Didn't clearly understand the boundary layer concept (today). What did this diagram mean?

A boundary layer will be present near solid surfaces when the Reynolds number of the flow is large compared to one. A boundary – layer is the region close to the solid surface where there is a large gradient and in which both the inertia and viscous forces are about the same order of magnitude. We will spend some time studying these in the next couple lectures and are of no importance for this test.

I am unsure how to apply the equations. Are these equations useful for modeling real systems?

The Navier – Stokes equations are (almost) exactly correct for incompressible Newtonian fluid flows. Thus if you can solve them, they tell you exactly what would be happening. Generally you solve ***any*** *problem to the level of accuracy necessary to give you what you need to know.*

In this class we use the equations to solve simple flows, we write down the non-zero terms for more complex flows, and we use the equations to gain insight for any flow.

In the chemical industry, what would a rotating sphere in a fluid be used for (practical use)?

In any system that has fluids and solids that are flowing, the solid particles will be rotating. The solids could be a catalyst, a solid that is dissolving, or a solid that is growing by condensation or crystallization. To get the rate of growth or the rate of reaction for the system, you will need the mass transfer coefficient. The flow field plays a big role in determining the mass transfer rate. If the particle is small enough for the Reynolds number to be small, the flow field from the rotation can be added to the flow field from any translation to get the total flow field. So the rotating flow problem gives insight into the rate of heat or mass transfer from a sphere and MOST IMPORTANTLY, TELLS HOW THESE WOULD CHANGE WITH CHANGES IN STIRRING OR OTHER FORCING. A lot of the insight I am trying to get you to gain in this course could not only be used to solve a new problem but will tell you what happens when things change.

How would you solve $\nabla^2 f = 0$?

LaPlace's equation is solvable by separation of variables if you want the answer on a finite domain. If you think of Cartesian coordinates, you would have f= X(x) Y(y). Try substituting this in to get a solution.

What is the dimensionless group governing elasticity?

Take a peak at Denn's book. (I just talked with him on Tuesday. He is well – if you are interested!)

Are the Euler equations used for anything if they are so difficult to solve?

The Euler equations, are sometimes solved if there is a ***rotational*** *inviscid flow. (This means that the fluid rotation was created in a viscous part of the flow field that you are not solving, and has been transported to a part of the flow field that you are solving). However,* *we will do just irrotational flows and thus we will just solve LaPlace's equation for the flow field and use Euler's equation to get the pressure field.*

Chapter four seems a little long and confusing. Are there any other major points we should be familiar with in that chapter other than the Navier-Stokes equations?

I will send a note on this as soon as I get access to a book.

The press has said that Davie has been calling the defensive schemes during the games ever since the second half of the Purdue game. Is that why our defensive has been slacking in the second half of recent games?

It is hard to know this answer for sure. I think that other teams have been a little stubborn and have tried to run against us even though the linebackers (in particular) and the defensive line are the strength of the defense. However, if they get behind, they try passing and it seems to often work. Fortunately, they have often been too far behind to catch up (WVU) or we just get enough luck (Air Force) to allow us to win. If I were an offensive coordinator and had some big receivers, I would run sets that spread us out enough to force man coverage or I would check to a pass anytime I saw man coverage on the wide receivers.

Despite being okay on homework, I still feel lost in lectures. What can I do?

I try to give you a framework for the subject in lectures that allows you to develop your own way of thinking that will you help you understand. I would suggest studying the notes and reading the book. However, if you can do the homework, you might actually understand better than you think.

What is the difference between $\nabla \cdot \mathbf{u}$ and $\nabla\mathbf{u}$?

The first, "del dot u" is also called the divergence and is the sum of the derivatives of the velocity (with respect to the same direction as the velocity) in all three directions. It produces a scalar number. The second, "grad u" produces a 3 by 3 array (a second order tensor) which gives all of the derivatives of the velocity with respect to all three directions. The "nabla", (∇) is called del or grad depending upon which of the two you are calculating. Note that in these equations, both the **nabla** and **u** represent vectors.

What useful design information do we get from Navier-Stokes eqs?

