State-space modeling of systems with frequency dependent parameters

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ABSTRACT: In this paper, a framework for studying the dynamics of systems with frequency dependent parameters, under stochastic loads, using the state-space modeling approach is presented. Particular emphasis is placed on the stochastic buffeting response of long span bridges under multi-correlated wind excitation. The buffeting response is expressed as the output of an integrated system driven by a vector-valued white noise process. The integrated state-space model allows direct evaluation of the covariance matrix of the response using the Lyapunov equation, which leads to higher computational efficiency than the conventional spectral analysis approach. The proposed model facilitates time domain simulation of the multi-correlated wind fluctuations, the associated frequency dependent aerodynamic forces and the induced structural motion. The effectiveness of this approach is demonstrated using a long span cable-stayed bridge.

1 INTRODUCTION

Systems with frequency dependent parameters are often encountered in engineering problems, for example, the interaction of structures with soils and structural response to hydrodynamic and aerodynamic excitations (e.g., Roger 1977, Wolf 1991, Spanos & Zeldin 1997, Damaren 2000). In case of bridge buffeting response under multi-correlated wind excitation, the aerodynamic forces on bridges are frequency dependent due to unsteady fluid memory effects. These characteristics are generally described in terms of experimentally identified flutter derivatives for the self-excited forces, and admittance function and spanwise coherence for the buffeting forces, respectively. They are essential for an accurate evaluation of structural response under wind excitation.

Although these frequency dependent characteristics can be easily incorporated in the frequency domain analysis (e.g., Katsuchi et al. 1999, Chen et al. 2000a), a special treatment is required in the time domain simulation. Most previous time domain studies regarding the bridge buffeting response have customarily utilized frequency independent quasi-steady assumption which is valid only at very high reduced velocities. Chen et al. (2000b) proposed a time domain analysis framework that includes the frequency dependent aerodynamic forces. This framework offers a rigorous representation of the frequency domain analysis of linear problems provided that the aerodynamic forces can be represented by a rational function approximation (RFA) exactly or with an acceptable level of error.

In this paper, an integrated state-space model of a multi-input and multi-output system with a vector-valued white noise input is presented to model the stochastic buffeting response of bridges under multi-correlated winds. Such a unified model has not been developed before due to a number of modeling difficulties faced by researchers. This approach is applied to a long span cable-stayed bridge to demonstrate its effectiveness. The proposed approach provides immediate applications to soil-structure and fluid-structure interaction problems.

2 STATE-SPACE MODELING OF MULTI-CORRELATED WINDS

The mathematical model for describing the dynamic response of a structure under winds is described schematically in Fig. 1. The multi-correlated wind fluctuations are considered as the output of a system with a vector-valued white noise excitation. The modeling of multi-correlated wind fluctuations in a state-space framework was addressed in Goβmann & Walter (1983), Suhardjo et al. (1992), Matsumoto et al. (1996) and Kareem (1997). This is based on factorization of the cross power spectral density (XPSD) matrix and subsequent realization of the transfer function matrix. These operations are non-trivial for the simulation of a large size wind field. In some cases, the mathematical difficulty and numerical error introduced by the calculation procedure precluded the use of this technique to realistic problems (Matsumoto et al. 1996). Kareem (1997) suggested some simplifications, but the approach remained tedious as attested by a lack of attempts to model wind for a feed-forward linkage in control problems. In Kareem & Mei (1999), the stochastic decomposition technique was utilized for state-space modeling. Chen & Kareem (2000) pointed out that with the truncation of higher modes of wind fluctuations this stochastic decomposition technique provides an efficient tool for state-space modeling of well-correlated random processes. Its effectiveness in modeling poorly-correlated random processes is rather limited, particularly, for representing high-frequency wind fluctuations.

