Evaluation of Equivalent Static Wind Loads on Buildings

Xinzhong Chen[1*] and Ahsan Kareem[2]

[1] *Assistant Professor of Civil Engineering, Texas Tech University, Texas, USA, xinzhong.chen@ttu.edu*
[2] *Professor of Engineering, University of Notre Dame, Indiana, USA, kareem@nd.edu*

ABSTRACT

Wind loads on buildings can be quantified through synchronous measurements of pressure over building model surfaces or by high frequency force balance (HFFB) measurements. Although this loading information can be directly utilized for building design applications, current design practice often requires the transformation of dynamic wind loading to the equivalent static wind loads (ESWLs). This paper addresses prediction of wind load effects and modeling of associated ESWLs of buildings with uncoupled mode shapes of vibration. Both building response in primary directions and that contributed by the combined actions of wind loads in different directions are studied. Various approaches used in the development of ESWLs associated with the peak wind load effects of buildings are critically evaluated with some new insights that lead to improved modeling of wind loading for design applications.

INTRODUCTION

Wind loads on buildings can be derived through synchronous measurements of pressure over building model surfaces or by high frequency force balance (HFFB) measurements (e.g., Kareem and Cermak 1979; Tschanz and Davenport 1983; Boggs and Peterka 1989; Isyumov 1999). Buildings with symmetric plan and coincident centers of mass and resistance generally have mode shapes which are one-dimensional, i.e., uncoupled in two orthogonal translational and rotational directions. This permits discussion of wind loads and their effects (building response) in each primary direction independently. On the other hand, for buildings characterized by three-dimensional (3-D) coupled mode shapes and/or closely-spaced modal frequencies, a 3-D coupled response analysis framework should be utilized that takes into account the cross-correlation of wind loads acting in different directions and the inter-modal coupling of modal responses (e.g., Kareem 1985; Chen and Kareem 2005a and b).

The wind load effects can be generally separated into the mean (static), background (quasi-static) and resonant components. Predictions of the mean and background response components using the static and quasi-static analyses involving influence functions result in more accurate estimates than the modal analysis restricted to the fundamental mode. Whereas, the modal analysis offers sufficiently accurate prediction of the resonant response component. The synchronous pressure measurements provide a detailed loading information for response analysis, while the HFFB technique requires empirical mode shape correction and assumption of neglecting higher mode contributions to the background response.

The measured dynamic wind loading information can be directly utilized for building design applications. However, current design practice often requires the transformation of dynamic wind loading to the equivalent static wind loads (ESWLs). The modeling of ESWLs seeks static load distributions whose static effects on buildings are equal to the actual dynamic wind load effects. This load representation allows designers to follow a relatively simple static analysis procedure for prediction of building response to spatiotemporally varying dynamic loads, and is often more suitable to current design practice. This format serves as pivotal information for estimating response under the combined action of wind and other loads, and is widely used in current building codes and standards worldwide.

This paper addresses prediction of wind load effects and modeling of associated ESWLs of buildings with uncoupled mode shapes. Both building response in primary directions and that contributed by the combined actions of wind loads in different directions are studied. Various approaches used in the development of ESWLs associated with the peak values of these responses are critically evaluated with some new insights that lead to improved modeling of wind loading for design applications.

*Corresponding author. Tel: +1-806-742-3476; Fax: +1-806-742-3446
Mailing address: Box 41023, Lubbock, TX 79409-1023, USA

PEAK RESPONSE IN PRIMARY DIRECTIONS

Consider building response along the primary translational x direction at a given wind speed and direction. The wind load per unit height at elevation z above the ground has a mean component of $\bar{P}_x(z)$ and fluctuating component of $P_x(z,t)$, which can be obtained through synchronous measurements of pressure over the building model surfaces. Any building response (e.g., displacement, bending moment, shear force and member forces), $R_x(t)$, can be separated into the mean (static), background (quasi-static), and resonant components, and are quantified by the static, quasi-static analysis, and modal analysis involving only the fundamental mode, respectively. The mean response and root mean square (RMS) background and resonant responses are expressed as (e.g., Chen and Kareem 2004)

$$\bar{R}_x = \int_0^H \mu_x(z)\bar{P}_x(z)dz \tag{1}$$

$$\sigma_{R_{xb}} = \sqrt{\int_0^H \int_0^H \mu_x(z_1)\mu_x(z_2)R_{P_{xx}}(z_1,z_2)dz_1dz_2} \tag{2}$$

$$\sigma_{R_{xr}} = \frac{\int_0^H m(z)\Theta_x(z)\mu_x(z)dz}{\int_0^H m(z)\Theta_x^2(z)dz}\sqrt{\frac{\pi}{4\xi_x}f_x S_{Q_x}(f_x)} \tag{3}$$

$$S_{Q_x}(f) = \int_0^H \int_0^H \Theta_x(z_1)\Theta_x(z_2)S_{P_{xx}}(z_1,z_2,f)dz_1dz_2 \tag{4}$$

where H = building height; $\mu_x(z)$ = influence function indicating the response R_x under unit load acting at the elevation z in x direction; $m(z)$ = mass per unit height; $\Theta_x(z)$ = fundamental mode shape; f_x and ξ_x = fundamental frequency and damping ratio (including aerodynamic damping), respectively; $R_{P_{xx}}(z_1,z_2)$ and $S_{P_{xx}}(z_1,z_2,f)$ = covariance and cross power spectral density (XPSD) function between $P_x(z_1,t)$ and $P_x(z_2,t)$; $S_{Q_x}(f)$ = power spectral density (PSD) function of the generalized modal force.

