
   - Quadratic interactions enhance the formation of the low mode for gas-liquid flows and cubic interactions enhance it for oil-water flows

6. Certainly the situation is complex because large ocean waves are not the most linearly unstable (5 cm waves are), but these do not receive much of their energy from nonlinear processes, wind must directly feed them.
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