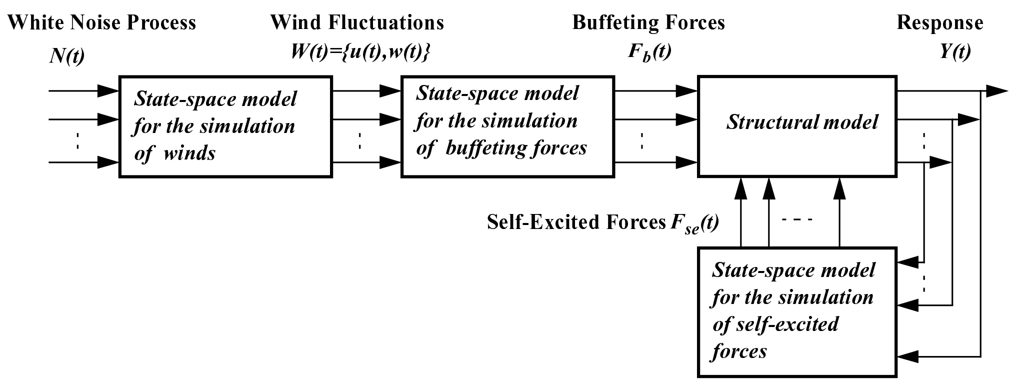

Figure 1 Integrated modeling of dynamic response of wind-excited structure

In this study, a multi-variate auto-regressive (AR) model is utilized for the state-space modeling of multi-correlated winds. This model provides an efficient and robust tool compared to the approach based on the factorization of the XPSD matrix and the approach based on the stochastic decomposition. Consider a structure represented by a finite element discretization. The wind fluctuations at the centers of elements, $\mathbf{W}(t)$, are represented by a multi-correlated random process. These can be represented as the output of a linear system with input of a vector-valued Gaussian white noise process $\mathbf{N}(t)$ with a zero mean and identity covariance matrix. This linear system can be described in terms of a multi-variate AR model as given below, which is considered as a special case of a general auto-regressive moving-average (ARMA) model (e.g., Mignolet and Spanos 1987, and Li and Kareem 1990a):

$$\mathbf{W}(t) = \sum_{k=1}^{p} \mathbf{P}_k \mathbf{W}(t - k\Delta t) + \mathbf{L}\mathbf{N}(t) \tag{1}$$

where Δt is the time interval; p is the model order; $\mathbf{P}_k$ $(k = 1, 2, ..., p)$ are the coefficient matrices which can be determined based on the wind correlation matrix.

Once the AR model is developed, there are several ways to express it in terms of a discrete-time state-space format. Here, the controllable canonical form is used (Ogata 1994):

$$\mathbf{X}_w(t) = \mathbf{A}_{dw}\mathbf{X}_w(t - \Delta t) + \mathbf{B}_{dw}\mathbf{N}(t); \quad \mathbf{W}(t) = \mathbf{C}_{dw}\mathbf{X}_w(t) + \mathbf{D}_{dw}\mathbf{N}(t) \tag{2}$$

where

$$\mathbf{X}_w(t) = \begin{bmatrix} \mathbf{X}_{w1}(t) \\ \mathbf{X}_{w2}(t) \\ \vdots \\ \mathbf{X}_{wp}(t) \end{bmatrix}; \ \mathbf{A}_{dw} = \begin{bmatrix} \mathbf{0} & \mathbf{I} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} & \dots & \mathbf{0} \\ \vdots & \vdots & \vdots & \dots & \vdots \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{I} \\ \mathbf{P}_p & \mathbf{P}_{p-1} & \mathbf{P}_{p-2} & \dots & \mathbf{P}_1 \end{bmatrix}; \ \mathbf{B}_{dw} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \vdots \\ \mathbf{I} \end{bmatrix};$$

$$\mathbf{C}_{dw} = [\ \mathbf{P}_p\mathbf{L} \quad \mathbf{P}_{p-1}\mathbf{L} \quad \dots \quad \mathbf{P}_1\mathbf{L}\ ]; \ \mathbf{D}_{dw} = \mathbf{L} \tag{3}$$

The equivalent continuous state-space representation can be given as

$$\dot{\mathbf{X}}_w = \mathbf{A}_w\mathbf{X}_w + \mathbf{B}_w\mathbf{N}; \quad \mathbf{W} = \mathbf{C}_w\mathbf{X}_w + \mathbf{D}_w\mathbf{N} \tag{4}$$

with the relationship

$$\mathbf{A}_{dw} = e^{\mathbf{A}_w\Delta t}; \ \mathbf{B}_{dw} = (e^{\mathbf{A}_w\Delta t} - \mathbf{I})\mathbf{A}_w^{-1}\mathbf{B}_w; \ \mathbf{C}_{dw} = \mathbf{C}_w; \ \mathbf{D}_{dw} = \mathbf{D}_w \tag{5}$$

