# Advances in Modeling of Aerodynamic Forces on Bridge Decks

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**Abstract:** Aerodynamic forces on bridges are commonly separated into static, self-excited, and buffeting force components. By delving into the relationships among force descriptors for static, self-excited, and buffeting components, novel perspectives are developed to unveil the subtle underlying complexities in modeling aerodynamic forces. Formulations for airfoil sections and those based on quasi-steady theory are both considered. The time domain modeling of unsteady aerodynamic forces including their frequency-dependent characteristics and spanwise correlation is presented, which are often neglected in current time domain analyses due to modeling difficulty. A nonlinear aerodynamic force model is proposed to take into account the nonlinear dependence of the aerodynamic forces on the effective angle of incidence. The nonlinear aerodynamics may become increasingly critical when the aerodynamic characteristics of innovative bridge deck designs, with attractive aerodynamic performance, exhibit significant sensitivity with respect to the effective angle of incidence and with the increases in the bridge span. Clearly, in these cases one may be pushing the envelope of the current linear aerodynamics which has successfully served thus far. The synergistic review of the writers' recent work in bridge aerodynamics presented here, in light of the current state-of-the-art in this field, may serve as a building block for developing new analysis tools and frameworks for the accurate prediction of the response of long span bridges under strong wind excitation.

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## Introduction

The increase in span length of long span bridges results in a remarkable decrease in their natural frequencies and the ratio between the fundamental torsional and vertical mode frequencies. This renders long span bridges very susceptible to the actions of strong wind. The wind load effects generally become the most critical external loads that need consideration in the design of long span bridges. While the wind loads acting on bluff bridge sections under turbulent winds are generally nonlinear functions of structural motions and incoming wind fluctuations, these can be represented for most cases by linear approximation and expressed in terms of time-averaged static and time-varying self-excited and buffeting force components (Davenport 1962; Scanlan 1978a,b). A very insightful review of the developments and problematic issues in the modeling of wind force on bridge decks has been presented in Scanlan (1993).

For most bridges, the aerodynamic coupling among modal response components resulting from the coupled self-excited forces can be neglected, and the flutter is dominated by a single torsional mode. Therefore, the mode-by-mode approach for the prediction of flutter and buffeting responses is valid and computationally efficient (Scanlan 1978a,b). However, this is not necessarily true for very long span bridges, for which the performance against winds has to be studied at higher-reduced velocities. In fact, the analysis of both flutter and buffeting responses requires consideration of the aerodynamic coupling among-modal responses by using the multimode coupled analysis approaches (Agar 1989; Jain et al. 1996; Diana et al. 1998; Jones et al. 1998; Katsuchi et al. 1999; Chen et al. 2000a,b; Chen and Kareem 2001a).

Both buffeting and self-excited forces are, in general, functions of the geometric configurations of bridge sections, the incoming wind fluctuations, and the reduced frequency. In the wind velocity range of interest for bridge design, the flow around bluff bridge sections is quite unsteady and not amenable to the quasi-steady theory, which neglects the unsteady fluid memory effect and is only valid at very high-wind velocities. The frequency-dependent aerodynamic characteristics of wind forces are generally described in terms of experimentally quantified flutter derivatives for the self-excited forces and in terms of admittance and spanwise coherence functions for the buffeting forces. Incorporating these unsteady characteristics of aerodynamic forces is essential for an accurate evaluation of these forces and the attendant bridge response.

Although the current linear aerodynamic force model has proven its utility for many applications, it may not be able to accommodate completely the issues related to aerodynamic nonlinearities and turbulence effects. These features may become increasingly critical when the aerodynamic characteristics of bridge decks exhibit significant sensitivity with respect to the effective angle of incidence and with the increase in the bridge span. Experimental studies have also shown that the aerodynamic characteristics of many innovative bridge deck designs with attractive aerodynamic performance are very sensitive to the angle of incidence (e.g., Zasso and Curami 1993; Matsumoto et al. 1998). For

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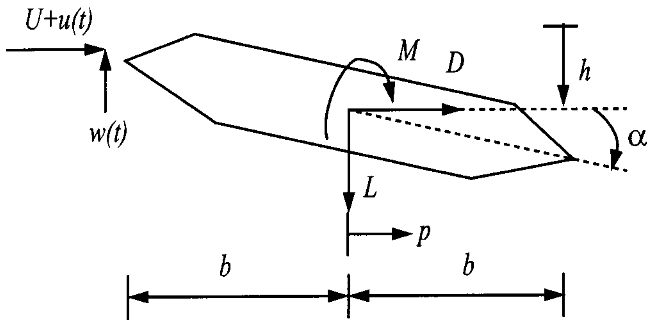



**Fig. 1.** Aerodynamic forces on cross section

these bridge sections, even for low levels of turbulence, structural motion and incoming wind fluctuations may vary the effective angle of incidence to such a degree that modeling of aerodynamic forces may not be realistic without taking into account the effects of aerodynamic nonlinearities. In such cases, the accuracy of conventional linear approaches in which the aerodynamic forces are linearized around the mean displaced position warrant further examination (Chen and Kareem 2000b).

In this paper, recent advances in the modeling of aerodynamic forces on bluff bridge decks in both frequency and time domains are reviewed. The relationships among the force descriptors for static, self-excited, and buffeting force components are discussed in detail including comparison with those for airfoil sections and those obtained based on the quasi-steady theory. By highlighting the relationships between force descriptors for static, self-excited, and buffeting components, and those obtained for airfoil sections and via quasi-steady theory, the subtle underlying complexities in modeling aerodynamic forces are ultimately revealed. Emphasis is also placed on recently introduced time domain modeling by the writers that captures the frequency-dependent aerodynamic force characteristics. The time domain modeling of spanwise correlation of the aerodynamic forces is also advanced for the overall bridge response analysis, and a nonlinear aerodynamic force model is proposed to take into account the dependence of aerodynamic force parameters on the effective angle of incidence.

## Forces on Bluff Bridge Sections

### *Time Domain*

Aerodynamic forces on bridge sections are commonly expressed as a sum of the mean static, self-excited, and buffeting force components. The mean static components, i.e., lift (downward), drag (downwind), and pitching moment (nose-up) components per unit length are expressed as (Fig. 1)

$$L_s=-\frac{1}{2}\rho U^2 B C_L(\alpha_s);\quad D_s=\frac{1}{2}\rho U^2 B C_D(\alpha_s);$$
$$M_s=\frac{1}{2}\rho U^2 B^2 C_M(\alpha_s) \tag{1}$$

where $\rho$=air density; $U$=mean wind velocity; $B=2b$ is the bridge deck width; $C_L$, $C_D$, and $C_M$=mean lift, drag, and pitching moment coefficients, respectively; and $\alpha_s$ is the mean static angle of attack of the bridge section.

The time-varying self-excited forces resulting from the structural motions can be expressed as a sum of components associated with each structural motion component in the vertical, lateral, and torsional directions. These are functions of oscillation frequency due to the unsteady aerodynamic memory effect, and can be represented in terms of convolution integrals of the impulse response functions as (Lin and Yang 1983; Bucher and Lin 1988; Dowell et al. 1989; Scanlan 1984, 1993; Chen et al. 2000b)

$$L_{\mathrm{se}}(t)=\frac{1}{2}\rho U^2\int_{-\infty}^{t}[I_{Lh}(t-\tau)h(\tau)+I_{Lp}(t-\tau)p(\tau)+I_{L\alpha}(t-\tau)\alpha(\tau)]d\tau \tag{2}$$

$$D_{\mathrm{se}}(t)=\frac{1}{2}\rho U^2\int_{-\infty}^{t}[I_{Dh}(t-\tau)h(\tau)+I_{Dp}(t-\tau)p(\tau)+I_{D\alpha}(t-\tau)\alpha(\tau)]d\tau \tag{3}$$

$$M_{\mathrm{se}}(t)=\frac{1}{2}\rho U^2\int_{-\infty}^{t}[I_{Mh}(t-\tau)h(\tau)+I_{Mp}(t-\tau)p(\tau)+I_{M\alpha}(t-\tau)\alpha(\tau)]d\tau \tag{4}$$

where $h$, $p$, and $\alpha$=vertical, lateral, and torsional displacement, respectively and $I_r$ ($r=Lh$, $Lp$, $L\alpha$, $Dh$, $Dp$, $D\alpha$, $Mh$, $Mp$, and $M\alpha$)=aerodynamic impulse response functions representing the influence of motion at a certain time instant on the generation of self-excited forces for a certain time period.