The expected peak response (excluding the mean response), R_{max}, is obtained by combining the background and resonant components:

$$R_{xmax} = \sqrt{g_b^2\sigma_{R_{xb}}^2 + g_r^2\sigma_{R_{xr}}^2} \tag{5}$$

where g_b and g_r = peak factors for the background and resonant response components, respectively, typically ranging in value between 3 and 4.

It is noted that the background response analysis using the influence function implicitly includes the contributions of all structural modes to the background response. Therefore, it results in a more accurate response prediction compared to the modal analysis involving only the fundamental mode. Consideration of the contributions of higher modes to the background response is important for low- and middle-rise buildings, although for high-rise buildings their contributions may be negligible as the resonant response generally dominates the total response.

In the case where the spatiotemporally varying wind loading is not available, but the integrated wind loading in terms of base bending moment, $M_x(t)$, is quantified through the HFFB measurements using scaled building model, the generalized force can then be estimated as

$$S_{Q_x}(f) = \frac{\eta_x^2(f)}{H^2}S_{M_x}(f) \tag{6}$$

$$\eta_x^2(f) = \frac{\int_0^H \int_0^H \Theta_x(z_1)\Theta_x(z_2)S_{P_{xx}}(z_1,z_2,f)dz_1dz_2}{\int_0^H \int_0^H (z_1/H)(z_2/H)S_{P_{xx}}(z_1,z_2,f)dz_1dz_2} \tag{7}$$

where $\eta_x^2(f)$ = the mode shape correction factor which has to be estimated by using an empirical formulation (e.g., Vickery et al. 1985; Boggs and Peterka 1989; Xu and Kwok 1993; Zhou et al. 2002; Holmes et al. 2003; Chen and Kareem 2004). Accordingly, the resonant response is estimated by Eq. (3), and the background response can be expressed in terms of the measured base bending moment as

$$\sigma_{R_{xb}} = \frac{\int_0^H m(z)\Theta_x(z)\mu_x(z)dz}{\int_0^H m(z)\Theta_x(z)(z/H)dz}\frac{\sigma_{M_x}}{H} \tag{8}$$

where σ_{M_x} = RMS base bending moment.

It is worth mentioning that while the above formulation is based on the modal analysis involving only the fundamental mode, as the measured base bending moment involves all mode contributions, the contribution of higher modes are consequently considered in the predicted background response, but with an assumption that the response contribution of higher modes is identical for all response components. However, bending moments at higher elevations and shear forces at all elevations are generally more affected by higher modes. In particular, if the first mode was linear, the higher modes would provide no contributions to the base bending moment but other response components. Therefore, amplification of response relative to the value computed from Eq. (8) may be introduced for improved response prediction. Similar consideration has been made in building codes for seismic design including International Building Code (IBC) 2000 (e.g., Chopra 2000), in which the response prediction was based on the base shear force and an reduction factor was introduced for predicted bending moment.

Formulations of the response in the other two directions can be accordingly developed. For building response in torsion, $R_\theta(t)$, the background and resonant components are expressed in terms of the base torque $M_\theta(t)$ as

$$\sigma_{R_{\theta b}} = \frac{\int_0^H I(z)\Theta_\theta(z)\mu_\theta(z)dz}{\int_0^H I(z)\Theta_\theta(z)dz}\sigma_{M_\theta} \tag{9}$$

$$\sigma_{R_{\theta r}} = \frac{\int_0^H m(z)\Theta_\theta(z)\mu_\theta(z)dz}{\int_0^H I(z)\Theta_\theta^2(z)dz}\eta_\theta(f_\theta)\sqrt{\frac{\pi}{4\xi_\theta}f_\theta S_{Q_\theta}(f_\theta)} \tag{10}$$

$$\eta_\theta^2(f) = \frac{\int_0^H\int_0^H \Theta_\theta(z_1)\Theta_\theta(z_2)S_{P_{\theta\theta}}(z_1,z_2,f)dz_1dz_2}{\int_0^H\int_0^H S_{P_{\theta\theta}}(z_1,z_2,f)dz_1dz_2} \tag{11}$$

where $\mu_\theta(z)$ = influence function indicating the response R_θ under unit load acting at the elevation z in torsion; $I(z)$ = polar moment per unit height; $\Theta_\theta(z)$ = fundamental mode shape in torsion; f_θ and ξ_θ = fundamental frequency and damping ratio (including aerodynamic damping) in torsion, respectively; $S_{M_\theta}(f)$ and σ_{M_θ} =PSD and RMS value of $M_\theta(t)$; and $S_{P_{\theta\theta}}(z_1,z_2,f)$ = XPSD between the dynamic torque per unit height at elevation z_1, i.e., $P_\theta(z_1,t)$, and at elevation z_2, i.e., $P_\theta(z_2,t)$.

PEAK RESPONSE TO COMBINED LOADING

Consider a response $R(t)$ that is influenced by the actions of loadings in three primary directions

$$R(t) = R_x(t) + R_y(t) + R_\theta(t) \tag{12}$$

where R_s $(s = x, y, \theta)$ = response component associate with loading in the s direction.