3 SATE-SPACE MODELING OF UNSTEADY BUFFETING FORCES

The buffeting forces acting on a beam element of length l, i.e. lift (downward), drag (downwind) and pitching moment (nose-up), induced by a sinusoidal wind fluctuation with circular frequency ω, are expressed in the frequency domain as

$$\mathbf{F}_b^e(i\omega) = (\rho U^2 Bl)\mathbf{A}_b^e(ik)\mathbf{W}^e(i\omega) \tag{6}$$

where

$$\mathbf{A}_b^e = \begin{bmatrix} C_{L1}\chi_{L_{bu}}J_{L_{bu}} & C_{L2}\chi_{L_{bw}}J_{L_{bw}} \\ C_{D1}\chi_{D_{bu}}J_{D_{bu}} & C_{D2}\chi_{D_{bw}}J_{D_{bw}} \\ BC_{M1}\chi_{M_{bu}}J_{M_{bu}} & BC_{M2}\chi_{M_{bw}}J_{M_{bw}} \end{bmatrix}; \ \mathbf{F}_b^e = \begin{Bmatrix} L_b^e \\ D_b^e \\ M_b^e \end{Bmatrix}; \ \mathbf{W}^e = \begin{Bmatrix} u^e/U \\ w^e/U \end{Bmatrix};$$

$$C_{L1} = -C_L; \quad C_{L2} = -0.5(C_L' + C_D); \quad C_{D1} = C_D;$$
$$C_{D2} = 0.5(C_D' - C_L); \quad C_{M1} = C_M; \quad C_{M2} = 0.5C_M' \tag{7}$$

ρ is the air density; U is the mean wind velocity; $B = 2b$ is the bridge deck width; C_D, C_L, C_M are the static coefficients; $C_L' = dC_L/d\alpha$ and $C_M' = dC_M/d\alpha$; χ_r ($r = L_{bu}, L_{bw}, D_{bu}, D_{bw}, M_{bu}, M_{bw}$) denote the aerodynamic transfer functions between the wind fluctuations and buffeting forces per unit span, and its absolute magnitude is referred to as the aerodynamic admittance function; J_r ($r = L_{bu}, L_{bw}, D_{bu}, D_{bw}, M_{bu}, M_{bw}$) denote the joint acceptance functions that describe the reduction effect of the buffeting forces due to the loss of spanwise correlation within the element compared to the fully correlated case; u^e and w^e are the wind fluctuations at the center of the element; superscript e indicates the component on the element; the subscript r indicates $L_{bu}, L_{bw}, D_{bu}, D_{bw}, M_{bu}$ and M_{bw}; $k = \omega b/U$ is the reduced frequency; ω is the circular frequency; and $i = \sqrt{-1}$. For tower and cable elements of cable-supported bridges, only the drag component is generally considered in the analysis of overall bridge response.

The buffeting forces are derived as the output of a system with wind fluctuations as input. The state-space modeling of $\mathbf{F}_b^e$ will be accomplished in two steps. The first step is to model the aerodynamic transfer function $\chi_r(ik)$, and the second step is to model the joint acceptance function $J_r(ik)$. These are expressed in terms of the following rational function approximations (Chen et al. 2000b), while other forms can also be utilized:

$$\chi_r(ik) = A_{r,1}^{\chi} + \sum_{j=1}^{m_r^{\chi}} \frac{(ik)A_{r,j+1}^{\chi}}{ik + d_{r,j}^{\chi}}; \quad J_r(ik) = A_{r,1}^{J} + \sum_{j=1}^{m_r^{J}} \frac{(ik)A_{r,j+1}^{J}}{ik + d_{r,j}^{J}} \tag{8}$$