The self-excited forces can be alternatively expressed in terms of indicial response functions $\Phi_r$ as

$$L_{\mathrm{se}}(t)=-\frac{1}{2}\rho U^2(2b)\int_{-\infty}^{t}\left((C_L'+C_D)\Phi_{Lh}(t-\tau)\frac{\ddot{h}(\tau)}{U}-2C_L\Phi_{Lp}(t-\tau)\frac{\ddot{p}(\tau)}{U}+(C_L'+C_D)\Phi_{L\alpha}(t-\tau)\dot{\alpha}(\tau)\right)d\tau \tag{5}$$

$$D_{\mathrm{se}}(t)=\frac{1}{2}\rho U^2(2b)\int_{-\infty}^{t}\left((C_D'-C_L)\Phi_{Dh}(t-\tau)\frac{\ddot{h}(\tau)}{U}-2C_D\Phi_{Dp}(t-\tau)\frac{\ddot{p}(\tau)}{U}+(C_D'-C_L)\times\Phi_{D\alpha}(t-\tau)\dot{\alpha}(\tau)\right)d\tau \tag{6}$$

$$M_{\mathrm{se}}(t)=\frac{1}{2}\rho U^2(2b)^2\int_{-\infty}^{t}\left(C_M'\Phi_{Mh}(t-\tau)\frac{\ddot{h}(\tau)}{U}-2C_M\Phi_{Mp}(t-\tau)\frac{\ddot{p}(\tau)}{U}+C_M'\Phi_{M\alpha}(t-\tau)\dot{\alpha}(\tau)\right)d\tau \tag{7}$$

where $C_L'=dC_L/d\alpha$, $C_D'=dC_D/d\alpha$, $C_M'=dC_M/d\alpha$; and the overdot denotes the derivative with respect to time $t$.

It is conventional to express the aerodynamic impulse and indicial response functions as functions of nondimensional time, $s=Ut/b$. For example, the lift component is expressed as

$$L_{\mathrm{seh}}(s)=\frac{1}{2}\rho U^2\int_{-\infty}^{s}[I_{Lh}(s-\sigma)h(\sigma)+I_{Lp}(s-\sigma)p(\sigma)+I_{L\alpha}(s-\sigma)\alpha(\sigma)]d\sigma$$

$$= -\frac{1}{2}\rho U^2(2b)\int_{-\infty}^{s}\left((C_L'+C_D)\Phi_{Lh}(s-\sigma)\frac{h''(\sigma)}{b} - 2C_L\Phi_{Lp}(s-\sigma)\frac{p''(\sigma)}{b} + (C_L'+C_D)\times\Phi_{L\alpha}(t-\tau)\alpha'(\tau)\right)d\sigma \tag{8}$$

where each prime denotes the derivative with respect to nondimension time $s$.

It can be seen that

$$I_r(s) = \frac{b}{U}I_r(t); \quad \Phi_r(s) = \Phi_r(t) \tag{9}$$

Neglecting initial conditions of motion, the relationship among impulse and indicial response functions can be expressed as follows, for example:

$$\begin{aligned} I_{Lh}(s) &= -2(C_L'+C_D)[\Phi_{Lh}(0)\delta'(s) + \Phi_{Lh}'(0)\delta(s) + \Phi_{Lh}''(s)] \\ I_{L\alpha}(s) &= -2b(C_L'+C_D)[\Phi_{L\alpha}(0)\delta(s) + \Phi_{L\alpha}'(s)] \end{aligned} \tag{10}$$

where $\delta$=Dirac delta function.

Similarly, buffeting forces per unit length can be expressed as a sum of force components induced by wind fluctuations in horizontal and vertical directions ($u$ and $w$) utilizing impulse response functions $I_{Lu}$, $I_{Lw}$, $I_{Du}$, $I_{Dw}$, $I_{Mu}$, and $I_{Mw}$ as (Scanlan 1993; Chen et al. 2000b)

$$L_b(t) = -\frac{1}{2}\rho U^2\int_{-\infty}^{t}\left(I_{Lu}(t-\tau)\frac{u(\tau)}{U} + I_{Lw}(t-\tau)\frac{w(\tau)}{U}\right)d\tau \tag{11}$$

$$D_b(t) = \frac{1}{2}\rho U^2\int_{-\infty}^{t}\left(I_{Du}(t-\tau)\frac{u(\tau)}{U} + I_{Dw}(t-\tau)\frac{w(\tau)}{U}\right)d\tau \tag{12}$$

$$M_b(t) = \frac{1}{2}\rho U^2\int_{-\infty}^{t}\left(I_{Mu}(t-\tau)\frac{u(\tau)}{U} + I_{Mw}(t-\tau)\frac{w(\tau)}{U}\right)d\tau \tag{13}$$

Alternatively, these can be expressed in terms of indicial response functions $\Phi_{Lu}$, $\Phi_{Lw}$, $\Phi_{Du}$, $\Phi_{Dw}$, $\Phi_{Mu}$, and $\Phi_{Mw}$ as

$$L_b(t) = -\frac{1}{2}\rho U^2(2b)\int_{-\infty}^{t}\left(2C_L\Phi_{Lu}(t-\tau)\frac{\dot{u}(\tau)}{U} + (C_L'+C_D)\Phi_{Lw}(t-\tau)\frac{\dot{w}(\tau)}{U}\right)d\tau \tag{14}$$

$$D_b(t) = \frac{1}{2}\rho U^2(2b)\int_{-\infty}^{t}\left(2C_D\Phi_{Dw}(t-\tau)\frac{\dot{u}(\tau)}{U} + (C_D'-C_L)\Phi_{Dw}(t-\tau)\frac{\dot{w}(\tau)}{U}\right)d\tau \tag{15}$$

$$M_b(t) = \frac{1}{2}\rho U^2(2b)^2\int_{-\infty}^{t}\left(2C_M\Phi_{Mu}(t-\tau)\frac{\dot{u}(\tau)}{U} + C_M'\Phi_{Mw}(t-\tau)\frac{\dot{w}(\tau)}{U}\right)d\tau \tag{16}$$

The following relationships among the impulse and indicial response functions can be deduced, for example:

$$\begin{aligned} I_{Lu}(s) &= 4bC_L[\Phi_{Lu}(0)\delta(s) + \Phi_{Lu}'(s)] \\ I_{Lw}(s) &= 2b(C_L'+C_D)[\Phi_{Lw}(0)\delta(s) + \Phi_{Lw}'(s)] \end{aligned} \tag{17}$$

### *Frequency Domain*

For complex sinusoidal motions with frequency $\omega$

$$h(t) = \overline{h}_0 e^{i\omega t}; \quad p(t) = \overline{p}_0 e^{i\omega t}; \quad \alpha(t) = \overline{\alpha}_0 e^{i\omega t} \tag{18}$$

the self-excited forces can be expressed in terms of the flutter derivatives $H_i^*$, $P_i^*$ and $A_i^*(i=1\sim6)$ as (Sarkar et al. 1994)

$$L_{se}(t) = \frac{1}{2}\rho U^2(2b)\left(kH_1^*\frac{\dot{h}}{U} + kH_2^*\frac{b\dot{a}}{U} + k^2H_3^*\alpha + k^2H_4^*\frac{h}{b} + kH_5^*\frac{\dot{p}}{U} + k^2H_6^*\frac{p}{b}\right) \tag{19}$$

$$D_{se}(t) = \frac{1}{2}\rho U^2(2b)\left(kP_1^*\frac{\dot{p}}{U} + kP_2^*\frac{b\dot{\alpha}}{U} + k^2P_3^*\alpha + k^2P_4^*\frac{p}{b} + kP_5^*\frac{\dot{h}}{U} + k^2P_6^*\frac{h}{b}\right) \tag{20}$$

$$M_{se}(t) = \frac{1}{2}\rho U^2(2b^2)\left(kA_1^*\frac{\dot{h}}{U} + kA_2^*\frac{b\dot{\alpha}}{U} + k^2A_3^*\alpha + k^2A_4^*\frac{h}{b} + kA_5^*\frac{\dot{p}}{U} + k^2A_6^*\frac{p}{b}\right) \tag{21}$$

where $k=\omega b/U$ is the reduced frequency.