The RMS background response of $R(t)$ is given by the combination of its components using the complete quadratic combination (CQC) rule:

$$\sigma_{R_b} = \sqrt{\sum_{s_1=x,y,\theta}\sum_{s_2=x,y,\theta}\sigma_{R_{s_1b}}\sigma_{R_{s_2b}}r_{R_{s_1s_2b}}} \tag{13}$$

where $\sigma_{R_{s_1b}}$ = RMS background response due to wind loading in the s_1 direction; $r_{R_{s_1s_2b}}$ = correlation coefficient between the background responses of $R_{s_1}(t)$ and $R_{s_2}(t)$, and is defined as

$$r_{R_{s_1s_2b}} = \int_0^H\int_0^H \mu_{s_1}(z_1)\mu_{s_2}(z_2)R_{P_{s_1s_2}}(z_1,z_2)dz_1dz_2/(\sigma_{R_{s_1b}}\sigma_{R_{s_2b}}) \tag{14}$$

It is noted that when the background response in each primary direction is approximated by the respective contribution of the corresponding fundamental mode, $r_{R_{s_1s_2b}}$ becomes independent of response and is identical to the correlation coefficient of generalized forces, $Q_{s_1}(t)$ and $Q_{s_2}(t)$:

$$r_{R_{s_1s_2b}} = r_{Q_{s_1s_2}} = \int_0^H\int_0^H \Theta_{s_1}(z_1)\Theta_{s_2}(z_2)R_{P_{s_1s_2}}(z_1,z_2)dz_1dz_2/(\sigma_{Q_{s_1}}\sigma_{Q_{s_2}}) \tag{15}$$

where σ_{Q_s} $(s = x, y, \theta)$ = RMS value of $Q_s(t)$. Furthermore, when the background response is directly evaluated based on the base bending moment or torque in Eq. (8) or (9), $r_{R_{s_1 s_2 b}}$ is then identical to the correlation coefficient of the base bending moments and torque, $M_{s_1}(t)$ and $M_{s_2}(t)$.

Similarly, the resonant response of $R(t)$ is given by

$$\sigma_{R_r} = \sqrt{\sum_{s_1=x,y,\theta} \sum_{s_2=x,y,\theta} \sigma_{R_{s_1 r}} \sigma_{R_{s_2 r}} r_{s_1 s_2 r}} \tag{16}$$

where $r_{s_1 s_2 r} = r_{jkr}$ = the correlation coefficient of the jth and kth resonant modal response components that correspond to the fundamental modes in s_1 and s_2 directions, which can be approximated by the following closed-form expressions (Der Kiureghian 1980; Chen and Kareem 2005a and b):

$$r_{jkr} = \alpha_{jkr} \rho_{jkr} \tag{17}$$

$$\alpha_{jkr} = \mathrm{Re}[S_{Q_{jk}}(f)] / \sqrt{S_{Q_j}(f) S_{Q_k}(f)}|_{f=f_j \ or \ f_k} \tag{18}$$

$$\rho_{jkr} = \frac{8\sqrt{\xi_j \xi_k}(\beta_{jk}\xi_j + \xi_k)\beta_{jk}^{3/2}}{(1-\beta_{jk}^2)^2 + 4\xi_j\xi_k\beta_{jk}(1+\beta_{jk}^2) + 4(\xi_j^2+\xi_k^2)\beta_{jk}^2} \tag{19}$$

where $\beta_{jk} = f_j/f_k$ with $0 \leq \rho_{jkr} \leq 1$, $\rho_{jjr} = \rho_{kkr} = 1$ and $\rho_{jkr} = \rho_{kjr} \ll 1$ when f_j, and f_k are well separated; $S_{Q_{jk}}(f)$ = XPSD between the generalized forces $Q_j(t)$ and $Q_j(t)$. It is worth mentioning that the parameter α_{jkr} represents the partially correlated feature of the generalized forces $Q_j(t)$ and $Q_k(t)$. In general, $|\alpha_{ijr}| \leq 1$, in the case of building response under partially correlated multiple inputs of wind loading. Consideration of this parameter for correct use of the CQC rule is critical. Unfortunately, it has neither been completely recognized in the literature concerning the analysis of wind load effects on buildings and structures nor it has been implemented in current wind tunnel practice (e.g., Xie et al. 2003). It is also noted that the correlation coefficients among background response components in three primary directions are generally different from those among the resonant response components.

EQUIVALENT STATIC WIND LOADING FOR RESPONSE IN PRIMARY DIRECTIONS

For a given peak response, a variety of ESWL distributions may be defined based on different considerations. The load distribution is not necessary unique simply because that different load distributions can result in identical building response. The challenge of equivalent loading representation for a given building is to develop load distributions that are physically meaningful, and are insensitive to individual response. Consequently, the number of loading distributions for a variety of important response components of consideration can be limited.