People solve the Navier-Stokes equations, or various simplifications of them for many flows in many industries. Flows in stirred tank reactors are a common application for the chemical industry. Other cases could be cyclone separators, (that separate solids from liquids or solids) flows in complex geometries in distillation columns or within the channels for plastic molding (where more general equations valid for visco-elastic fluids could be solved). Probably, you can find cases where people have solved them in almost any kind of process equipment.

I'm still not sure about nondimensionality PDE's. How do you pick a characteristic length and velocity so that terms are no bigger than 1?

You ask a question that is not always easy to answer. However for the problems that we deal with, it will be straightforward. The length scale needs to represent the main geometric length. Thus if it is flow through something, it would be the diameter or radius. (Note that the factor of 2 is of no consequence in the arguments.) The velocity just needs to be some sort of average velocity over a region or the nominal velocity that is impinging on the object. Again, a factor of 2 would not be significant.

Once you do this, the terms will automatically not be bigger than one. This is because the numerator is just a change in nondimensional velocity, which is no bigger than one and the denominator is the change in length over which the change occurs, or the square of this, which will also be about one. Certainly the velocity variation inside a pipe will change over the length that you choose (the radius). As for flow around something, you might expect the velocity to start slowing down a couple of diameters away, but not 20 diameters away. Thus the length scale argument works as well.

The situations in which this question is not easy to answer are when there are regions in the flow where the velocity changes a lot in a small distance. This may create the need to use two different length scales. In this class we will deal with boundary layers, which are regions of large change in a small distance. This will be discussed specifically in a future lecture.

Still not sure how to solve some of the more involved PDE's after simplification--just use computer?

You will be getting some practice in this if you work through the solutions that I have given on the computer. These involve separation of variables, as shown above. In Cartesian coordinates, you can probably work one out. If I ask about this on a test, I will give hints.

Kind of fuzzy on how to use dimensionless groups to find most important terms.

There are just two cases for the nondimensional equations. If the Reynolds number is large, the Reynolds number will multiply the inertia and pressure terms. Thus these terms will be retained because they are of order 1 and multiplied by a big number. The viscous terms are neglected.

If the Reynolds number is much smaller than 1, the Reynolds number multiplies just the inertia terms. Since these are about 1 and multiplied by a small number, they can be neglected.

The "rules" are then.

1. *Always keep pressure.*
2. *If Re is large, viscous terms are omitted*
3. *If Re is small, inertia terms are omitted.*

Are most of the fluids we will deal with in our careers of the form Re--> infinity or Re-->0?

A simple answer is yes. You could be dealing with very viscous materials such as suspensions of particles or plastic melts. In these flows, inertia is likely to be small and the Reynolds number is 0. Alternatively, most process scale flows of less viscous materials are very high Reynolds number. The problem with the latter case is that the flow is turbulent which makes detailed solutions difficult, however, the scaling arguments that we make are still valid.

I'm still not clear on when and how to use a non-dimensional group/nondimensionalization. Is it when the Navier-Stokes eqs are still unsolvable after canceling of terms?

If we can solve these or any equations exactly, we don't need to nondimensionalize. However, we might find this convenient to see how a solution depends on a dimensionless parameter, rather than each of several variables. It could, for example save use from recalculating the numbers many times of times and most likely would give some physical insight – which would help us understand our answers and may allow us to predict if small changes in one variable will lead to a big change in something else.

If we cannot solve the problem exactly, looking at the nomdimensional equations for terms that can be canceled might help. The ones we have considered are low Reynolds number flows in which the inertia terms can be neglected and inviscid flows (very high Reynolds number) which allow the viscous terms to be neglected.

What do we use all of the eqs for, that we saw in class today (Laplace, etc.)?

Publishers use equations to take up space in books, thus leading to greater profits for the publishers because they are not very densely packed. Engineers use them to solve problems. Better yet, we use them to get as much insight as possible without actually having to solve them.

On tests and homework, I will tell you which equations are to be solved. The only exception would be that you will need to find the non-zero terms in the Navier-Stokes equations for a particular flow.

How do you know what terms to cancel in Navier-Stokes? How do you know exactly which terms to use in the Navier-Stokes when setting up a problem and knowing the best orientation?

This depends on the geometry and the direction of the flow or the forcing that is creating the flow. I don't know if it is the best way since I have never tried it before this year, but I would make a table of derivative directions and velocity directions and check the ones that could be non-zero. Once you have done this, you can immediately see the terms that are non-zero.