The relationship among the impulse or indicial response functions and the flutter derivatives can be obtained by substituting Eq. (18) into Eqs. (2)–(4) or Eqs. (5)–(7) and comparing to Eqs. (19)–(21) (Lin and Yang 1983; Bucher and Lin 1988; Chen et al. 2000b)

$$\begin{aligned} \overline{I}_{Lh} &= -2(ik)(C_L'+C_D)C_{Lh} = 2k^2(H_4^*+iH_1^*); \\ \overline{I}_{Lp} &= 4(ik)C_LC_{Lp} = 2k^2(H_6^*+iH_5^*); \\ \overline{I}_{L\alpha} &= -2b(C_L'+C_D)C_{L\alpha} = 2k^2b(H_3^*+iH_2^*); \\ \overline{I}_{Dh} &= 2(ik)(C_D'-C_L)C_{Dh} = 2k^2(P_6^*+iP_5^*); \\ \overline{I}_{Dp} &= -4(ik)C_DC_{Dp} = 2k^2(P_4^*+iP_1^*); \\ \overline{I}_{D\alpha} &= 2b(C_D'-C_L)C_{D\alpha} = 2k^2b(P_3^*+iP_2^*); \\ \overline{I}_{Mh} &= 4(ik)bC_M'C_{Mh} = 2k^2b(A_4^*+iA_1^*); \\ \overline{I}_{Mp} &= -8(ik)bC_MC_{Mp} = 2k^2b(A_6^*+iA_5^*); \\ \overline{I}_{M\alpha} &= 4b^2C_M'C_{M\alpha} = 2k^2b^2(A_3^*+iA_2^*) \end{aligned} \tag{22}$$

where $i=\sqrt{-1}$; $\overline{I}_r$ and $C_r$ ($r=Lh$, $Lp$, $L\alpha$, $Dh$, $Dp$, $D\alpha$, $Mh$, $Mp$, and $M\alpha$) are given by

$$\overline{I}_r = \int_0^\infty I_r(t)e^{-i\omega t}dt = \int_0^\infty I_r(s)e^{-iks}ds; \quad C_r = (ik)\int_0^\infty \Phi_r(s)e^{-iks}ds \tag{23}$$

and $C_r$ can be referred to as an equivalent Theodorsen function for different force components.

The buffeting forces induced by the complex sinusoidal horizontal and vertical wind fluctuations

$$u(t)=\overline{u}_0 e^{i\omega t}; \quad w(t)=\overline{w}_0 e^{i\omega t} \tag{24}$$

are given by (Davenport 1962; Scanlan 1993; Chen et al. 2000b)

$$L_b(t)=-\frac{1}{2}\rho U^2(2b)\left(2C_L\chi_{Lu}\frac{u(t)}{U}+(C_L'+C_D)\chi_{Lw}\frac{w(t)}{U}\right) \tag{25}$$

$$D_b(t)=\frac{1}{2}\rho U^2(2b)\left(2C_D\chi_{Du}\frac{u(t)}{U}+(C_D'-C_D)\chi_{Dw}\frac{w(t)}{U}\right) \tag{26}$$

$$M_b(t)=\frac{1}{2}\rho U^2(2b)^2\left(2C_M\chi_{Mw}\frac{u(t)}{U}+C_M'\chi_{Mw}\frac{w(t)}{U}\right) \tag{27}$$

where $\chi_{Lu}$, $\chi_{Lw}$, $\chi_{Du}$, $\chi_{Dw}$, $\chi_{Mu}$, and $\chi_{Mw}$=aerodynamic transfer functions between fluctuating wind velocities and buffeting forces (absolute magnitudes of these functions are also referred to as aerodynamic admittance functions).

The following relationships hold among the unsteady force parameters of buffeting forces (Chen et al. 2000b):

$$\overline{I}_{Lu}/(4bC_L)=\Theta_{Lu}=\chi_{Lu}; \quad \overline{I}_{Lw}/[2b(C_L'+C_D)]=\Theta_{Lw}=\chi_{Lw};$$
$$\overline{I}_{Du}/(4bC_D)=\Theta_{Du}=\chi_{Du}; \quad \overline{I}_{Dw}/(2bC_D'-C_L)=\Theta_{Dw}=\chi_{Dw}; \tag{28}$$
$$\overline{I}_{Mu}/(8b^2C_M)=\Theta_{Mu}=\chi_{Mu}; \quad \overline{I}_{Mw}/(4b^2C_M')=\Theta_{Mw}=\chi_{Mw}$$

where $\Theta_r$ ($r=Lu$, $Lw$, $Du$, $Dw$, $Mu$, and $Mw$) can be referred to as an equivalent Sears functions for different force components, and are defined as

$$\Theta_r=(ik)\int_0^\infty \Phi_r(s)e^{-iks}ds \tag{29}$$

## Forces on Airfoil Section

Since the modeling of aerodynamic forces acting on bluff bridge sections is strongly influenced by analogous expressions used in two-dimensional airfoil theory (Scanlan 1993), a review of airfoil aerodynamics will improve one's understanding of the aerodynamic force models tailored for bluff bridge sections. For an airfoil section, neglecting the terms containing acceleration terms $\ddot{h}$ and $\ddot{\alpha}$, the lift and moment around the midchord of the section are given as (Feng 1955)

$$L_{\mathrm{se}}(t)=-\frac{1}{2}\rho U^2(2b)(2\pi)\left(\frac{b\dot{\alpha}}{2U}+\int_{-\infty}^{t}\Phi(t-\tau)\dot{\alpha}_e(\tau)d\tau\right) \tag{30}$$

$$M_{\mathrm{se}}(t)=\frac{1}{2}\rho U^2(2b)^2(\pi/2)\left(-\frac{b\dot{\alpha}}{2U}+\int_{-\infty}^{t}\Phi(t-\tau)\dot{\alpha}_e(\tau)d\tau\right) \tag{31}$$

where $\alpha_e$=effective angle of incidence

$$\alpha_e=\frac{\dot{h}}{U}+\alpha+\frac{b\dot{\alpha}}{2U} \tag{32}$$

and $\Phi(t)$=Wanger function given in nondimensional form in terms of the Jones approximation as

$$\Phi(s)=1-0.165e^{-0.0455s}-0.335e^{-0.3s} \tag{33}$$

For the airfoil section, the terms $\dot{h}/U$, $\alpha$, and $b\dot{\alpha}/(2U)$ have equal contribution to the effective angle of incidence. This implies that the associated indicial response functions are dependent on each other and can be expressed in terms of the Wagner function as

$$\Phi_{Lh}(s)=\Phi(s); \quad \Phi_{L\alpha}=0.5[\Phi(0)+1]\delta(s)+\Phi(s)+0.5\Phi'(s)$$
$$\Phi_{Mh}(s)=\Phi_{Lh}(s); \tag{34}$$
$$\Phi_{M\alpha}=0.5[\Phi(0)-1]\delta(s)+\Phi(s)+0.5\Phi'(s)$$

where $C_L'=2\pi$ and $C_M'=\pi/2$.

In the frequency domain, the flutter derivatives are also related through the Theodorsen function $C(k)=F(k)-iG(k)$ as

$$H_1^*=-\frac{2\pi F(k)}{k}; \quad H_2^*=-\frac{\pi}{k}\left(1+F(k)-\frac{2G(k)}{k}\right);$$
$$H_3^*=-\frac{2\pi}{k^2}\left(F(k)+\frac{kG(k)}{2}\right); \quad H_4^*=-\frac{2\pi G(k)}{k};$$
$$A_1^*=\frac{\pi F(k)}{k}; \quad A_2^*=\frac{\pi}{2k}\left(-1+F(k)-\frac{2G(k)}{k}\right); \tag{35}$$
$$A_3^*=\frac{\pi}{k^2}\left(F(k)+\frac{kG(k)}{2}\right); \quad A_4^*=\frac{\pi G(k)}{k}$$

The Theodorsen function is related to the Wagner function as

$$C(k)=(ik)\int_0^\infty \Phi(\tau)e^{-ik\tau}d\tau \tag{36}$$

and is given in terms of the Jones approximation as

$$C(k)=1-\frac{0.165(ik)}{ik+0.0455}-\frac{0.335(ik)}{ik+0.30} \tag{37}$$

For the airfoil section, only the lift due to the vertical component of turbulence is important, which is expressed as

$$L_{\mathrm{bw}}(t)=-\frac{1}{2}\rho U^2(2b)(2\pi)\int_{-\infty}^{t}\Psi(t-\tau)\frac{\dot{w}(\tau)}{U}d\tau \tag{38}$$

where $\Psi(t)$ is the Kussner function, which is given in a nondimensional form in terms of the Jones approximation as

$$\Psi(s)=1-0.5e^{-0.15s}-0.5e^{-s} \tag{39}$$

The impulse and indicial response functions are related to the Kussner function as

$$L_{Lw}(s)=(2b)(2\pi)[\Psi(0)\delta(s)+\Psi'(s)]; \quad \Phi_{Lw}(s)=\Psi(s) \tag{40}$$

In the frequency domain, we have

$$\chi_{Lw}(k)=\Theta_{Lw}(k)=\Theta(k)=(ik)\int_0^\infty \Psi(\tau)e^{-ik\tau}d\tau \tag{41}$$

which is expressed in terms of the Jones approximation as

$$\Theta(k)=1-\frac{0.5(ik)}{ik+0.130}-\frac{0.5(ik)}{ik+1} \tag{42}$$

It should be emphasized that even for the airfoil section, the self-excited force term associated with the effective angle of incidence $\alpha_e$ is different from the buffeting force term associated with the incoming vertical fluctuation $w/U$. This implies that the generation of the self-excited forces due to body motion is differ-