Accordingly, the state-space equations for $\mathbf{F}_b^e$ are then written as

$$\dot{\mathbf{X}}_b^e = \mathbf{A}_b^e\mathbf{X}_b^e + \mathbf{B}_b^e\mathbf{W}^e; \quad \mathbf{F}_b^e = \mathbf{C}_b^e\mathbf{X}_b^e + \mathbf{D}_b^e\mathbf{W}^e \tag{9}$$

Based on the finite element procedure, the total nodal force $\mathbf{F}_b(t)$ can be finally expressed in terms of the state-space equations with wind fluctuations as input, $\mathbf{W}(t)$,

$$\dot{\mathbf{X}}_b = \mathbf{A}_b\mathbf{X}_b + \mathbf{B}_b\mathbf{W}; \quad \mathbf{F}_b = \mathbf{C}_b\mathbf{X}_b + \mathbf{D}_b\mathbf{W} \tag{10}$$

4 STATE-SPACE MODELING OF SELF-EXCITED FORCES

The self-excited forces acting on a beam element of length l induced by a sinusoidal motion with circular frequency ω are expressed as follows by assuming the self-excited forces to be fully correlated:

$$\mathbf{F}_{se}^e(i\omega) = \frac{1}{2}\rho U^2\Big(\mathbf{A}_s^e(ik) + (ik)\mathbf{A}_d^e(ik)\Big)\mathbf{Y}^e(i\omega) \tag{11}$$

where

$$\mathbf{A}_s^e(ik) = \begin{bmatrix} 2k^2lH_4^* & 2k^2lH_6^* & 2k^2lbH_3^* \\ 2k^2lP_6^* & 2k^2lP_4^* & 2k^2lbP_3^* \\ 2k^2blA_4^* & 2k^2blA_6^* & 2k^2b^2lA_3^* \end{bmatrix}; \quad \mathbf{A}_d^e(ik) = \begin{bmatrix} 2klH_1^* & 2klH_5^* & 2klbH_2^* \\ 2klP_5^* & 2klP_1^* & 2klbP_2^* \\ 2kblA_1^* & 2kblA_5^* & 2kb^2lA_2^* \end{bmatrix};$$

$$\mathbf{F}_{se}^e(t) = [\ L_{se}^e \quad D_{se}^e \quad M_{se}^e\ ]^T; \quad \mathbf{Y}^e = [\ h^e \quad p^e \quad \alpha^e\ ]^T \tag{12}$$

and h^e, p^e and α^e are the vertical, lateral and torsional displacement at the center of the element, respectively; H_i^*, P_i^* and $A_i^*(i = 1, 2, ..., 6)$ are the flutter derivatives; and the over-dot denotes the differentiation with respect to time.

The self-excited forces are modeled as the output of a system with structural response as the input. Their transfer function is defined in terms of the flutter derivatives. Much research effort has been performed in the area of state-space modeling of unsteady self-excited aerodynamic forces in the aeronautical field by using a technique based on the rational function approximation (RFA) (e.g., Roger 1977). Among these schemes, Roger's RFA is the most widely utilized because of the accuracy, simplicity and robustness of the method. The application of RFA to bluff body bridge aerodynamics can be found in the representation of the self-excited forces (e.g., Scanlan et al. 1975; Wilde et al. 1996; Chen et al. 2000a and 2000b). By fitting the tabular data $\mathbf{H}_{se}^e(ik_j)$ defined at a set of discrete reduced frequencies k_j $(j = 1, 2, ...)$, the transfer matrix between the self-excited forces and the structural motion can be represented in terms of the following RFA in terms of the reduced frequency k (Roger 1977; and Chen et al. 2000a and 2000b):

$$\mathbf{H}_{se}^e(ik) = \mathbf{A}_s^e + (ik)\mathbf{A}_d^e = \mathbf{A}_1^e + (ik)\mathbf{A}_2^e + (ik)^2\mathbf{A}_3^e + \sum_{j=1}^{m^e}\frac{(ik)\mathbf{A}_{j+3}^e}{ik + d_j^e} \tag{13}$$

where $\mathbf{A}_1^e,\ \mathbf{A}_2^e,\ \mathbf{A}_3^e,\ \mathbf{A}_{j+3}^e$ and d_j^e $(d_j^e \geq 0; j = 1, 2, ..., m^e)$ are the frequency independent matrices and parameters; and m_e represents the order of RFA.