The "gust loading factor" (GLF), or "gust response factor" (GRF) approach (Davenport 1967) has been used in most major building codes and standards around the world. In this scheme, the equivalent wind loading used for design is equal to the mean wind load multiplied by a GRF, often for the building top displacement. The GRF is defined as the ratio of peak dynamic response to it mean value. Although the traditional GRF method is simple to use in the building design process, the GRF may widely vary for different response components of a structure and may have significantly different values for structures with similar geometric configuration and associated wind load characteristics but different structural systems. As illustrated in Chen and Kareem (2004), among others, for alongwind response of buildings, the GRFs for the top displacement and base bending moment are almost the same which are generally larger than that for the base shear force. The GRF for building response at higher elevation generally is markedly larger than that for the top displacement or base bending moment. Therefore, the equivalent loading given by the mean load multiplied by the GRF for the top displacement or base bending moment generally leads to underestimation of building response at higher elevations. Furthermore, for the crosswind and torsional responses, which are typically characterized by low values of mean wind loading and associated response, particularly, in the cases of symmetric buildings, although similar GRF concepts may be invoked (Kareem and Zhou 2003), the corresponding GRFs may not have the same physical meaning as the traditional GRF for the alongwind response.

Similar to the GRF approach, the dynamic response factor (DRF) approach has been adopted in some building codes (Holmes 2002a). The DRF has been defined as the ratio of peak dynamic response (including the mean, background, and resonant components) to the response caused by the peak dynamic load that includes the mean and the background load effects but excludes any reduction due to loss of spatial correlation of wind loading. The DRF for response R_x is expressed as

$$D_{R_x} = \frac{\bar{R}_x + \sqrt{g_b^2 \sigma_{R_{xb}}^2 + g_r^2 \sigma_{R_{xr}}^2}}{\bar{R}_x + g_b \sigma'_{R_{xb}}} \tag{20}$$

$$\sigma'_{R_{xb}} = \int_0^H \mu_x(z) \sqrt{R_{P_x}(z)} dz = \sigma_{R_{xb}} / B_z \tag{21}$$

$$B_z = \frac{\sqrt{\int_0^H \int_0^H \mu_x(z_1)\mu_x(z_2) R_{P_{xx}}(z_1, z_2) dz_1 dz_2}}{\int_0^H \mu_x(z_1) R_{P_x}(z) dz} \tag{22}$$

where $R_{P_x}(z) = R_{P_{xx}}(z, z)$; B_z = background factor representing the reduction effect with respect to the response R_x due to loss of spatial correlation of wind loading.

The relationship between the DRF and GRF can be expressed as

$$D_{R_x} = \frac{1 + \sqrt{G_{R_{xb}}^2 + G_{R_{xr}}^2}}{1 + G_{R_{xb}} / B_z} \tag{23}$$

where $G_{R_{xb}} = g_b \sigma_{R_{xb}} / \bar{R}_x$ and $G_{R_{xr}} = g_r \sigma_{R_{xr}} / \bar{R}_x$ = background and resonant GRFs, respectively.

The DRF approach leads to an ESWL description which is similar to the peak dynamic pressure/wind load (including the mean load) but scaled by the DRF:

$$F_{eR_x} = D_{R_x} \left(\bar{P}_x(z) + F'_{ebx}(z) \right) \tag{24}$$

where $F'_{ebx}(z) = g_b R_{P_x}(z)$ = gust loading envelope.

Separation of the ESWLs into background and resonant components provides a physically more meaningful description of loading (Davenport 1985; Kasperski 1992; Holmes et al. 2000; Zhou and Kareem 2000; Chen and Kareem 2001 and 2004; Kareem and Zhou 2003). It is straightforward to express the resonant ESWL (RESWL) as the modal inertial load, which depends on the mass distribution and mode shape:

$$F_{exr}(z) = \frac{g_r m(z) \Theta_x(z)}{\int_0^H m(z) \Theta_x^2(z) dz} \sqrt{\frac{\pi}{4\xi_x} f_x S_{Q_x}(f_x)} \tag{25}$$

The advantage of this load description over the traditional GRF approach is that it leads to a universal load distribution for all response components. Within the traditional GRF approach, different GRFs and associated loads have to be assigned for accurate predictions of distinct response components. This advantage is attributed to the presumption that the resonant response is only contributed by the fundamental mode and the higher mode contributions are negligible.

Compared to the straightforwardness of RESWL, modeling of background ESWL (BESWL) is relatively complex which is attributed to the nature of partially correlated multiple inputs of wind loading. Under the action of dynamic loading, different background response components generally reach their peaks at different time instants. When the BESWL for a given peak background response is directly derived from the conditional sampling and subsequent ensemble average of dynamic pressures over the building surface at the instant when the desired peak load effect occurs, the load distribution varies with individual response of consideration (e.g., Tamura et al. 2003a). The ensemble averages of this conditional sampling of dynamic pressures is very close to the load distribution provided by using the load-response-correlation (LRC) approach (Kisperski 1992), which results in a most probable load distribution for a given peak response. According to the LRC approach, the BESWL for $R_{xbmax} = g_b \sigma_{R_{xb}}$ can be expressed as

$$F_{eR_{xb}}(z) = \frac{g_b}{\sigma_{R_{xb}}} \int_0^H \mu_x(z_1) R_{P_{xx}}(z, z_1) dz_1 \tag{26}$$