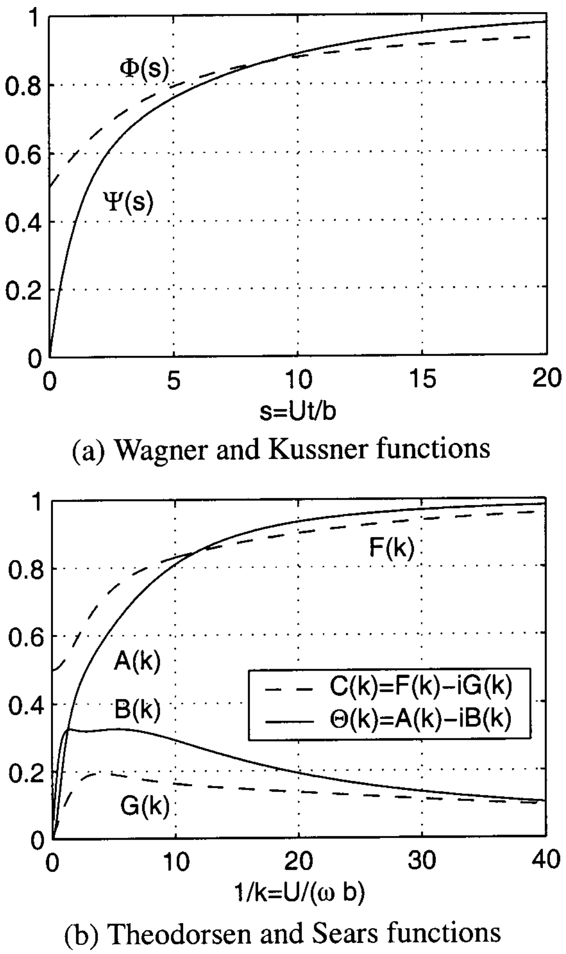



**Fig. 2.** Comparison of unsteady aerodynamic force functions for airfoil section

ent from the generation of the buffeting forces due to turbulence. These forces are characterized in terms of different functions, i.e., the Wagner function and Kussner function in the time domain, and the Theodorsen function and Sears function in the frequency domain, respectively. The relationship between the Theodorsen function and Sears function is given by

$$\Theta(k)=C(k)[J_0(k)-J_1(k)]+iJ_1(k) \tag{43}$$

where $J_0$ and $J_1$ are Bessel functions of reduced frequency $k$. Figs. 2(a and b) show a comparison of the Wagner and Kussner functions, and the Theodorsen and Sears functions.

## Quasi-Steady Aerodynamic Forces

The quasi-steady theory is utilized in most time domain buffeting analysis studies (e.g., Miyata et al. 1995). However, it should be noted that the quasi-steady theory is only applicable when the frequency-dependent fluid memory effect is negligible at very high-reduced velocities, i.e., very low-reduced frequencies. The aerodynamic forces including the time-averaged and time-varying components are expressed in light of the quasi-steady theory as (Fig. 3)

$$L=F_L\cos\phi-F_D\sin\phi;\quad D=F_L\sin\phi+F_D\cos\phi;$$

$$M=\frac{1}{2}\rho U_r^2B^2C_M(\alpha_e) \tag{44}$$

$$F_L=-\frac{1}{2}\rho U_r^2BC_L(\alpha_e);\quad F_D=\frac{1}{2}\rho U_r^2BC_D(\alpha_e) \tag{45}$$

where $U_r$=relative velocity

$$U_r=\sqrt{(U+u-\dot{p})^2+(w+\dot{h}+m_1b\dot{\alpha})^2} \tag{46}$$

and $\alpha_e$=effective angle of incidence

$$\alpha_e=\alpha_s+\alpha+\phi;\quad \phi=\tan^{-1}\left(\frac{w+\dot{h}+m_1b\dot{\alpha}}{U+u-\dot{p}}\right) \tag{47}$$

where $m_1$ is a constant, which is defined later.

These nonlinear quasi-steady forces can be linearized around the statically deformed position when the instantaneous effective angle of incidence is small. By assuming

$$\phi\doteq\frac{w+\dot{h}+m_1b\dot{\alpha}}{U+u-\dot{p}};\quad \sin\phi\doteq\phi;\quad \cos\phi\doteq 1$$

$$C_{L,D,M}(\alpha_e)=C_{L,D,M}(\alpha_s)+C'_{L,D,M}(\alpha_s)\alpha+C'_{L,D,M}(\alpha_s)\phi \tag{48}$$

and neglecting the products of small variables, the quasi-steady forces can be expressed as a sum of static, self-excited, and buffeting forces

$$L(t)=L_s+L_{se}(t)+L_b(t);\quad D(t)=D_s+D_{se}(t)+D_b(t);$$

$$M(t)=M_s+M_{se}(t)+M_b(t) \tag{49}$$

where the static components are the same as those given by Eq. (1), and the self-excited and buffeting force components are given as

$$L_{se}(t)=\frac{1}{2}\rho U^2(2b)\left(-(C'_L+C_D)\frac{\dot{h}+m_1b\dot{\alpha}}{U}-C'_L\alpha+2C_L\frac{\dot{p}}{U}\right) \tag{50}$$

$$D_{se}(t)=\frac{1}{2}\rho U^2(2b)\left(-2C_D\frac{\dot{p}}{U}+C'_D\alpha+(C'_D-C_L)\frac{\dot{h}+m_1b\dot{\alpha}}{U}\right) \tag{51}$$

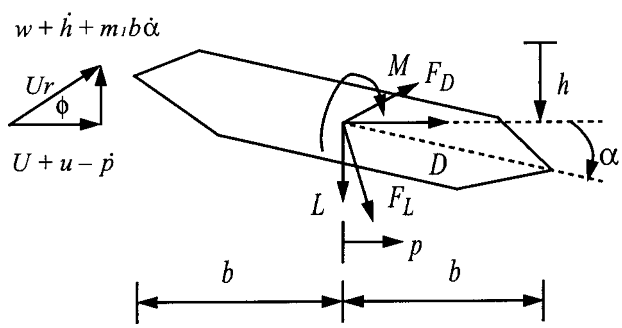



**Fig. 3.** Quasi-steady forces on cross section

$$M_{se}(t)=\frac{1}{2}\rho U^2(2b^2)\left(2C_M'\frac{\dot{h}+m_1 b\dot{\alpha}}{U}\right.$$
$$\left.+2C_M'\alpha-4C_M\frac{\dot{p}}{U}\right) \tag{52}$$

and

$$L_b(t)=-\frac{1}{2}\rho U^2(2b)\left(2C_L\frac{u(t)}{U}+(C_L'+C_D)\frac{w(t)}{U}\right) \tag{53}$$

$$D_b(t)=\frac{1}{2}\rho U^2(2b)\left(2C_D\frac{u(t)}{U}+(C_D'-C_L)\frac{w(t)}{U}\right) \tag{54}$$

$$M_b(t)=\frac{1}{2}\rho U^2(2b)^2\left(2C_M\frac{u(t)}{U}+C_M'\frac{w(t)}{U}\right) \tag{55}$$

Based on the definition of the flutter derivatives [Eqs. (19), (20), and (21)], the nonzero flutter derivatives can then be expressed in terms of the static force coefficients as

$$H_1^*=-(C_L'+C_D)/k;\quad H_2^*=-m_1(C_L'+C_D)/k;$$
$$H_3^*=-C_L'/k^2;\quad H_5^*=2C_L/k;$$
$$P_1^*=-2C_D/k;\quad P_2^*=m_1(C_D'-C_L)/k;\quad P_3^*=C_D'/k^2;$$
$$P_5^*=(C_D'-C_L)/k; \tag{56}$$
$$A_1^*=2C_M'/k;\quad A_2^*=2m_1C_M'/k;\quad A_3^*=2C_M'/k^2;$$
$$A_5^*=-4C_M/k$$

and the indicial response functions $\Phi_r$, equivalent Theodorsen function $C_r$, and the equivalent Sears function $\Theta_r$ are equal to unity except that

$$\Phi_{L\alpha}(s)=m_1\delta(s)+C_L'/(C_L'+C_D);$$
$$\Phi_{D\alpha}(s)=m_1\delta(s)+C_D'/(C_D'-C_L);$$
$$\Phi_{M\alpha}(s)=m_1\delta(s)+1 \tag{57}$$
$$C_{L\alpha}(k)=m_1(ik)+C_L'/(C_L'+C_D);$$
$$C_{D\alpha}(k)=m_1(ik)+C_D'/(C_D'-C_L);$$
$$C_{M\alpha}(k)=m_1(ik)+1 \tag{58}$$

In the quasi-steady theory, the admittance functions are all equal to unity. It is also noteworthy that the self-excited terms related to $-\dot{p}/U$ and $\dot{h}/U$ are the same as those in the buffeting terms related to $u/U$ and $w/U$, respectively.