Replacing the Fourier transform by a Laplace transform through analytic continuation with $\bar{s}$ (where $\bar{s} = (-\xi + i)k$, ξ is the damping ratio of the motion) substituted for ik, and by taking an inverse Laplace transform, the self-excited forces induced by arbitrary motion can be expressed in terms of following state-space equations:

$$\dot{\mathbf{X}}_{sej}^e(t) = -\frac{d_j^e U}{b}\mathbf{X}_{sej}^e(t) + \mathbf{A}_{j+3}^e\dot{\mathbf{Y}}^e(t) \qquad (j = 1, 2, ..., m^e) \tag{14}$$

$$\mathbf{F}_{se}^{e}(t)=\frac{1}{2}\rho U^{2}\left(\mathbf{A}_{1}^{e}\mathbf{Y}^{e}(t)+\frac{b}{U}\mathbf{A}_{2}^{e}\dot{\mathbf{Y}}^{e}(t)+\frac{b^{2}}{U^{2}}\mathbf{A}_{3}^{e}\ddot{\mathbf{Y}}^{e}(t)+\sum_{j=1}^{m^{e}}\mathbf{X}_{sej}^{e}(t)\right) \tag{15}$$

where $\mathbf{X}_{sej}^{e}$ $(j=1,...,m^{e})$ are the augmented new variables representing the aerodynamic states.

Based on the finite element procedure, the total self-excited forces can be finally expressed in terms of the nodal motion $\mathbf{Y}$ as

$$\dot{\mathbf{X}}_{sej}(t)=-\frac{d_{j}U}{b}\mathbf{X}_{sej}(t)+\mathbf{A}_{j+3}\dot{\mathbf{Y}}(t) \qquad (j=1,2,...,m) \tag{16}$$

$$\mathbf{F}_{se}(t)=\frac{1}{2}\rho U^{2}\left(\mathbf{A}_{1}\mathbf{Y}(t)+\frac{b}{U}\mathbf{A}_{2}\dot{\mathbf{Y}}(t)+\frac{b^{2}}{U^{2}}\mathbf{A}_{3}\ddot{\mathbf{Y}}(t)+\sum_{j=1}^{m}\mathbf{X}_{sej}(t)\right) \tag{17}$$

where, $\mathbf{A}_1,\ \mathbf{A}_2,\ \mathbf{A}_3,\ \mathbf{A}_{j+3}$ and d_j $(d_j\geq 0; j=1,2,...,m)$ are the frequency independent matrices and parameter; m represents the order of RFA; and $\mathbf{X}_{sej}(t)$ $(j=1,2,...,m)$ are the augmented aerodynamic states.

5 REDUCED-ORDER STATE-SPACE MODEL OF INTEGRATED BRIDGE SYSTEM

For linear structures, the reduced-order equations of motion in terms of the generalized modal coordinates $\mathbf{q}$ can be utilized for computational convenience:

$$\mathbf{M_0}\ddot{\mathbf{q}}+\mathbf{C_0}\dot{\mathbf{q}}+\mathbf{K_0}\mathbf{q}=\mathbf{Q}_{se}+\mathbf{Q}_{b} \tag{18}$$

where $\mathbf{M_0}=\mathbf{\Psi}^{T}\mathbf{M}\mathbf{\Psi},\mathbf{C_0}=\mathbf{\Psi}^{T}\mathbf{C}\mathbf{\Psi}$ and $\mathbf{K_0}=\mathbf{\Psi}^{T}\mathbf{K}\mathbf{\Psi}$ are the generalized mass, damping and stiffness matrices, respectively; $\mathbf{Q}_{se}=\mathbf{\Psi}^{T}\mathbf{F}_{se}$ and $\mathbf{Q}_{b}=\mathbf{\Psi}^{T}\mathbf{F}_{b}$ are the generalized self-excited and buffeting force vectors, respectively; $\mathbf{\Psi}$ is the mode shape matrix; and $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are the mass, damping and stiffness matrices in physical coordinates, respectively.

