Model for predicting the acrosswind response of buildings

A. Kareem

Department of Civil Engineering, University of Houston, Houston, Texas, USA
(Received December 1982; revised July 1983)

A model is presented for predicting the acrosswind response of isolated square cross-section buildings to typical atmospheric boundary layers, over different terrains. Closed-form expressions for the auto- and co-spectra of the acrosswind force fluctuations are formulated, based on wind tunnel measurements. A statistical integration scheme is used to develop a mode-generalized acrosswind spectrum for any desired approach flow condition, i.e. open country, suburban and urban. A simplified expression based on random vibration analysis is used to compute the modal response. The model provides flexibility in the selection of appropriate input parameters, broadening the scope of its application and serving as a useful tool for tailoring the preliminary design of tall buildings.

Keywords: buildings, wind loadings, acrosswind response

It is important to recognize the unsteady nature of wind loads in the design of buildings to ensure structural safety and serviceability requirements. The alongwind response of buildings can be computed using the gust factor approach.[1] However, lack of a suitable transfer function between the velocity fluctuations in the approach flow and the pressure fluctuations on the faces of a building with separated flow has prohibited any acceptable formulation, to date, of the acrosswind response based on a gust factor approach.

Physical modelling of wind–structure interaction in a boundary layer wind tunnel, therefore, continues to serve as the practical approach relating structural response and aerodynamic loading to properties of local wind climates.[2] Quantitative description of wind loads would permit the numerical estimation of a building response at the preliminary design stages, allowing early assessment of the structural requirements to resist oscillatory response to ensure occupancy comfort, and assessing the need for detailed aeroelastic wind tunnel tests. The experimental measurement of aerodynamic forces on a scale model can be introduced in lieu of the intractable solution of the equations of fluid motion around the model.[3–8]

Reference 4 presents a detailed account of the measurement of total fluctuating loads on a building using surface pressures. A covariance (statistical) integration procedure was used to derive the spectra of the acrosswind loads from the numerical estimates of the auto- and co-spectra of pressure fluctuations on the building model surface.

The model developed was limited to quasi-static loading and did not include motion-induced aerodynamic loads. Such motion induced loads result from a combination of negative aerodynamic damping and an increase in the correlation of the fluctuating pressure on the building surface. It is a general consensus that in most of the tall buildings the influence of motion-induced loading is insignificant for typical design wind speeds. However, for exceptionally slender, flexible and lightly-damped structures at high reduced velocities, the quasi-static forcing function developed in reference 4 may underestimate the structural response. For these exceptional structures, the motion-induced aerodynamic loads can easily be incorporated through the modification of the building mechanical admittance function using appropriate values of aerodynamic damping.[9] On the other hand, in view of the significance of the motion induced loads, such structures, which are exceptionally sensitive to wind excitation, are generally subjected to extensive wind tunnel aeroelastic model testing. Here, closed form expressions are derived for the auto- and co-spectra of acrosswind forces at various levels to develop overall mode-generalized spectra. This procedure would provide the flexibility of changing the power law exponent, turbulence intensity, acrosswind force coefficients, mean wind speed and structural properties of a building, i.e. mass, stiffness, damping, frequency and mode shapes, influence coefficients for shear and moment as the input parameters. In this paper the validity of some commonly used simplifications in the development of forcing functions are examined.[10,11] Response estimates are compared with the values obtained from the spectra derived in reference 9. This procedure can be adopted

0141-0296/84/02136-06/$03.00

conveniently for desk-top computers in a small design office.

Development of forcing function

The fluctuating acrosswind force on a building is given by:

$$F_y(z,t) = \tfrac{1}{2}\rho U_h^2 C_L(z,t) A \qquad (1)$$

where $C_L(z, t)$ is the random acrosswind force coefficient, ρ is air density and U_h is the wind velocity at the building height. The power spectral density function of $F_y(z, t)$ from experimental measurements indicates energy concentration near the vortex shedding frequency and the spectral bandwidth depends on the geometry of the building and the approach flow characteristics.[9-13] $S_{F_y}(z, n)$ is generally described by a Gaussian-type expression for slender (very high aspect ratio) structures, e.g. chimneys.[10-12] Such a Gaussian description best describes the narrow band region around the Strouhal frequency but fails to adequately represent the spectral energy at frequencies other than the Strouhal frequency.