The quasi-steady formulations can be referred to as a special case of unsteady forces when $k\to 0$ or $U/(fB)\to\infty$. However, the definition of the value of $m_1$ is critical since it is related to the contribution of $\dot{\alpha}$ to the effective angle of incidence. It can be selected as 0.5 indicating that the downward velocity at the leeward three-quarter-chord point is selected for the calculation of the effective angle of incidence as that for airfoil section. However, this will result in $A_2^*\geqslant 0$ when $C_M'\geqslant 0$ that implies a negative torsional aerodynamic damping and the existence of a torsional flutter. This is inconsistent with the airfoil theory and the wind tunnel derived data concerning bluff bridge sections. For most bluff sections it is found that $C_M'\leqslant 0$ indicates the potential of a torsional flutter. Alternatively, a negative value of $m_1$ may be assumed such as $-0.5$ corresponding to the downward velocity at the forward three-quarter-chord point (Miyata et al. 1995), which may lead to an inconsistency in the sign of $H_2^*$ and $P_2^*$. It is well understood that while the quasi-steady assumption can to a certain extent describe the aerodynamic forces associated with the vertical and lateral motions at very high-reduced velocities (Parkinson and Brooks 1961; Novak 1972), it may not be conveniently invoked for the torsional motion (e.g., van Oudheusden 2000). This is because of the existence of flow memory effect which leads to aerodynamic forces lagging the structural motions. This plays an important role in the generation of aerodynamic forces due to torsional motion even at higher-reduced velocity range, which are neglected in the quasi-steady theory.

A "quasi-static corrected theory" was proposed by Diana et al. (1993), in which different relative velocities and effective angles for the lift and drag, and for the moment have been assumed by using different values of $m_1$. Related issues about the flutter derivatives with respect to static force coefficients were also discussed by Larose and Livesey (1997). While the quasi-steady assumption neglects the essential frequency dependence of the unsteady forces, it can nonetheless provide insight into the global trends and enable preliminary estimates of the flutter derivatives based on the static coefficients. For instance, a negative value of $C_L'$ or $C_M'$ indicates a potential for galloping or torsional flutter. Low values of $C_L'$ and $C_M'$ generally correspond to low-unsteady aerodynamic forces and low-absolute values of flutter derivatives. In addition, the drag component of the self-excited forces is commonly evaluated based on the quasi-steady assumption.

## Approximate Relationships Among Force Parameters

The interrelationships among the impulse or indicial response functions and among flutter derivatives as noted for airfoil sections are not necessarily valid for bluff bridge sections (Scanlan 1993). Instead, different functions are required for featuring the unsteady force components associated with each component of structural motion and wind fluctuations. In some cases, certain approximations may be utilized for the sake of simplicity. Based on experimental results, Matsumoto et al. (1995) suggested that it can be assumed approximately that the contribution of $\dot{\alpha}$ to the effective angle of incidence is negligible, and $\dot{h}/U$ and $\alpha$ contribute equally to the effective angle of incidence. This results in the following approximate expressions for the lift and moment in the time and frequency domains:
time domain

$$L_{se}(t)=\frac{1}{2}\rho U^2\int_{-\infty}^{t}I_{L\alpha}(t-\tau)\left[\alpha(\tau)+\frac{\dot{h}(\tau)}{U}\right]d\tau \tag{59}$$

$$M_{se}(t)=\frac{1}{2}\rho U^2\int_{-\infty}^{t}I_{M\alpha}(t-\tau)\left[\alpha(\tau)+\frac{\dot{h}(\tau)}{U}\right]d\tau \tag{60}$$

frequency domain

$$L_{se}(t)=\frac{1}{2}\rho U^2(2b)k^2(H_3^*+iH_2^*)\left(\alpha+\frac{h}{U}\right) \tag{61}$$

$$M_{se}(t)=\frac{1}{2}\rho U^2(2b^2)k^2(A_3^*+iA_2^*)\left(\alpha+\frac{h}{U}\right) \tag{62}$$

It is equivalent to introducing the following interrelationships among the impulse and indicial response functions, and among flutter derivatives

$$I_{Lh}(s)=[I'_{L\alpha}(s)+I_{L\alpha}(0)\delta(s)]/b;$$

$$I_{Mh}(s)=[I'_{M\alpha}(s)+I_{M\alpha}(0)\delta(s)]/b \tag{63}$$

$$\Phi_{Lh}(s)=\Phi_{L\alpha}(s); \quad \Phi_{Mh}(s)=\Phi_{M\alpha}(s)$$

$$H_1^*=kH_3^*; \quad H_4^*=-kH_2^*; \quad A_1^*=kA_3^*; \quad A_4=-kA_2^* \tag{64}$$

Similar interrelationships among flutter derivatives have been discussed by Scanlan et al. (1974) and Scanlan et al. (1997) using a different approach.

Further assuming that the significance of $-\dot{p}/U$ and $\dot{h}/U$ to the generation of self-excited forces is equal to that of $u/U$ and $W/U$, respectively, in generating buffeting forces, as observed in the quasi-steady theory, the relationship among the admittance functions and the flutter derivatives can be derived from Eqs. (19)–(21) and Eqs. (25)–(27) as

$$2C_L\chi_{Lu}=k(H_5^*-iH_6^*); \quad 2C_D\chi_{Du}=-k(P_1^*-iP_4^*)$$

$$4C_M\chi_{Mu}=-k(A_5^*-iA_6^*); \quad (C_L'+C_D)\chi_{Lw}=-k(H_1^*-iH_4^*) \tag{65}$$

$$(C_D'-C_L)\chi_{Dw}=k(P_5^*-iP_6^*); \quad 2C_M'\chi_{Mw}=k(A_1^*-iA_4^*)$$

It is noteworthy that when the admittance functions become unity the preceding equations will result in the same formulations as those derived on the basis of the quasi-steady theory. Similar formulations relating the flutter derivatives and admittance functions have been suggested by Scanlan and Jones (1999) and Scanlan (2000) using a different procedure.

In Tanaka and Hatanaka (2000), two different functions referred to as "equivalent Theodorsen functions" were introduced for describing lift and pitching moment components of the self-excited forces on bridge sections. The effective angle of incidence was defined as for the airfoil section including the contribution of $\dot{\alpha}$. Using those two independent aerodynamic functions instead of the generally used four independent functions is equivalent to introducing new relationships between the self-excited forces associated with vertical and torsional motions. In addition, these equivalent Theodorsen functions are identified based on measured flutter derivatives in their study, which are then used to determine the admittance functions based on the approximate relationship among the self-excited and buffeting forces. It is emphasized that the admittance functions predicted according to this approach include two kinds of approximations. One is introduced by the inter-relationship among flutter derivatives, and another comes from the relationship between the flutter derivatives and admittance functions.

It is noted that on the one hand these relations may be a good approximation for some bridge sections and also help to improve our understanding of the generation mechanisms of aerodynamic forces. On the other hand, they do not permit a plenary application to every bluff section. In fact, even for an airfoil section, while the flutter derivatives are dependent on each other and related to the Theodorsen function, the inter-relationships between the flutter derivatives as shown in Eq. (64) are not strictly valid. In addition, the Theodorsen function is not equal to the Sears function which implies that the generation of lift force due to vertical fluctuation $w/U$ is not the same as self-excited forces due to vertical motion $\dot{h}/U$. Application of such relationships to the modeling of aerodynamic forces results in the introduction of a level of error or uncertainty in the aerodynamic forces for such sections where these relationships are not strictly applicable. Therefore, their application to flutter and buffeting analysis should be handled carefully unless they are well validated through wind tunnel tests. In light of this discussion, it is emphasized that an experimental evaluation of all flutter derivatives and admittance functions using wind tunnel models is still considered to be a most accurate means of estimating the unsteady forces and attendant response of long span bridges.