Its efficacy can be readily illustrated as

$$g_b \sigma_{R_{xb}} = \int_0^H \mu_x(z) F_{eR_{xb}}(z) dz \tag{27}$$

As this load distribution depends on the influence function of response under consideration, each response component corresponds to a distinct spatial load distribution. This feature may limit its potential application to design standards or practice. To eliminate the dependence of load distribution on individual response, an approximate modeling of BESWL has been suggested in Holmes (1996). This scheme provided an identical load distribution for any building response at the same building elevation, but the response components at different elevations have distinct load distributions. In light of this scheme, the BESWL for building response at elevation z_0 may be defined as that for the shear force. The influence function of the shear force is given as $\mu_x(z) = 1$ when $z \geq z_0$ and $\mu_x(z) = 0$ when $z \leq z_0$. Accordingly, the BESWL can be approximated as

$$F_{ebx}(z) = \frac{g_b \int_{z_0}^H R_{P_{xx}}(z, z_1) dz_1}{\sqrt{\int_{z_0}^H \int_{z_0}^H R_{P_{xx}}(z_1, z_2) dz_1 dz_2}} \tag{28}$$

The LRC-based BESWL can be further expressed in terms of the loading modes derived from the proper orthogonal decomposition (POD) of the loading correlation function. Akin to the modal analysis in structural dynamics, the POD loading modes serve as intrinsic functions for characterizing the spatial variations of dynamic loading and associated equivalent load distribution (e.g., Chen and Kareen 2005c). The jth loading mode $\Phi_j(z)$ with an eigenvalue λ_j is defined by the following eigenvalue problem:

$$\int_0^H \Phi_j(z_1) R_{P_{xx}}(z, z_1) dz_1 = \lambda_j \Phi_j(z) \tag{29}$$

$$\int_0^H \Phi_i(z) \Phi_j(z) dz = \delta_{ij} \quad (i, j = 1, 2, ...) \tag{30}$$

where δ_{ij} = Kronecker delta.

By expressing the load correlation function in terms of loading modes, the background response and associated equivalent loading based on the LRC approach can be expressed as

$$R_{P_{xx}}(z_1, z_2) = \sum_j \lambda_j \Phi_j(z_1) \Phi_j(z_2); \qquad \sigma_{R_{xb}} = \sqrt{\sum_j \lambda_j c_j^2} \tag{31}$$

$$F_{eR_{xb}}(z) = g_b \sum_j \lambda_j c_j \Phi_j(z) / \sqrt{\sum_j \lambda_j c_j^2}; \qquad c_j = \int_0^H \mu_x(z) \Phi_j(z) dz \tag{32}$$

where c_j = building response R_x under the action of load distribution $\Phi_j(z)$. Clearly, the contribution of each loading mode depends on individual response. When only the first loading mode with the largest eigenvalue is considered, the equivalent loading can be reduced to the first loading mode, which is independent of response under consideration:

$$F_{eR_{xb}}(z) = g_b \sqrt{\lambda_1} \Phi_1(z) \tag{33}$$

A universal load distribution for all background response components has been suggested in Katsumura et al. (2004) with application to a large span cantilever roof. According to this scheme, the BESWL for any peak background response is expressed as a linear combinations of the first N loading modes with larger eigenvalues:

$$F_{exb}(z) = \sum_{j=1}^N a_j \Phi_j(z) \tag{34}$$

where the combination factors a_j $(j = 1, 2, ..., N)$ are determined based on the M peak response components of interests (where $M >> N$) in a least square sense to ensure the response components of interests under this universal loading distribution are close to their actual values. It is noted that a universal load distribution for any response component (including both background and resonant components) was also suggested by Repetto and Solari (2004), in which a polynomial expansion was utilized for describing the load distribution over the building height.

Chen and Kareem (2004) has proposed the gust loading envelope (GLE) approach for modeling the BESWL:

$$F_{eR_b}(z) = B_z F'_{ebx}(z) = B_z g_b \sqrt{R_{P_x}(z)} \tag{35}$$

This approach results in a load distribution similar to the gust loading envelope but scaled by a background factor. The background factor depends on individual response. For a global response, the background factor is much less than unity, while for a local response, it is close to unity. The background factor is potentially insensitive to the individual response thus simplification of equivalent loading may be achieved. It is obvious that when the wind loads are fully correlated, i.e., $R_{P_{xx}}(z_1, z_2) = \sqrt{R_{P_x}(z_1) R_{P_x}(z_2)}$, B_z reduces to unity and the BESWLs based on the LRC and GLE approaches converge to the gust loading envelope.

Once the RESWL and the BESWL have been determined, the corresponding peak resonant and background response components are calculated following a static analysis procedure. These are then combined for the total peak response (excluding the mean component) using the square root of the sum of squares (SRSS) approach. It is important to note that the ESWL for the total peak response cannot be determined by directly combining the background and resonant loading components using the SRSS approach. However, the following linear combination scheme offers an alternative (Boggs and Peterka 1989; Chen and Kareem 2001 and 2004; Holmes 2002):

$$F_{eR_x}(z) = \Big(g_b \sigma_{R_{xb}} F'_{eR_{xb}}(z) + g_r \sigma_{R_{xr}} F_{erx}(z) \Big) / \sqrt{g_b^2 \sigma_{R_{xb}}^2 + g_r^2 \sigma_{R_{xr}}^2} \tag{36}$$

When both the background and resonant response components are approximated by the fundamental mode response, the ESWL for any total peak response becomes identical to the modal inertial loading involving contributions of both background and resonant response components. It can be expressed by distributing the base bending moment response over the building height as

$$F_{ex}(z) = \frac{m(z)\Theta_x(z)}{\int_0^H m(z)\Theta_x(z)(z/H)dz} \frac{\sigma_{M_x}}{H} \sqrt{g_b^2 + g_r^2 k_x^2 \frac{\pi}{4\xi_x} \frac{f_x S_{M_x}(f_x)}{\sigma_{M_x}^2}} \tag{37}$$

$$k_x = \frac{\int_0^H m(z)\Theta_x(z)(z/H)dz}{\int_0^H m(z)\Theta_x^2(z)dz} \eta_x(f_x) \tag{38}$$

where k_x = modified mode shape correction factor.