To predict the structural response for non-resonant conditions, $nD/U_h \neq S$, which is generally the case for buildings, the function used to represent $S_{F_y}(z, n)$ must be able to describe the spectral contents over the entire range of frequencies of interest along the height of the structure. A number of expressions representing filtered white noise were used to model $S_{F_y}(z, n)$. Due to the asymmetry of the spectral density function, $S_{F_y}(z, n)$, about the Strouhal frequency the following two expressions, which are functions of the power law exponent and turbulent intensity profile of the approach flow and and the building height, best describe the measured values:

$$\frac{nS_{F_y}(z,n)}{\sigma^2} = \alpha\alpha \times \beta\beta \times (n/n_s)^{0.9} \qquad n \leqslant n_s$$
$$= \alpha\alpha \times \beta\beta \times (n/n_s)^{3.0} \qquad n \geqslant n_s \qquad (2)$$

$$\alpha\alpha = \frac{B}{\left[1-\left(\frac{n}{n_s}\right)^2\right]^2 + \left[2B\left(\frac{n}{n_s}\right)\right]^2}$$

$$\beta\beta = 1.32\left[\left(\frac{1}{3\alpha}\right)^{1/2} + 0.154\left(1-\frac{z}{H}\right)^{3.5}\right]$$

in which n_s is shedding frequency $= SU(z)/D$; σ^2 is the mean square value of fluctuating acrosswind force; S is the Strouhal number; α is the exponent term in the power law; B is the bandwidth coefficient $= 2^{1/2}I(z)$; and $I(z)$ is the turbulence intensity at height z.

The above expression is plotted in *Figure 1* along with the measured spectrum at the mid-height of the building. The spectrum of the fluctuating overall acrosswind force in the ith mode is given by:

$$S_{F_i}(n) = \int_0^H \int_0^H \int_0^\infty (F_y(z_1,t)\,F_y(z_2,t+\tau)\,e^{-j2\pi n\tau}\,d\tau) \times \phi_i(z_1)\,\phi_i(z_2)\,dz_1\,dz_2 \qquad (3)$$

The term in the brackets is the co-spectrum of the acrosswind force fluctuations at z_1 and z_2 locations; $\phi_i(z_1)$ is the ith mode shape evaluated at z_1; and H is the building

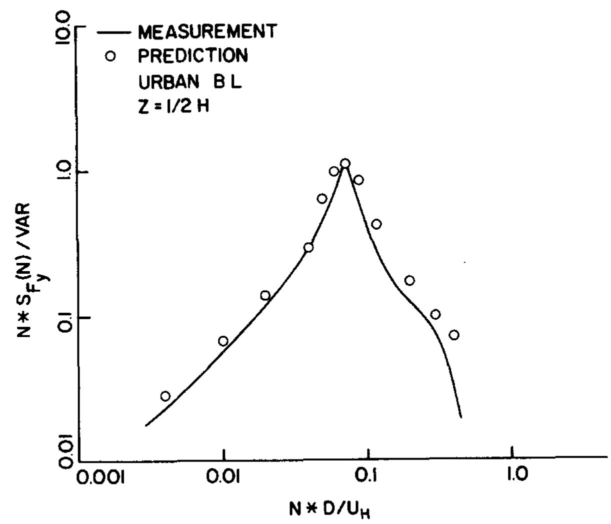

Figure 1 Predicted and measured normalized reduced spectra of acrosswind force

height. Equation (3) is rewritten in terms of the normalized co-spectrum:

$$S_{F_i}(n) = \int_0^H \int_0^H \sqrt{S_{F_y}(z_1,n)\,S_{F_y}(z_2,n)}\,\mathrm{Co}(z_1,z_2,n) \times \phi_i(z_1)\,\phi_i(z_2)\,dz_1\,dz_2 \qquad (4)$$

in which $\mathrm{Co}(z_1, z_2, n)$ is the normalized co-spectrum, and $S_{F_y}(z_1, n)$ is the lift force spectrum at location z_1. Further rearranging terms in equation (4) leads to the following expression for the reduced normalized spectrum of the acrosswind loading in the ith mode:

$$\frac{nS_{F_i}(n)}{[\frac{1}{2}\rho U_h^2 DH]^2} = \frac{1}{H^2}\int_0^H \int_0^H \sqrt{\frac{nS_{F_y}(z_1,n)}{\sigma_F^2(z_1)}}\sqrt{\frac{nS_{F_y}(z_2,n)}{\sigma_F^2(z_2)}} \times \mathrm{Co}(z_1,z_2,n)\,C_L(z_1)\,C_L(z_2)\,\phi_i(z_1)\,\phi_i(z_2)\,dz_1\,dz_2 \qquad (5)$$

in which:

$$C_L(z) = \frac{\sigma_F(z_1)}{\frac{1}{2}\rho D U_h^2}$$

the acrosswind force coefficient.