## Rational Function Approximation of Force Parameters

The assessment of unsteady aerodynamic forces in the time domain requires identification of aerodynamic impulse or indicial response functions. A direct determination of these functions for bluff bridge sections is laced with difficulties, and the techniques based on wind tunnel tests have not been well established. Instead, the techniques for identifying the frequency domain force parameters such as flutter derivatives and admittance functions have been fully established, and a large data set for a host of geometric configurations of bridge sections has been developed (e.g., Walshe and Wyatt 1983; Sarkar et al. 1994; Bosch 1995; Matsumoto et al. 1995; Larose and Mann 1998). However, the flutter derivatives and admittance functions are normally known only at discrete values of reduced frequency *k*. It is difficult to directly use the aforementioned relationships to quantify the impulse or indicial response functions by means of the inverse Fourier transform. Therefore, approximate continuous functions of the reduced frequency are required for describing frequency-dependent force parameters for future analysis. For the self-excited forces, the rational function approximation technique known as Roger's approximation can be utilized for this context. Like the Jones approximation of the Theodorsen function, the aerodynamic transfer functions or the equivalent Theodorsen functions in terms of flutter derivatives can be approximated in terms of rational functions (Scanlan et al. 1974; Lin and Yang 1983; Xie and Xiang 1985; Bucher and Lin 1988; Matsumoto et al. 1994; Wilde et al. 1996; Boonyapinyo et al. 1999; Chen et al. 2000a,b). For example, the aerodynamic transfer function between the lift force and the vertical motion is expressed as

$$2k^2(H_4^*+iH_1^*)=A_{Lh,1}+(ik)A_{Lh,2}+(ik)^2A_{Lh,3}+\sum_{j=1}^{m_{Lh}}\frac{(ik)A_{Lh,j+3}}{ik+d_{Lh,j}} \tag{66}$$

where $A_{Lh,1}$, $A_{Lh,2}$, $A_{Lh,3}$, $A_{Lh,j+3}$, and $d_{Lh,j}$ ($d_{Lh,j}\geqslant 0$; $j=1,2,\ldots,m_{Lh}$)=frequency-independent coefficients; the first and second terms=noncirculatory static-aerodynamics and the aerodynamic damping, respectively; the third term=additional aerodynamic mass which is normally negligible; and the rational terms=unsteady components which lag the velocity of body motion and permit an approximation of the time delays through positive values of the parameter $d_{Lh,j}$ ($j=1,2,\ldots,m_{Lh}$). The value of $m_{Lh}$ determines the level of accuracy of this approximation and the size of additional equations representing the aerodynamic states. These coefficients can be determined by curve fitting the experimentally obtained flutter derivatives at different reduced frequencies.

Eq. (66) is equivalent to expressing the impulse response function $I_{Lh}(s)$ and the indicial function $\Phi_{Lh}(s)$ as the following exponential time-series functions including aerodynamic stiffness, damping, and inertial terms

$$I_{Lh}(s)=\left(A_{Lh,1}+\sum_{j=1}^{m_{Lh}}A_{Lh,j+3}\right)\delta(s)+A_{Lh,2}\delta'(s)$$

$$+A_{Lh,3}\delta''(s)-\sum_{j=1}^{m_{Lh}}A_{Lh,j+3}d_{Lh,j}e^{-d_{Lh,j}s} \qquad (67)$$

$$-2(C_L'+C_D)\Phi_{Lh}(s)=A_{Lh,2}+\sum_{j=1}^{m_{Lh}}A_{Lh,j+3}/d_{Lh,j}+A_{Lh,1}s$$

$$+A_{Lh,3}\delta(s)-\sum_{j=1}^{m_{Lh}}A_{Lh,j+3}/d_{Lh,j}e^{-d_{Lh,j}s} \qquad (68)$$

Accordingly, the unsteady frequency dependent aerodynamic forces $L_{\mathrm{seh}}(t)$ can then be expressed in the time domain as

$$L_{\mathrm{seh}}(t)=-\frac{1}{2}\rho U^2\left(A_{Lh,1}h(t)+A_{Lh,2}\frac{b}{U}\dot{h}(t)\right.$$

$$\left.+A_{Lh,3}\frac{b^2}{U^2}\ddot{h}(t)+\sum_{j=1}^{m_{Lh}}\phi_{Lh,j}(t)\right) \qquad (69)$$

$$\dot{\phi}_{Lh,j}(t)=-\frac{d_{Lh,j}U}{b}\phi_{Lh,j}(t)+A_{Lh,j+3}\dot{h}(t)$$

$$(j=1,2,\ldots,m_{Lh}) \qquad (70)$$

where $\phi_{Lh,j}(t)$ $(j=1,2,\ldots,m_{Lh})$=augmented aerodynamic states.

Similar formulations for other self-excited force components can be obtained with analogous definitions. For example, the aerodynamic transfer function between the pitching moment and torsion is expressed as

$$2k^2(A_3^*+iA_2^*)=A_{M\alpha,1}+(ik)A_{M\alpha,2}$$

$$+(ik)^2A_{M\alpha,3}+\sum_{j=1}^{m_{M\alpha}}\frac{(ik)A_{M\alpha,j+3}}{ik+d_{M\alpha,j}} \qquad (71)$$

and accordingly the indicial function $\Phi_{M\alpha}(s)$ is expressed as

$$2C_M'\Phi_{M\alpha}(s)=A_{M\alpha,1}+A_{M\alpha,2}\delta(s)$$

$$+A_{M\alpha,3}\delta'(s)+\sum_{j=1}^{m_{M\alpha}}A_{M\alpha,j+3}e^{-d_{M\alpha,j}s} \qquad (72)$$

Similarly, for the buffeting force component, the aerodynamic transfer function, for example, $\chi_{Lw}$ can be recast using a rational function approximation (Matsumoto and Chen 1996; Matsumoto et al. 1996; Chen et al. 2000b; Chen and Kareem 2001a):

$$\chi_{Lw}=A_{Lw,1}+\sum_{j=1}^{m_{Lw}}\frac{(ik)A_{Lw,j+1}}{ik+d_{Lw,j}} \qquad (73)$$

where $A_{Lw,1}$, $A_{Lw,j+1}$, and $d_{Lw,j}$ $(d_{Lw,j}\geqslant 0;\ j=1,\ldots,m_{Lw})$ =frequency-independent coefficients determined by curve-fitting $\chi_{Lw}$ at discrete reduced frequencies. Accordingly, the corresponding impulse response function $I_{Lw}(s)$, indicial function $\Phi_{Lw}(s)$, and unsteady buffeting force $L_{bw}(t)$, the lift induced by vertical wind fluctuation, are expressed as

$$I_{Lw}(s)=2b(C_L'+C_D)\left(\left(A_{Lw,1}+\sum_{j=1}^{m_{Lw}}A_{Lw,j+1}\right)\delta(s)\right.$$

$$\left.-\sum_{j=1}^{m_{Lw}}A_{Lw,j+1}d_{Lw,j}e^{-d_{Lw,j}s}\right) \qquad (74)$$

$$\Phi_{Lw}(s)=A_{Lw,1}+\sum_{j=1}^{m_{Lw}}A_{Lw,j+1}e^{-d_{Lw,j}s} \qquad (75)$$

$$L_{bw}(t)=\frac{1}{2}\rho U^2(2b)(C_L'+C_D)\left(\left(A_{Lw,1}+\sum_{j=1}^{m_{Lw}}A_{Lw,j+1}\right)\right.$$

$$\left.\times\frac{w(t)}{U}-\sum_{j=1}^{m_{Lw}}\frac{d_{Lw,j}U}{b}\phi_{Lw,j}(t)\right) \qquad (76)$$

$$\dot{\phi}_{Lw,j}(t)=-\frac{d_{Lw,j}U}{b}\phi_{Lw,j}(t)+A_{Lw,j+1}\frac{w(t)}{U}$$

$$(j=1,2,\ldots,m_{Lw}) \qquad (77)$$

where $\phi_{Lw,j}(t)$ $(j=1,2,\ldots,m_{Lw})$ is the augmented aerodynamic state vector. Similar expressions for other buffeting force components can be given with analogous definitions and are omitted here for the sake of brevity.

It is noted that the condition of the rational function approximation at $k\rightarrow 0$ or $U/fb\rightarrow\infty$ (found by invoking the quasi-steady theory) may be used for improving the accuracy of the curve fitting, which results in more realistic modeling at higher-reduced velocity. For example, $\Phi_{Lh}(\infty)=1$ results in

$$A_{Lh,1}=0;\quad A_{Lh,2}+\sum_{j=1}^{m_{Lh}}A_{Lh,j+3}/d_{Lh,j}=-2(C_L'+C_D) \quad (78)$$

The frequency-dependent force parameters at low-reduced velocity range (generally less than 20) is most important for the buffeting and flutter analysis of long span bridges. Since data at only lower velocities is commonly measured and available for bridge sections, using such a condition at $U/fb\rightarrow\infty$ may reduce the accuracy of the curve fitting at low-reduced velocities of interest. Rational function approximations should be determined in order to achieve higher accuracy at the reduced velocity range of interest. These approximated indicial functions based on limited data at lower velocities are referred to as the representatives of the low-reduced velocity components of the original functions. Figs. 4(a and b) show rational function approximations of $k^2(A_3^*+iA_2^*)$ for a set of rectangular sections with different side ratios $B/D=5,10,20$ ($B$: body width, $D$: body depth). For comparison, the results for the airfoil section using the Jones approximation of the Theodorsen function are also included. The symbols indicate the data obtained from wind tunnel tests (Matsumoto et al. 1995). For $B/D=15$ rectangular section, we have

$$k^2(A_3^*+iA_2^*)=2.8877-1.5091(ik)$$

$$-\frac{0.6162(ik)}{ik+0.1739}+\frac{0.5135(ik)}{ik+0.9871} \qquad (79)$$

Fig. 5 shows the rational function approximation of Davenport's aerodynamic admittance function with a decay factor $\lambda=8$

$$\chi_D^2=2(c-1+e^c)/c^2 \qquad (80)$$

where $c=\lambda fD/U=\lambda k_1/\pi$; $k_1=\omega D/U$; and $\chi_D$ is approximated as