The state-space equations of $\mathbf{Q}_b$ is given below based on the state-space representation of $\mathbf{F}_b$ (Eqs. 10)

$$\dot{\mathbf{X}}_{b}=\mathbf{A}_{b}\mathbf{X}_{b}+\mathbf{B}_{b}\mathbf{W}; \quad \mathbf{Q}_{b}=\mathbf{C}_{b0}\mathbf{X}_{b}+\mathbf{D}_{b0}\mathbf{W} \tag{19}$$

where $\mathbf{C}_{b0}=\mathbf{\Psi}^{T}\mathbf{C}_{b}$, $\mathbf{D}_{b0}=\mathbf{\Psi}^{T}\mathbf{D}_{b}$.

The state-space equations of $\mathbf{Q}_{se}$ can be given as follows based on the state-space model of $\mathbf{F}_{s}e$ (Eqs. 16 and 17)

$$\dot{\mathbf{q}}_{sej}(t) = -\frac{d_{j}U}{b}\mathbf{q}_{sej}(t)+\mathbf{Q}_{j+3}\dot{\mathbf{q}}(t) \qquad (j=1,2,...,m) \tag{20}$$

$$\mathbf{Q}_{se}(t) = \frac{1}{2}\rho U^{2}\left(\mathbf{Q}_{1}\mathbf{q}(t)+\frac{b}{U}\mathbf{Q}_{2}\dot{\mathbf{q}}(t)+\frac{b^{2}}{U^{2}}\mathbf{Q}_{3}\ddot{\mathbf{q}}(t)+\sum_{j=1}^{m}\mathbf{q}_{sej}(t)\right) \tag{21}$$

where $\mathbf{Q}_{1}=\mathbf{\Psi}^{T}\mathbf{A}_{1}\mathbf{\Psi},\ \mathbf{Q}_{2}=\mathbf{\Psi}^{T}\mathbf{A}_{2}\mathbf{\Psi},\ \mathbf{Q}_{3}=\mathbf{\Psi}^{T}\mathbf{A}_{3}\mathbf{\Psi},\ \mathbf{Q}_{j+3}=\mathbf{\Psi}^{T}\mathbf{A}_{j+3}\mathbf{\Psi}$ and $\mathbf{q}_{sej}(t)=\mathbf{\Psi}^{T}\mathbf{X}_{sej}(t)$ $(j=1,2,...,m)$.

An alternative approach for modeling the generalized self-excited forces is to directly fit the generalized modal aerodynamic matrices calculated at discrete reduced frequencies. If a smaller number of lag terms can thus be used, this will reduce the number of added aerodynamic states.

By augmenting the state-space equations of the structural motion with the corresponding state-space representations of the loading components and wind fluctuations as stated above, an integrated state-space model, given below, is established that synthesizes the unsteady characteristics of multi-correlated wind field, frequency dependent unsteady aerodynamic forces and the dynamics of the bridge.

$$\dot{\mathbf{X}}_{0}=\mathbf{A}_{0}\mathbf{X}_{0}+\mathbf{B}_{0}\mathbf{N}; \quad \mathbf{Y}=\mathbf{\Psi}^{T}\mathbf{G}_{0}\mathbf{X}_{0} \tag{22}$$

where

$$\mathbf{X}_0 = \begin{bmatrix} \mathbf{X}_{sse0} \\ \mathbf{X}_b \\ \mathbf{X}_w \end{bmatrix}; \quad \mathbf{A}_0 = \begin{bmatrix} \mathbf{A}_{sse0} & \mathbf{B}_{sse0}\mathbf{C}_{b0} & \mathbf{B}_{sse0}\mathbf{D}_{b0}\mathbf{C}_w \\ \mathbf{0} & \mathbf{A}_b & \mathbf{B}_b\mathbf{C}_w \\ \mathbf{0} & \mathbf{0} & \mathbf{A}_w \end{bmatrix};$$

$$\mathbf{B}_0 = \begin{bmatrix} \mathbf{B}_{sse0}\mathbf{D}_{b0}\mathbf{D}_w \\ \mathbf{B}_b\mathbf{D}_w \\ \mathbf{B}_w \end{bmatrix}; \quad \mathbf{G}_0 = \begin{bmatrix} \mathbf{C}_{sse0} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}^T;$$