The co-spectrum for the acrosswind force is generally an exponentially decaying function with a peak near the Strouhal frequency.[13,14] The following analytical expressions, as a function of frequency and the separation distance, are derived to match the measured values in various boundary layer flows which are characterized by their power law exponents and the building height:

$$\mathrm{Co}\left(\frac{\Delta z}{H}, n\right) = \exp\left\{\left(-\frac{\Delta z}{H}\right)^{1/3}\left[1-\frac{\Delta z}{H}\right]^{1/2}\left(\frac{nD}{U_H}\right)\right\}$$
$$\times\left\{\exp\left(\frac{2\Delta z}{H}\right)\alpha(0.88+\alpha)^2 + \frac{2}{5}\left(1-\exp\left(\frac{\Delta z}{H}\right)\right)\right\}$$
$$\times\left[1+5(\tfrac{1}{3}-\alpha)\left(1-\exp\left(-\left(\frac{\Delta z}{H}\right)^2\right)\right)\right]$$

$$\times\left[\cos\left(\left(\frac{20\pi}{1-\pi}\left(\frac{nD}{U_H}\right)-1\right)\right]; \quad \frac{nD}{U_h}\leqslant F^*$$

$$=\mathrm{Co}\left(\frac{\Delta z}{H}, F^*\right)\exp\left(-20\left(\frac{\Delta z}{H}\right)^{1.2}\left(\frac{nD}{U_h}\right)^{1/2}\right); \quad \frac{nD}{U_h}\geqslant F^* \tag{6}$$

in which $\Delta z = |z_1-z_2|$ and $F^* = 1.25(1-\alpha)/10$.

The plots for $\mathrm{Co}(\Delta z/H, n)$ are given in *Figures 2* and *3* for $\alpha = 0.12$ and $\alpha = 0.34$, respectively.

Equations (2) and (6) are substituted in equation (5) to obtain the spectra of the acrosswind loading for the linear mode shape. The predicted as well as the experimental spectra are given in *Figures 4* and *5* for open country and urban environments. The measured and the predicted values are in good agreement, especially in the frequency range of tall buildings.

An important feature in the model presented here is the choice of approach flow characteristics and the building height. This broadens the scope of applications and makes this model highly useful for tailoring the preliminary design of an isolated tall building in any desired environment. In *Figure 6* a set of typical spectra, based on equation (5) for the fundamental mode, are presented for a building with an aspect ratio of 1:6 in open country, suburban, and urban flow conditions.

In the following section some commonly used simplifications in the formulation of mode-generalized spectra are examined. The first such simplification introduces a pressure correlation length scale in equation (5) to facilitate evaluation of a two-fold integral by reducing it to single-fold. The second simplification is a procedure for adjusting a mode-generalized spectrum from one mode shape to another without re-evaluating equation (5).

Equation (5) can be simplified on the assumption that the spanwise correlation of the acrosswind force is small compared with the building height and that it is frequency independent:

$$\frac{nS_{F_i}(n)}{[\frac{1}{2}\rho U_h^2 DH]^2}=\frac{1}{H^2}\int_0^H\int_0^H\sqrt{\frac{nS_{F_y}(z_1,n)}{\sigma_F^2(z_1)}}\sqrt{\frac{nS_{F_y}(z_2,n)}{\sigma_F^2(z_2)}}$$

$$\times\,\delta(z_1-z_2)\,\lambda C_L(z_1)\,C_L(z_2)\,\phi_i(z_1)\,\phi_i(z_2)\,\mathrm{d}z_1\,\mathrm{d}z_2 \tag{7}$$

In equation (7) the co-spectrum is replaced by a function of separation distance $\delta(z_1-z_2)\lambda$, in which $\delta(\ )$ is a Dirac delta function and λ is the length scale of the spanwise