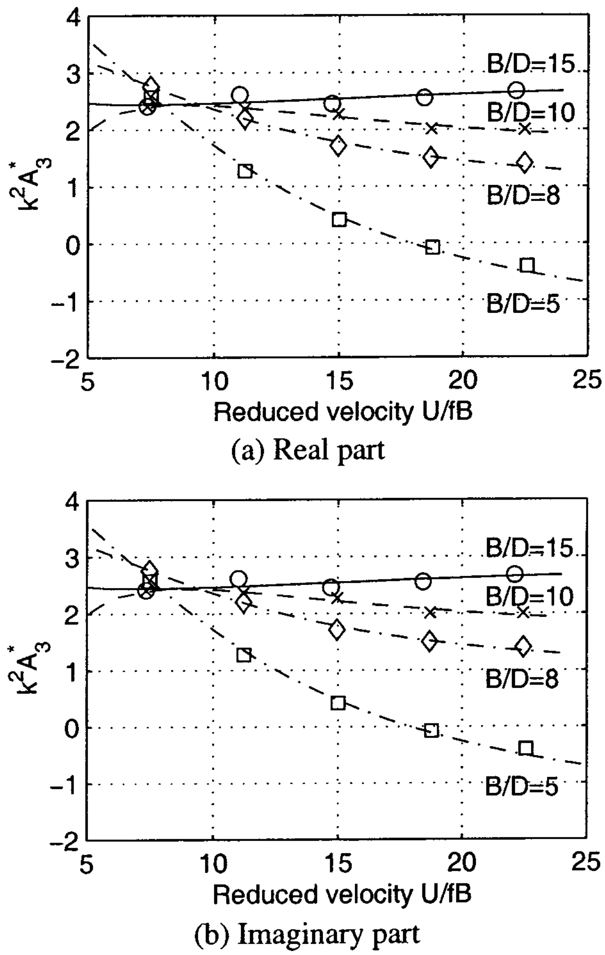



**Fig. 4.** Rational function approximations of aerodynamic transfer function $k^2(A_3^* + iA_2^*)$ for rectangular sections with different side rations $B/D$ ($B$: body width, $D$: body depth)

$$\chi_D(k_1) = 0.998 - \frac{0.1203(ik_1)}{ik_1 + 0.2957} - \frac{0.1621(ik_1)}{ik_1 + 0.5648} - \frac{0.2304(ik_1)}{ik_1 + 2.0875} - \frac{0.1868(ik_1)}{ik_1 + 2.9832} \quad (81)$$

Generally, the aerodynamic transfer functions in terms of flutter derivatives and admittance functions can be expressed in terms of the following rational function with negative poles suggesting that the aerodynamic force lags the body motion in phase

$$H(k) = \frac{N(ik)}{D(ik)} = \frac{b_0(ik)^n + b_1(ik)^{n-1} + \cdots + b_n}{(ik)^n + a_1(ik)^{n-1} + \cdots + a_n} \quad (82)$$

The coefficients $a_1,\ldots,a_n$ and $b_1,\ldots,b_n$ can be evaluated by minimizing the error

$$\sum_{j=1}^{m} \left[ H(k_j) - \frac{N(ik_j)}{D(ik_j)} \right]^2 \quad (83)$$

where $H(k_j)$ $(j=1,2,\ldots,m)$ represents the measured tabular data of aerodynamic transfer functions.

Once these aerodynamic transfer functions have been expressed in terms of the rational function format, the frequency-dependent unsteady aerodynamic forces can be calculated through a set of linear differential equations or through a state-space model (Ogawa 1994; Chen and Kareem 2001a). A controllable canonical form of the state-space representation is given as follows:

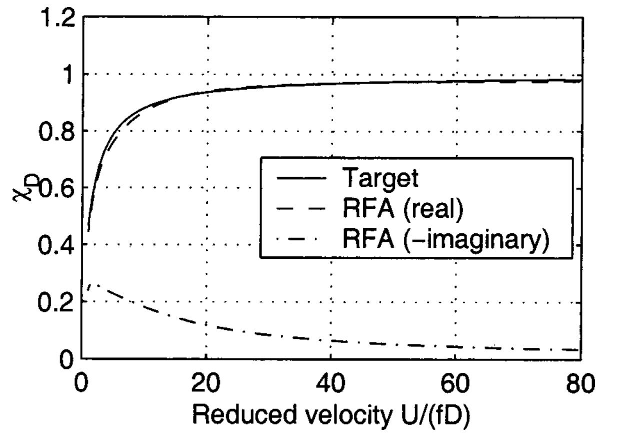



**Fig. 5.** Rational function approximations of admittance function

$$\dot{\mathbf{X}} = \mathbf{A}\mathbf{X} + \mathbf{B}u; \quad y = \mathbf{C}\mathbf{X} + \mathbf{D}u \quad (84)$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \\ -a_n & -a_{n-1} & -a_{n-2} & \cdots & -a_1 \end{bmatrix}; \quad \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}; \quad (85)$$

$$\mathbf{C} = [b_n - a_n b_0 \vdots b_{n-1} - a_{n-1} b_0 \vdots \ldots \vdots b_1 - a_1 b_0]; \quad \mathbf{D} = b_0 \quad (86)$$

It is noted that if the frequency dependent aerodynamic force parameters can be represented exactly or with an acceptable error by rational functions of reduced frequency, corresponding formulations of aerodynamic forces in the time domain would lead to exact or near exact representations of their frequency domain counterpart. These formulations lead to a more accurate estimate of the unsteady forces and attendant response of bridges in the time domain in comparison with those based on the routinely used frequency-independent quasi-steady assumption.

Time domain modeling of frequency-dependent aerodynamic forces results in augmented aerodynamic states. The number of aerodynamic states depends on the number of the denominator coefficients in the rational terms of the rational function approximation such as $d_{Lh,j}$ in Eq. (66) and $d_{Lw,j}$ in Eq. (73). Efforts for reducing the augmented aerodynamic states have been conducted by using common coefficients for different force components and by using minimum-state unsteady aerodynamic approximations (Hoadley and Karpel 1991; Wilde et al. 1996). Different forms of rational functions have also been utilized for accurate approximations (Sternberg 1991; Eversman and Tewari 1991). Similar applications using the rational function approximation technique are noted in engineering problems such as the interaction of structures with soil (Wolf 1991), structural response under hydrodynamic excitations (Damaren 2000), and random vibration of systems with frequency-dependent parameters (Spanos and Zeldin 1997).

## Spanwise Correlation of Aerodynamic Forces

The overall response analysis of long span bridges requires consideration of the spanwise correlation of the aerodynamic forces.

The self-excited forces are commonly assumed to be fully correlated in the spanwise direction. It has been noted that a loss of spanwise correlation of the self-excited forces stabilizes the single-mode torsional flutter (Scanlan 1997). Although the stabilizing effect of spanwise correlation loss may be apparent for single-mode torsional flutter, it is not obvious that this will apply to multimode coupled flutter cases. Correlation loss along the span may stabilize a bridge by reducing unfavorable negative aerodynamic damping effects, and yet it may destabilize a bridge by reducing favorable aerodynamic damping. A recent experimental study has identified that turbulence only slightly influences the spanwise correlation of self-excited forces (Haan et al. 1999; Haan 2000). This preliminary study tends to support the assumption of fully correlated self-excited forces in current approaches.

It has been commonly assumed that the buffeting forces have the same spanwise correlation as the incoming wind fluctuations based on strip theory. This assumption is questioned by the ''rapid distortion theory'' of turbulence and measurements in the separated flow regions suggest that the pressure field may have higher-correlation scales than the incident turbulence (e.g., Kareem 1990). Several studies have reported that the buffeting forces have a higher-spanwise correlation than that of the incident wind fluctuations and have been found to be a function of spanwise separation, scale of turbulence and the deck width (e.g., Larose and Mann 1998).

Taking into account the spanwise correlation of forces, the buffeting forces acting on an element of length $l$ can be referred to as the filtered output of the forces per unit length. The filter is characterized in terms of the spanwise coherence in the frequency domain and the impulse response function in the time domain. For example, the buffeting lift force is given in terms of the following double convolution integral in the time domain as

$$L_b(t) = -\frac{1}{2}\rho U^2 l \int_{-\infty}^{t}\int_{-\infty}^{\tau_2}\left( J_{Lu}(t-\tau_2) I_{Lu}(\tau_2-\tau_1)\frac{u(\tau_1)}{U} + J_{Lw}(t-\tau_2) I_{Lw}(\tau_2-\tau_1)\frac{w(\tau_1)}{U}\right) d\tau_1 d\tau_2 \quad (87)$$

where $J_{Lu}$ and $J_{Lw}$ are the impulse response functions.