$$\mathbf{X}_{sse0} = \begin{bmatrix} \mathbf{q} \\ \dot{\mathbf{q}} \\ \mathbf{q}_{se1} \\ \vdots \\ \mathbf{q}_{sem} \end{bmatrix}; \quad \mathbf{A}_{sse0} = \begin{bmatrix} \mathbf{0} & \mathbf{I} & \mathbf{0} & \dots & \mathbf{0} \\ -\bar{\mathbf{M}}_0^{-1}\bar{\mathbf{K}}_0 & -\bar{\mathbf{M}}_0^{-1}\bar{\mathbf{C}}_0 & \frac{1}{2}\rho U^2\bar{\mathbf{M}}_0^{-1} & \dots & \frac{1}{2}\rho U^2\bar{\mathbf{M}}_0^{-1} \\ \mathbf{0} & \mathbf{Q}_4 & -\frac{U}{b}d_1\mathbf{I} & \dots & \mathbf{0} \\ \vdots & \vdots & \vdots & & \vdots \\ \mathbf{0} & \mathbf{Q}_{3+m} & \mathbf{0} & \dots & -\frac{U}{b}d_m\mathbf{I} \end{bmatrix};$$

$$\mathbf{B}_{sse0} = \begin{bmatrix} \mathbf{0} & \bar{\mathbf{M}}_\mathbf{0}^{-1} & \mathbf{0} & \dots & \mathbf{0} \end{bmatrix}^T; \quad \mathbf{C}_{sse0} = \begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} \end{bmatrix}^T;$$

$$\bar{\mathbf{M}}_0 = \mathbf{M}_0 - \frac{1}{2}\rho b^2\mathbf{Q}_3; \quad \bar{\mathbf{C}}_0 = \mathbf{C}_0 - \frac{1}{2}\rho U b\mathbf{Q}_2; \quad \bar{\mathbf{K}}_0 = \mathbf{K}_0 - \frac{1}{2}\rho U^2\mathbf{Q}_3 \tag{23}$$

When considering a linear aerodynamic problem, the state-space modeling of $\mathbf{Q}_b$ can be established alternatively based on their XPSD matrix which is subsequently expressed in terms of an AR model, similar to the case of multi-correlated wind fluctuations.

The solution of Eq. 22 can be obtained by (Soong and Grigoriu 1993)

$$\mathbf{X}_0(t) = e^{\mathbf{A_0}(t-t_0)}\mathbf{X}_0(t_0) + \int_{t_0}^{t} e^{\mathbf{A}_0(t-\tau)}\mathbf{B}_0\mathbf{N}(\tau)d\tau \tag{24}$$

The covariance matrix $\mathbf{R}_\mathbf{X}$ can be directly calculated by solving the following Lyapunov equation:

$$\mathbf{A}_0\mathbf{R}_{\mathbf{X}0} + \mathbf{R}_{\mathbf{X}0}\mathbf{A}^T + \mathbf{B}_0\mathbf{B}_0^T = 0 \tag{25}$$

The recasting of the overall system equations in the state-space format allows the use of tools based on the linear system theory for the response analysis, optimization, and active suppression of flutter and alleviation of buffeting. By using this model, the wind load information can be incorporated in a structural control design as a feed-forward link with the potential to enhance the control effectiveness (Suhardjo et al. 1992). Direct calculation of the covariance matrix of the response by using the Lyapunov equation provides higher computational efficiency than the conventional spectral analysis approach. In addition, the structural and aerodynamic coupling effects can be automatically included in the analysis.

6 EXAMPLE

An example was used to illustrate the integrated state-space analysis framework formulated in this study. The example bridge is a cable-stayed bridge with a main span of approximately 1000 m. For simplicity and without loss of generality, only wind forces acting on the bridge deck were considered. The von Karman spectra were used for describing the power spectra of wind fluctuations. For u and w components, turbulence intensities and integral length scales equal to 10 % and 7.5 %, and 80 m and 40 m, respectively, were considered. Higher length scales were used in the calculation of coherence functions in order to account for the stronger spanwise correlation in the buffeting forces than those of the wind fluctuations. Length scales were chosen as 160 m and 80 m for the buffeting force component associated with the u and w components, respectively.