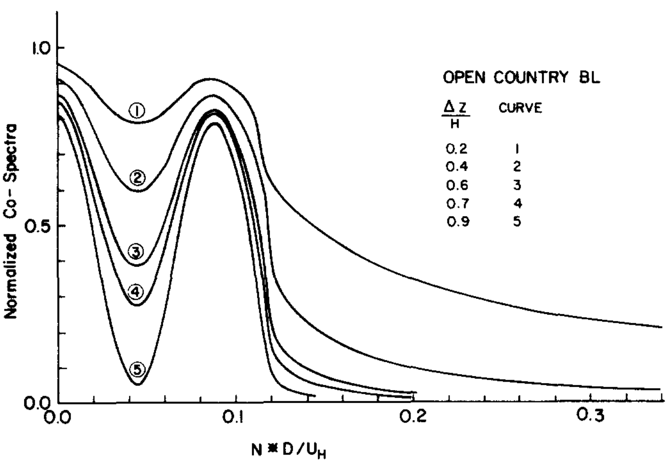

Figure 2 Normalized predicted co-spectra for open country flow

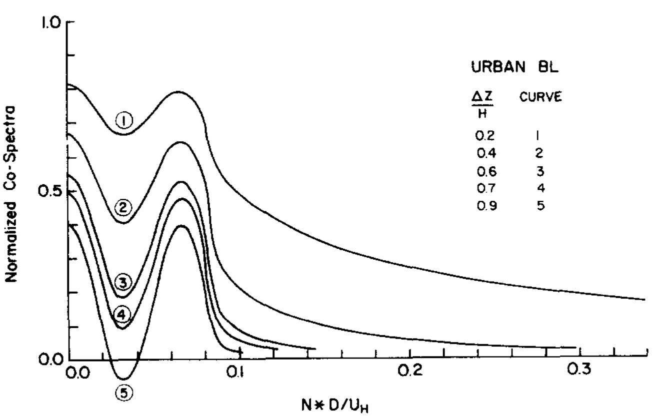

Figure 3 Normalized predicted co-spectra for urban flow

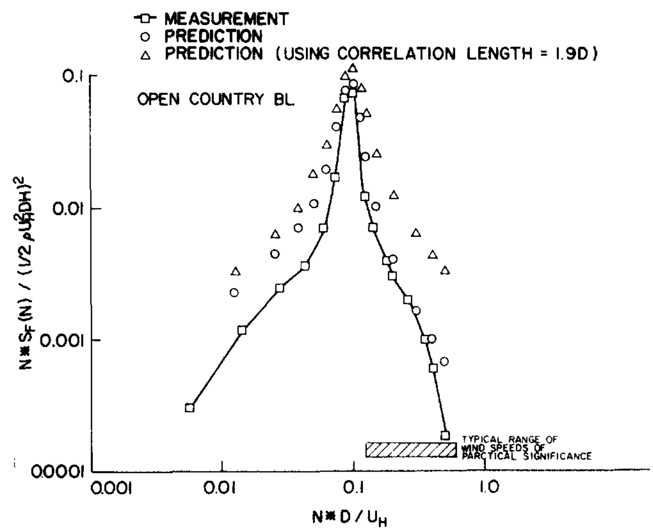

Figure 4 Predicted and measured mode-generalized spectra for open country flow

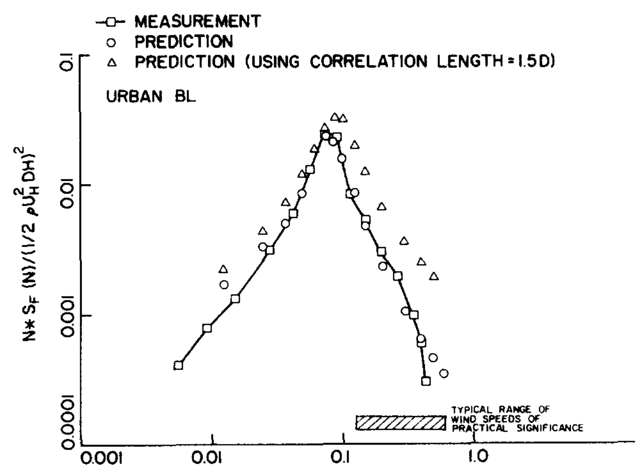

Figure 5 Predicted and measured mode-generalized spectra for urban flow

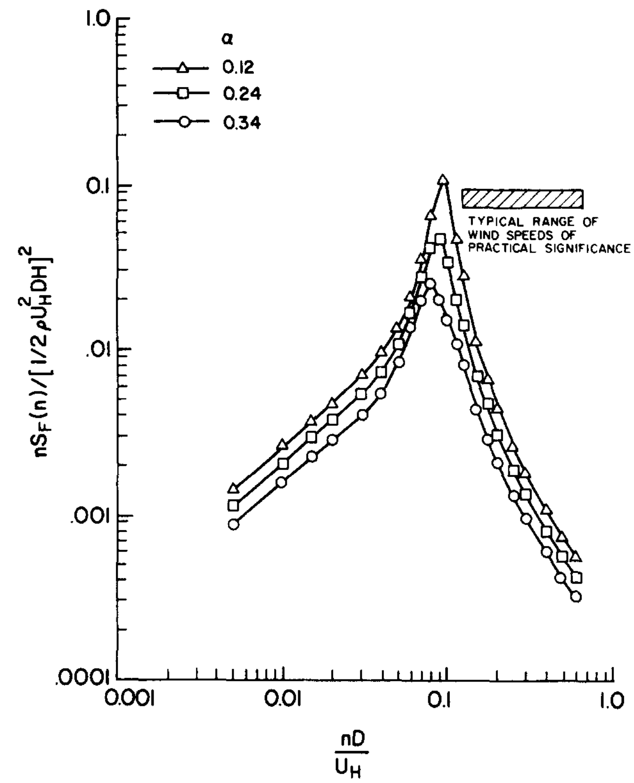

Figure 6 Mode-generalized spectra for open country, suburban and urban flow conditions

acrosswind force. The Dirac delta function is used for mathematical convenience here, since integration of the co-spectrum, assumed to be only a function of separation distance, with respect to height, reduces to a pressure length scale. This substitution reduces two-fold integration in equation (5) to the following single integration:

$$\frac{nS_{F_i}(n)}{[\frac{1}{2}\rho U_h^2 DH]^2} = \frac{\lambda}{H^2} \int_0^H \frac{nS_{F_y}(z,n)}{\sigma_F^2(z)} C_L^2(z)\, \phi_i^2(z)\, \mathrm{d}z \quad (8)$$

Acrosswind spectra obtained from the simplified procedure (equation (8)) and the full-equation approach (equation (5)) are compared in *Figures 4* and *5*. The simplified procedure provides extremely conservative values. In as much as the co-spectrum is a function of separation distance and the frequency, the assumption which led to equation (8) may have oversimplified the procedure. Methods based on such assumptions should therefore be viewed carefully.