In the frequency domain, it can be expressed as

$$L_b(t) = -\frac{1}{2}\rho U^2 (2b) l \left( 2C_L \overline{J}_{Lu}\chi_{Lu}\frac{u(t)}{U} + (C_L' + C_D)\overline{J}_{Lw}\chi_{Lw}\frac{w(t)}{U}\right) \quad (88)$$

where $\overline{J}_{Lu}$ and $\overline{J}_{Lw}$ are the Fourier transform counterparts of $J_{Lu}$ and $J_{Lw}$, respectively, and are referred to as the joint acceptance functions given by

$$\overline{J}_r = \int_0^l \int_0^l \mathrm{coh}_r(x_1, x_2, f)\, dx_1 dx_2 \quad (r = Lu, Lw) \quad (89)$$

where $\mathrm{coh}_r$ is the coherence function; and $x_1$ and $x_2$ are the spatial coordinates.

Similar to the frequency-dependent forces per unit length, using rational function approximations of the joint acceptance functions allows the frequency-dependent forces on an element of finite length to be calculated in the time domain using frequency-independent linear differential equations or a state-space model with augmented aerodynamic states.

## Nonlinear Aerodynamic Force Model

A nonlinear unsteady force model has been proposed based on the so-called ''quasi-static corrected theory'' by Diana et al. (1993) and Diana et al. (1999). A different nonlinear force model has also been developed by Chen and Kareem (2001b), which is based on the static force coefficients, flutter derivatives, and admittance functions along with the spanwise correlations at varying angles of incidence. The latter model has a clear relationship with the conventional linear force model. In this model, the turbulence and associated aerodynamic forces and responses have been separated into low-frequency (large-scale) and high-frequency (small-scale) components based on a critical frequency. For the low-frequency force component, the quasisteady formulation is utilized for modeling the aerodynamic forces because of the high-reduced velocities. The high-frequency component is linearized around the effective angle of incidence (low-frequency component) just as the conventional linear force model is linearized around the statically displaced position. For example, the high-frequency component of the self-excited lift force due to vertical motion is expressed as

$$L_{\mathrm{seh}}(t) = \frac{1}{2}\rho U^2 \int_{-\infty}^{t} I_{Lh}(\alpha_e, t-\tau) h(\tau) d\tau = -\frac{1}{2}\rho U^2 (2b)(C_L' + C_D) \times \int_{-\infty}^{t} \Phi_{Lh}(\alpha_e, t-\tau)\frac{\ddot{h}(\tau)}{U} d\tau \quad (90)$$

where the impulse and indicial response functions are functions of both effective angle of incidence and time. These are related to the flutter derivatives which are functions of both reduced frequency and angle of incidence.

The instantaneous effective angle of incidence is determined based on the low-frequency component of turbulence and structural motions (the low-frequency components are indicated by the superscript $l$) as

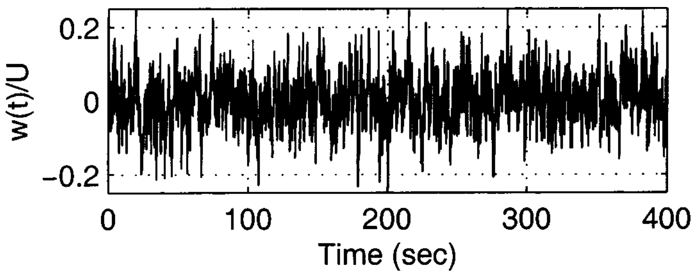
$$\alpha_e = \alpha_s + \alpha^l + \phi^l; \quad \phi^l = \tan^{-1}\left(\frac{w^l + \dot{h}^l + 0.5b\dot{\alpha}^l}{U + u^l - \dot{p}^l}\right) \quad (91)$$

When the low-frequency response is relatively small as is the case for long-span bridges, $\alpha_e$ can be approximated as

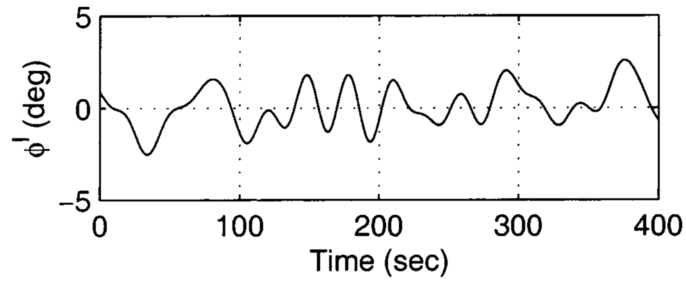
$$\alpha_e = \alpha_s + \tan^{-1}\left(\frac{w^l}{U+u^l}\right) \approx \alpha_s + \frac{w^l}{U+u^l} \quad (92)$$

Figs. 6(a) and 6(b) show an example of the vertical wind fluctuations with a turbulence intensity of $\sigma_w/U = 7.5\%$ and the corresponding low-frequency effective angle of incidence. Fig. 7 shows the flutter derivatives $A_2^*$ of a twin-box section measured at different angles of incidence (Matsumoto et al. 1998). $A_2^*$ for this section is very sensitive to the angle of incidence, and the consideration of this dependence will be important for an accurate estimation of aerodynamic forces.

Utilizing this nonlinear model, both the dependence of aerodynamic forces on frequency and effective angle of incidence can be considered. Furthermore, the effects of turbulence on flutter and the interaction of flutter and buffeting can be studied. The proposed analytical framework with nonlinear aerodynamics provides a unique tool for examining the effect of aerodynamic nonlinearities on bridge response. A coordinated experimental investigation is in progress for a comprehensive validation of this approach. This involves seeking an understanding of turbulence-




(a) Vertical fluctuation $w(t)/U$




(b) Low frequency effective angle of incidence

**Fig. 6.** An example of vertical wind fluctuations and associated effective angle of incidence ($\sigma_w/U=7.5\%$)

induced modifications of the magnitude and spanwise coherence of both the buffeting and the self-excited forces.

A number of analytical studies using stochastic approaches to randomize the dynamic pressure term have been conducted to predict some global trends in the turbulence-induced changes in flutter stability (e.g., Bucher and Lin 1988; Lin and Li 1993; Shinozuka and Billah 1993). These models have not been addressed in this paper. A shortcoming of these approaches lies in their inability to capture the effects of turbulence on the unsteady aerodynamics. Nonetheless, these contributions provide an elegant framework of stochastic stability analysis that may offer a useful avenue of analysis once the effects of turbulence are better understood.

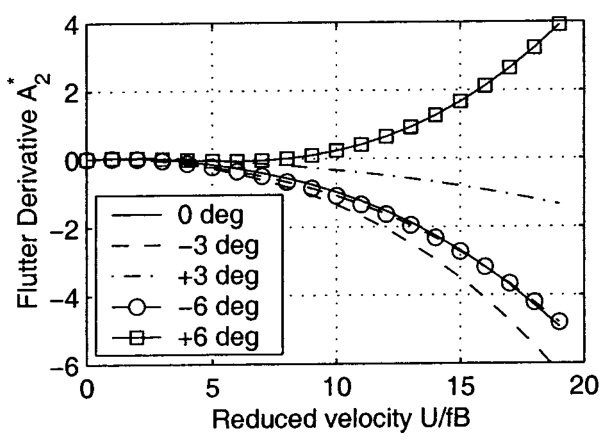



**Fig. 7.** Dependence of flutter derivatives $A_2^*$ on angle of incidence (twin-box section)

## Concluding Remarks

Recent advances in the modeling of aerodynamic forces on bridge decks were presented. Approximate relationships among aerodynamic force descriptors for static, self-excited, and buffeting force components, and the interrelationship among force parameters for self-excited forces were comprehensively discussed. Their comparison with those based on airfoil theory and those derived on the basis of quasi-steady theory was presented. Caution in using such approximate relationships for simplified modeling of aerodynamic forces and bridge response estimation was emphasized. The importance of modeling the frequency dependence of aerodynamic forces was addressed. Central to this modeling is the rational function approximation of the frequency domain force parameters as continuous functions of reduced frequency. This technique allows the unsteady aerodynamic forces to be expressed as a set of linear differential or state-space equations. A nonlinear aerodynamic force model was proposed to take into consideration the dependence of aerodynamic forces on frequency and the effective angle of incidence. This nonlinear force model provides a unique tool for investigating the effects of nonlinear aerodynamics on the overall bridge response. The nonlinear aerodynamics may become increasingly critical when the aerodynamic characteristics of innovative bridge deck designs, with attractive aerodynamic performance, exhibit significant sensitivity with respect to the effective angle of incidence and with the increase in the bridge span. These issues may not be addressed by utilizing the current linear aerodynamic force model. A coordinated experimental validation of the model is in progress. The synergistic review of the writers' recent work in bridge aerodynamics, in light of the current state-of-the-art of this field, presented here may serve a critical role in the development of new analysis tools and frameworks for the accurate prediction of the response of long span bridges under strong wind excitation.

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