The bridge deck was discretized into 43 elements along the span. In this study, the u and w components were assumed to be independent. These were expressed using two separate state-space models with 258 states. The modeling is straightforward for the cases considering the correlation between u and w components of wind fluctuations.

For each element, two different admittance functions for u and w components were used, which were all expressed using RFAs with two rational terms. Davenport's formula with a decay factor of 8 was used for drag, and Sears function was used for the lift and pitching moment. Two different joint acceptance functions were used for buffeting force components associated with u and w components. These were also expressed using RFAs with two rational terms. The dimensions of the state-vector for the buffeting forces acting on each element and the overall structure were 6 and 258, respectively, and these were the same for both components corresponding to u and w components of wind fluctuations.

The drag component of the self-excited forces due to lateral motion was evaluated based on the quasi-steady theory, and the components relevant to the vertical and torsional motions were neglected. The lift and pitching moment components of the self-excited forces were calculated based on the Theodorsen function. The generalized self-excited forces on the first 12 bridge deck dominated natural modes were expressed using RFA with two rational terms. The natural frequencies range from 0.07 to 0.6 Hz. The logarithmic decrement for each mode was assumed to be 0.02. The total dimension of the state-vectors of the integrated system corresponding to u and w components were both equal to 564. For each of these two integrated systems, the covariance matrix was calculated using the Lyapunov equation to obtain the covariance of the total response.

Figure 2 shows the root mean square (RMS) buffeting response in lateral, vertical and torsional directions along the span at mean wind velocities of 40, 60 and 80 m/s compared with those based on the spectral analysis approach. Excellent agreement demonstrated the accuracy of the proposed framework, while the state-space model is computationally more efficient. For this example, computational effort using the proposed scheme is less than half of that needed for the conventional spectral approach. The critical flutter velocity was found to be 113.8 m/s utilizing a stability analysis of this integrated system through the complex eigenvalue analysis.

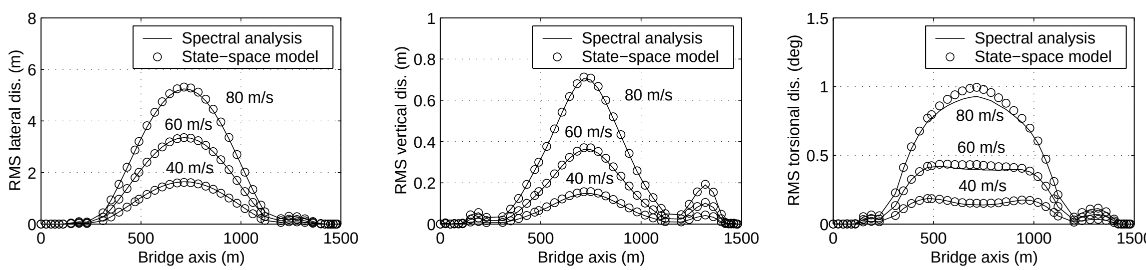

Figure 2 Comparison of the buffeting response

7 CONCLUDING REMARKS

An integrated state-space model of a multi-input and multi-output system with a vector-valued white noise input was presented to describe the dynamic response of bridges under multi-correlated winds. The state-space modeling of multi-correlated winds was established using a multi-variate AR model. The unsteady buffeting and self-excited forces were modeled using rational function approximations of their frequency dependent characteristics. The proposed approach helps to glean a clear insight into the modeling of wind-induced vibration problems.

This approach facilitates the use of tools based on linear system theory for the response analysis and structural control design. This procedure allows the time domain simulation of the response to incorporate the frequency dependent aerodynamic forces instead of the generally assumed quasi-steady forces. This feature enhances the accuracy of the predicted responses. This framework can be utilized in a structural control design by incorporating the wind loading information as a feed-forward link which improves the effectiveness of control actions. Direct evaluation of the covariance matrix of response using the Lyapunov equation offers computational efficiency over the conventional spectral analysis approach.

Although emphasis was placed in this study on the response of bridges under multi-correlated wind excitations, the proposed approach provides immediate applications to other structures under wind excitations. It has immediate applications to systems with frequency dependent parameters such as those involved in soil-structure and fluid-structure interactions.

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