However, it is important to note that these models may predict accurately the resonant response of a structure excited by a narrow band forcing function. Hansen[10] has used a similar approach for predicting the resonant response of a tall chimney due to vortex shedding. In such a case, at the resonant frequency the co-spectrum is only a function of separation distance which justifies the use of equation (8).

The mode-generalized spectra, i.e. those obtained from equation (5), can be modified to any desired mode shape without re-evaluating the two-fold integral. This modification also becomes necessary when only a mode-generalized spectrum is available in a mode shape which differs from the desired shape, and it is given by:

$$S_{F_\psi}(n) = \frac{1}{C} \int_0^H S_{F_\phi}(n)\, \psi^2(z)\, \mathrm{d}z$$

$$C = \int_0^H \phi^2(z)\, \mathrm{d}z \quad (9)$$

in which $\phi(z)$ is the original mode shape; $\psi(z)$ is the modified mode shape, and the second subscript in $S_F(n)$ represents the mode shape with respect to which the spectrum has been weighted.

However, it appears that the validity of this simplification is influenced by an increase in the nonlinearity of the mode shape, i.e. departure from linear mode shape. A similar expression for adjusting the linear mode generalized spectra to any desired mode shape has been given in reference 15.

In *Figure 7*, the modified values and that obtained from the solution of the complete equation (5) are compared for two different mode shapes. Results show good agreement. Nevertheless, for most of the building structures the procedure given in equation (9) can be used to adjust a given spectrum for any desired mode shape. A commonly occurring problem in structural dynamics, in which such a modification becomes necessary, has led to the verification of the procedure given in equation (9) for which equation (5) provides a good reference for comparison.