When $m(z) = m_0$ and $\Theta_x(z) = (z/H)^\beta$, then

$$F_{ex}(z) = (\beta + 2) \Big(\frac{z}{H} \Big)^\beta \frac{\sigma_{M_x}}{H^2} \sqrt{g_b^2 + g_r^2 \frac{(2\beta+1)^2}{(\beta+2)^2} \eta_x^2(f_x) \frac{\pi}{4\xi_x} \frac{f_x S_{M_x}(f_x)}{\sigma_{M_x}^2}} \tag{39}$$

Based on this framework, the formulations of the equivalent loading associated with building response in the other two directions is immediate. For building response, when both background and resonant response components are approximated by the corresponding fundamental response in torsion, the equivalent loading in terms of torque per unit height can be expressed by distributing the base torque response over the building height as

$$F_{e\theta}(z) = \frac{I(z)\Theta_\theta(z)}{\int_0^H I(z)\Theta_\theta(z)dz} \sigma_{M_\theta} \sqrt{g_b^2 + g_r^2 k_\theta^2 \frac{\pi}{4\xi_\theta} \frac{f_\theta S_{M_\theta}(f_\theta)}{\sigma_{M_\theta}^2}} \tag{40}$$

$$k_\theta = \frac{\int_0^H I(z)\Theta_\theta(z)dz}{\int_0^H I(z)\Theta_\theta^2(z)dz} \eta_\theta(f_\theta) \tag{41}$$

When $I(z) = I_0$ and $\Theta_\theta(z) = (z/H)^\beta$, then

$$F_{e\theta}(z) = (\beta+1)\left(\frac{z}{H}\right)^\beta \frac{\sigma_{M_\theta}}{H}\sqrt{g_b^2 + g_r^2\frac{(2\beta+1)^2}{(\beta+1)^2}\eta_\theta^2(f_\theta)\frac{\pi}{4\xi_\theta}\frac{f_\theta S_{M_\theta}(f_\theta)}{\sigma_{M_\theta}^2}} \tag{42}$$

By further introducing the nondimensional bending moment coefficients, $C_{M_x}(t) = M_x(t)/(q_H BH^2)$ and $C_{M_y}(t) = M_y(t)/(q_H DH^2)$, and torque coefficient, $C_{M_\theta}(t) = M_\theta(t)/(q_H BDH)$ (where q_H = wind speed pressure at the building top; B and D = the representative width and depth of the building), the aforementioned formulations can be readily used to develop equivalent static loading in each primary direction on a variety of tall buildings for design applications. The force coefficients can be quantified using empirical formulas developed based on extensive wind tunnel tests using HFFB measurements or synchronous measurements of pressure over scaled building models, or directly estimated from aerodynamic loading database (Zhou et al. 2003).

EQUIVALENT LOADING FOR COMBINED WIND LOAD EFFECTS

Corresponding to Eq. (13), the 3-D equivalent static loading associated with $R_{bmax} = g_b\sigma_{R_b}$ can be expressed in terms of the loadings in each primary direction as

$$F'_{eR_bs_1}(z) = W_{s_1R_b}F_{eR_{s_1b}}(z); \quad W_{s_1R_b} = \Big(\sum_{s_2=x,y,\theta}\sigma_{R_{s_2b}}r_{R_{s_1s_2b}}\Big)/\sigma_{R_b} \quad (s_1 = x, y, \theta) \tag{43}$$

Similarly, corresponding to Eq. (16), the 3-D equivalent static loading associated with $R_{rmax} = g_r\sigma_{R_r}$ can be expressed in terms of the modal inertial loading in each primary direction as

$$F'_{eR_rs_1}(z) = W_{s_1R_r}F_{es_1r}(z); \quad W_{s_1R_r} = \Big(\sum_{s_2=x,y,\theta}\sigma_{R_{s_2r}}r_{s_1s_2r}\Big)/\sigma_{R_r} \quad (s_1 = x, y, \theta) \tag{44}$$

In fact, there are infinite combinations of peak equivalent loads in three primary directions for a given peak wind load effect. However, the aforementioned combination scheme leads to a most probable load distribution. When the background loading in each primary direction is presented in terms of gust loading envelope or approximated in terms of the modal inertial loading, or other universal loading, the equivalent static loading for any response have similar spatial distribution but scaled by different weighting factors. These weighting factors generally depend on individual response.