$\frac{\partial}{\partial \phi}$ v_ϕ

$$\frac{\partial}{\partial \theta} \qquad v_\theta$$

$$\frac{\partial}{\partial r} \qquad v_r$$

As far as orientation goes, if there is rotation, this must be aligned with the θ *axis for cylindrical coordinates and* ϕ *for spherical coordinates. These are also the axes of symmetry. There really are not a lot of possible problems that you would not find difficult to orient within a coordinate system. We already did the normal cases that exist.*

On this next test, will there be questions on the Magic School Bus? Will we be asked to cross out the terms that are =0, find boundary conditions, and entirely solve the differential equations resulting from the Navier-Stokes equations?

I wish I had the Magic School Bus tape on friction for one of the remaining Wednesday nights. You will be asked write the non-zero terms of the N-S equations for some problems. (I like the half full viewpoint.) You will need to know the boundary conditions. You will need to solve the differential equations – at least the ODE's or a simple PDE.

I wonder how much stuff we are going to cover--there seems to be SO much stuff in fluids. How are we doing on course goals/outline from first of year?

I will give the book sections as soon as I can. Otherwise, the notes since the last test and the homework is what you need to know.

I have a hard time picking which dimensions there are no changes in and which velocity terms are zero. Any advice?

Try making the table that I list above for your coordinate system.

What is *fugacity*? Why is it important, and what are some real life uses other than to torture undergraduate students? What does it physically represent?

In some ways, fugacity means different things to different people. (I should have asked Sandler when he was telling election jokes yesterday in the workout room). If you look closely at his book when he talks about fugacity he make a big point that it is not just some sort of corrected pressure – a comment in direct response to a book by John Prausnitz (Brennecke's academic grandfather and my academic uncle) who states that it is a corrected pressure. At least this was in the first edition.

Fugacity is a function of the excess Gibbs Free Energy, which represents a substances chemical "activity" in comparison with other substances. Matching excess Gibbs free energy for a substance in different phases is the criterion for phase (and chemical) equilibrium. Thus equal fugacity for a substance in different phases is also a criterion for phase equilibria. The *problem with Excess Gibbs Free energy (and the corresponding beauty of fugacity) is that it is not possible to express the Gibbs Free energy in terms of simple functions of concentration or mole fraction. However, fugacity is quite easily expressed. For ideal VLE you would write*

$f_i^L = x P^{vap}$

$f_i^V = y P$

And at equilibrium they must match. To me this gives me the best concept of fugacity. In terms of great engineering equations, it is a close 2^{nd} *to* $N = k (C- C^*)$ *for heat and mass transfer.*

Note that if the liquid phase is nonideal, you need an activity coefficient. If the pressure is high you need a Poynting correction, etc..

One problem the chemical engineers get right (or should) and the other novice engineers mess up is gas solubility using Henry's law. You will see people write

$xH = y.$

The question is, what is the effect of pressure? Since we know about fugacity, we would write

$x_i H = y_i P$. *Then we get the correct answer for any pressure less than, perhaps, 50 ATM.*

Are the problems (inviscid flow) that use the equations (e.g., LaPlace) we learned today long and complicated?

The inviscid flow problems are easier than low Reynolds number flow because the underlying differential equation is only second order.

How much deriving is going to be on the test? What is going to be given?

You will get photocopied pages of the Navier-Stokes equations and all of the other main equations that we have worked with. As we discussed earlier, "deriving" to me is getting these equations and there will be none of that. However, if "deriving" to you is solving the equations, there will be a lot of this.

I didn't understand non-rotation of fluid particles.

Think of a hockey puck as a fluid particle. If you wish to make it rotate while exerting only a surface force, this must be a tangential force (we are excluding a body force that could cause rotation without touching.) If there is no friction, a tangential force cannot be exerted and the puck would not rotate. "Friction" is analogous to viscosity of a fluid. If there is no viscosity, there is no way to transmit a tangential force. Thus for an inviscid flow, the fluid "particles" and hence the fluid does not rotate.

Will we have to solve the diff. eqs. in the exam when they have several variables?

You may need to solve a simple PDE that will be separable.

In thermo, it seems like every equation we learn is an approximation. How accurate are equations in fluids? Do the Navier-Stokes equations apply to all fluid flow?