Consider a resonant response contributed by the modal responses in two primary translational x and y directions, e.g., the first two modes. In light of Eq. (44), the combination (weighting) factors become

$$W_{xR_r} = \frac{1 + (\sigma_{R_{yr}}/\sigma_{R_{xr}})r_{xyr}}{\sqrt{1 + (\sigma_{R_{yr}}/\sigma_{R_{xr}})^2 + 2r_{xyr}(\sigma_{R_{yr}}/\sigma_{R_{xr}})}} \tag{45}$$

$$W_{yR_r} = \frac{r_{xyr} + (\sigma_{R_{yr}}/\sigma_{R_{xr}})}{\sqrt{1 + (\sigma_{R_{yr}}/\sigma_{R_{xr}})^2 + 2r_{xyr}(\sigma_{R_{yr}}/\sigma_{R_{xr}})}} \tag{46}$$

For the case in which $\sigma_{R_{xr}} = \sigma_{R_{yr}}$ and $r_{xyr} = 0$, it results in $W_{xR_r} = W_{yR_r} = 0.707$, which corresponds to the combination factor of 0.75 adopted in ASCE 7-02 standard and NBCC code.

Alternatively, the following combinations can be defined:

$$W_{xR_r}^{(1)} = 1; \quad W_{yR_r}^{(1)} = \Big(\sqrt{1 + (\sigma_{R_{yr}}/\sigma_{R_{xr}})^2 + 2r_{xyr}(\sigma_{R_{yr}}/\sigma_{R_{xr}})} - 1\Big)(\sigma_{R_{xr}}/\sigma_{R_{yr}}) \tag{47}$$

$$W_{xR_r}^{(2)} = \Big(\sqrt{1 + (\sigma_{R_{xr}}/\sigma_{R_{yr}})^2 + 2r_{xyr}(\sigma_{R_{yr}}/\sigma_{R_{xr}})} - 1\Big)(\sigma_{R_{yr}}/\sigma_{R_{xr}}); \quad W_{yR_r}^{(2)} = 1 \tag{48}$$

When $\sigma_{R_{xr}} = \sigma_{R_{yr}}$, Eqs. (47) and (48) lead to

$$W_{yR_r}^{(2)} = W_{xR_r}^{(1)} = \sqrt{2 + 2r_{xyr}} - 1 \tag{49}$$

which corresponds to the combination rule that takes into account the modal correlation adopted in AIJ-RLB-2004 (Asami 2000; Tamura et al. 2003b). By further setting $r_{xyr} = 0$, it leads to $W^{(2)}_{yR_r} = W^{(1)}_{xR_r} \approx 40\%$, i.e., the 40% rule, which has been widely adopted in building codes for earthquake loadings.

These simplified combination rules eliminate the dependence of the weighting factors on the individual response, and further on the modal correlation coefficient. Their performance can be investigated by comparing their estimates with those according to the CQC rule for different r_{xyr} and $c_{xyr} = \sigma_{R_{yr}}/\sigma_{R_{xr}}$. The combination rule that takes into account the modal correlation leads to more conservative results than the CQC rule, and offers a better performance than the 40% and 75% rules, provided the modal correlation coefficient can be adequately estimated. The 40% and 75% rules result in very conservative estimates in some cases but nonconservative in others. For example, when $r_{xyr} = -0.6$ and $c_{xyr} = 1$, or when $r_{xyr} = 0.6$ and $c_{xyr} = -1$, the response ratios with respect to CQC rule are 1.57 and 1.68, respectively, for the 40% and 75% rules, which means an overestimate of the response. On the other hand, when $r_{xyr} = 0.6$ and $c_{xyr} = 1$, or when $r_{xyr} = -0.6$ and $c_{xyr} = -1$, the response ratios are 0.78 and 0.84, respectively, which means underestimation of the response. Further investigation on the combination factors for a variety of response components, which are important for building design, are needed in order to limit the number of equivalent static loads for design applications.

CONCLUDING REMARKS

The prediction of wind loads and modeling of associated ESWLs were addressed for buildings with uncoupled mode shapes, based on the loading information obtained through either synchronous measurements of pressure over building model surfaces or HFFB measurements. Separation of wind load effects and ESWLs into background and resonant components led to physically more meaningful modeling. The modal inertial load distribution significantly simplifies the equivalent static loading for resonant response. Various approaches proposed in the literature for better modeling of background equivalent static loading were critically evaluated. Some new insights were provided on the correct use of the CQC rule for predicting wind load effects influenced by wind loads in different primary directions, and on their combinations. It is envisaged that the results presented in this study would aid in improved modeling of wind loading on buildings for design applications.

ACKNOWLEDGMENT

The support for this work provided in part by the NSF Grant CMS 03-24331 is gratefully acknowledged. The first author also gratefully acknowledges the support of the new faculty start-up funds provided by the Texas Tech University.

REFERENCES

Asami, Y. (2000). "Combination method for wind loads on high-rise buildings." *Proc. of 16th National Symposium on Wind Eng.*, Japan Association for Wind Engineering (JAWE), Tokyo, Japan, 531-534 (In Japanese).

ASCE 7-02, *Minimum Design Loads for Buildings and Other Structures.* ASCE, 2000.

Boggs, D. W. and Peterka, J. A. (1989). "Aerodynamic model tests of tall buildings." *J. Eng. Mech.*, 115(3), 618-635.

Chen, X. and Kareem, A. (2001). "Equivalent static wind loads for buffeting response of bridges." *J. Struct. Eng.*, 127(12), 1467-1475.

Chen, X. and Kareem, A. (2004). "Equivalent static wind loads on tall buildings: new model." *J. Struct. Eng.*, 130(10), 1425-1435.