The Navier-Stokes equations are exact for simple liquids and gases at constant temperature. The velocity must be less than a substantial fraction of the speed of sound and the density must be such that the mean free path is small compared to the dimensions of the flow. If the fluid is a polymer melt or a suspension, more complicated (than Newton's law) relations are used for the viscous stresses. These all are approximations to varying degrees.

I still don't understand what exactly we are supposed to do with the nondimensional equation once we have it. How does Re just go into the equation?

The nondimensional equations are solved the same as dimensional equations. However, you may have used nondimensionalization to make an argument that some terms can be neglected even if they are not exactly 0.

Can you go over practice exam 2?

I can answer specific questions.

How can we solve the equations derived from the Navier-Stokes equation?

If you need to solve a PDE, this will likely be by separation of variables that requires you to guess the angle function from the boundary conditions and when you substitute, you get an Euler differential equation that is readily solved. If you just need to solve an ODE, it will be straightforward.

How can you truly see flow around a sphere since you can't actually suspend a sphere with flow going around it?

Actually, you can do this. You would use wires as tethers. The wire diameter needs to be very much smaller than the sphere diameter and actually, much smaller than the thickness of any flow visualization smoke or die streaks you are using. The problem with the sphere is getting a photo of just the planar cross section down the center axis is hard If you see anything out of the plane, the picture will be distorted. Most pictures of flow past something like a sphere are really flow past a circular cylinder. All aspects of the experiment are easier. However, the mathematical solution for creeping flow does not exist.

Besides a rotating sphere or fluid past a sphere what other situations can exist? What kind of situations are possible for use of Navier-Stokes equations? I know of rotating cylinder, flow past sphere?

The pure radial flwo inside a bubble is a good one. You can have a rotating drop of liquid in another liquid, a rising bubble that has flow inside the bubble and a cylinder rotating in a *container where the walls are very far away. You could have a flowing liquid with a lot of solid particles in it.*

When do you include the pressure term when canceling out terms in the Navier-Stokes equations? When can you eliminate it?

The pressure is never eliminated a priori because of the Reynolds number being much less than or much greater than one. The only time it will be missing from the equation is when there is symmetry and the pressure does not change in a particular direction or the flow is driven by a moving wall and again the pressure does not change.

By simplifying down to $\nabla^2 f = 0$, will that really help us for that awful $\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + g$ for any other situation?

Yes. This simplification that results from the lack of fluid rotation makes this problem substantially simpler.

In Navier-Stokes, is left side all inertial and right side all viscous components?
What exactly does inertial forces mean?

Maybe I should more properly say that these are the acceleration terms that arise when a fluid flow changes inertia either by changing speed or direction. The left side of the NS equations is the "m ***a****" of m****a*** *=* ***F.***

When will we use the things we're learning once we graduate?

You don't need to wait until you graduate. You can amaze your family at thanksgiving explaining the way that cooking time for a turkey depends on its weight or even guessing how long it would be. You will build on this course next semester in the second transport class and use info from this course in lab.

When you go to work you will find that mass and heat transfer problems exist in many different kinds of equipment and often limit production. You understanding of fluid mechanics is not separate from the heat and mass transfer problems, but it will also help you understand where and how fluid flows in packed bed reactors, other process equipment, within porous media either in the ground or in tumors.

Can you go over exactly how to eliminate the parts of the Navier-Stokes eqn one more time? And does it get any "more harder" after this semester?

Unfortunately we have run out of chances, but you can send email or come see me. Try making the table that I mention above.

I think that the hardest intellectual parts of the chemical engineering curriculum are just about over. You will find lab time consuming and you will find that you will be writing about things that you do not understand completely – but you must try to attain this understand.

You will also find that lab is a lot of work. Transport II will probably seem easier that this semester. After that it is all downhill – but please, do the work.

What is the most effective way to study for the next exam?

You need to work on the homework and make sure that you know it. Study the lecture notes and try to understand what I was trying to get across. This should make more sense upon further reflection. Then read the book sections.

A person does not spin only in an ideal fluid right? In any real fluid the person spins?

For the ideal (invsicid) fluid, there is no fluid rotation. Thus the person does not spin. In a real fluid if there is a boundary somewhere, there will be a velocity gradient and hence some fluid rotation.