Chen, X. and Kareem, A. (2005a). "Coupled dynamic analysis and equivalent static wind loads on buildings with 3-D modes." *J. Struct. Eng.*, 131(7).

Chen, X. and Kareem, A. (2005b). "Dynamic wind load effects on buildings with 3-D coupled modes: application of HFFB measurements." *J. Eng. Mech.* (in press).

Chen, X. and Kareem, A. (2005c). "Proper orthogonal decomposition-based modeling, analysis, and simulation of dynamic wind load effects on structures." *J. Eng. Mech.* 131(4), 1-18.

Chopra, A. K., *Dynamics of Structures: Theory and Applications to Earthquake Engineering.*, (2nd edition), 2000, Prentice-Hall.

Davenport, A. G. (1967). "Gust loading factors." *J. Struct. Eng. Div.*, ASCE, 93, 11-34.

Davenport, A. G. (1985). "The representation of the dynamic effects of turbulent wind by equivalent static wind loads." *AISC/CISC Int. Symp. on Struct. Steel*, Chicago, 3-1-3-13.

Der Kiureghian, A. (1980). "Structural response to stationary excitation." *J. Eng. Mech. Div.*, 106(6), 1195-1213.

Holmes, J. D. (1996). "Along-wind of lattice towers: Part III - Effective load distribution.", *Eng. Struct.*, 18, 483-488.

Holmes, J. D. (2002a). "Effective static load distributions in wind engineering." *J. of Wind Eng. and Ind. Aerodyn.*, 90, 91-109.

Holmes, J. D. (2002b). "Gust loading factor to dynamic response factor (1967-2002)." *Symposium Preprints, Engineering Symposium to Honor Alan G. Davenport for His 40 Year of Contributions*, the University of Western Ontario, June 20-22, 2002, London, Ontario, Canada, A1-1-A1-8, to appear in the *J. of Wind Eng. and Ind. Aerodyn.*

Isyumov, N. (1999). "Overview of wind action on tall buildings and structures." *Proc. of the Tenth Int. Conf. on Wind Eng.*, Copenhagen, Denmark, 15-18.

Kareem, A. (1985). "Lateral-torsional motion of tall buildings to wind loads." *J. Struct. Eng.*, 111(11), 2479-2496.

Kareem, A. and Cermak, J. E. (1979). "Wind tunnel simulation of wind-structure interactions." *ISA Transactions*,18(4), 23-41.

Kareem, A. and Zhou, Y. (2003). "Gust loading factors - past, present and future." *J. Wind Eng. and Ind. Aerodyn.*, 91(12-15), 1301-1328.

Kasperski, M. (1992). "Extreme wind load distributions for linear and nonlinear design." *Eng. Struct.*, 14, 27-34.

Katsumura, A., Tamura, Y., Nakamura, O. and Yoshida T. (2004). "Universal equivalent static wind load distribution on a large span cantilevered roof."*Proc. of the 18th National Symposium on Wind Engineering*, Japan Association for Wind Engineering, Tokyo, Japan, 461-466.

Melbourne, W. H. (1975). "Probability distributions of response of BHP house to wind action and model comparisons." *J. Ind. Aerodyn.*, 1(2), 167-175.

National Building Code of Canada 1995 (NBC), Institute for Research in Construction, National Research Council Canada.

Repetto, M. P. and Solari, G. (2004). "Equivalent static wind actions on vertical structures." *J. of Wind Eng. and Ind. Aerodyn.*, 92(5), 335-357.

Tamura, Y., Kikuchi, H. and Hibi, K. (2003a). "Quasi-static wind load combinations for low- and middle-rise buildings." *J. Wind Eng. and Ind. Aerodyn.*, 91(12-15), 1613-1625

Tamura, Y., Kawai, H., Uematsu, Y., Kondo, K. and Ohkuma, T. (2003b). "Revision of AIJ Recommendation for wind loads on buildings." *The Int. Wind Eng. Symposium, IWES 2003*, Tamsui, Taipei County, Taiwan.

Tschanz, T. and Davenport, A. G. (1983). "The base balance technique for the determination of dynamic wind loads." *J. Wind Eng. and Ind. Aerodyn.*, 13(1-3), 429-439.

Vickery, P. J., Steckley, A. C., Isyumov, N., and Vickery, B. J. (1985). "The effect of mode shape on the wind induced response of tall buildings." *Proc. of the 5th U.S. National Conf. on Wind Eng.*, Texas Tech. University, Lubbock, TX, 1B-41-1B-48.

Xie, J., Kumar, S. and Gamble, S. (2003). "Wind loading study for tall buildings with similar dynamic properties in orthogonal directions." *Proc. of the 10th Int. Conf. on Wind Eng.*, Copenhagen, Denmark, 2389-2396.

Xu, Y. L. and Kwok, K. C. S. (1993). "Mode shape corrections for wind tunnel tests of tall buildings." *Eng. Struct.*, 15, 618-635.

Zhou, Y. and Kareem, A. (2001). "Gust loading factor: new model." *J. Struct. Eng.*, 127(2), 168-175.

Zhou, Y., Kareem, A. and Gu, M. (2002). "Mode shape corrections for wind load effects." *J. Eng. Mech.*, 128(1), 15-23.

Zhou, Y., Kijewski, T. and Kareem, A. (2003). "Aerodynamic loads on tall buildings: an interactive database." *J. Struct. Eng.*, 129(3), 394-404.