Prediction of response

The acrosswind dynamic response can be computed using random vibration theory. The mean square response is given by:

$$\sigma_{y(r)}^2(z) = \int_0^\infty S_{y(r)}(z,n)\, \mathrm{d}n \quad (10a)$$

$$S_{y(r)}(z,n) = \sum_i \phi_i^2(z)\, H_i^{(r)}(n)^2 S_{F_i}(n) \quad (10b)$$

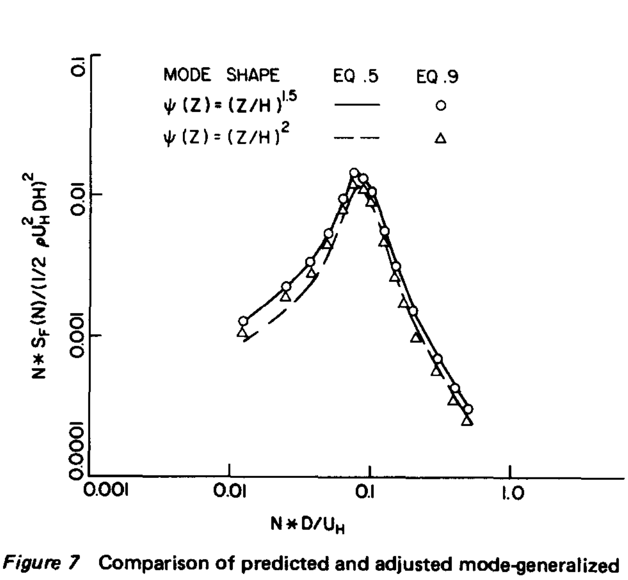

Figure 7 Comparison of predicted and adjusted mode-generalized spectra

in which $|H_i^{(r)}(n)|^2$ is the frequency response function in the ith mode for rth derivative of displacement response; $r = 0, 1, 2, 3$, represents displacement, velocity, acceleration and jerk components of response. $S_{F_i}(n)$ is given by equation (5).[9] If only the fundamental mode of vibration is of interest then $i = 1$ and the summation of equation (10) is reduced to a single value. Equation (10) involves a two-fold integration to evaluate $S_{F_i}(n)$ and a single integral to compute the mean square value. This integration can be expeditiously carried out using a Monte Carlo procedure.[16] The integration in equation (10a) can be also performed using the residue theorem:

$$\sigma_{y_r}^2(z) = \sum_i \frac{\phi_i^2(z)\,\pi n_i S_{F_i}(n_i)\,(2\pi n_i)^{2r}}{4(2\pi n_i)^4 \xi_i m_i^2} \tag{11}$$

in which $S_{F_i}(n_i)$ is the generalized forcing function in the ith mode evaluated at the ith frequency; ξ_i is damping ratio in the ith mode:

$$m_i = \int_0^H m(z)\,\phi^2(z)\,\mathrm{d}z$$

the generalized mass in the ith mode; and r denotes the derivatives of displacement response.

Equation (11) not only eliminates the computational effort required in integrating equation (10), but also limits the evaluation of equation (5) to the resonant frequency. This leads to a very significant reduction in the computational time for evaluating the acrosswind response and makes this procedure attractive for microcomputers.

The expected peak value of acrosswind response can be obtained from theoretical considerations of the probability of extreme values and is generally expressed as a product of a peak factor and the rms value for each mode (e.g. peak factor generally varies between 3 and 4). For structures with well-separated frequencies, the SRSS (square-root of sum of squares) method of combining modal maxima gives acceptable values. In the absence of well-separated frequencies, which is unlikely for typical tall buildings, a more realistic method of modal combination can be used which is based on random vibration theory. The procedure of modal combination is essentially a cross-modal coefficient (normalized response covariance matrix) weighted summation of modal maxima. This approach, however, requires the computation of the response covariance matrix.

Example

Consider a square cross-section building with linear mode shape for which $H = 180$ m; $D = 31$ m; natural frequency, $n = 0.2$ Hz; building density $= 192$ kg/m^3; air density, $\rho = 0.973$ kg/m^3; and ratio of critical damping, $\xi = 0.01$.

Equation (10) is used along with the spectra in *Figure 6* to obtain estimates of acrosswind response. The predicted response for a range of wind velocities and corresponding response estimates given in reference 9 are plotted in *Figure 8*. The results indicate excellent agreement and suggest the proposed model is valid.

The response of the aforementioned building was also computed assuming a different mode shape, $\psi = (z/H)^{1.5}$, in suburban environment ($\alpha = 0.25$) for comparing results with the numerical and experimental estimates given by Kwok,[8] and Saunders and Melbourne.[15] Equation (5) was used for adjusting the mode generalized spectrum to conform to the new mode shape. The predicted response is plotted in *Figure 9* and it shows good agreement with the predicted and experimental estimates given in references 8 and 15.

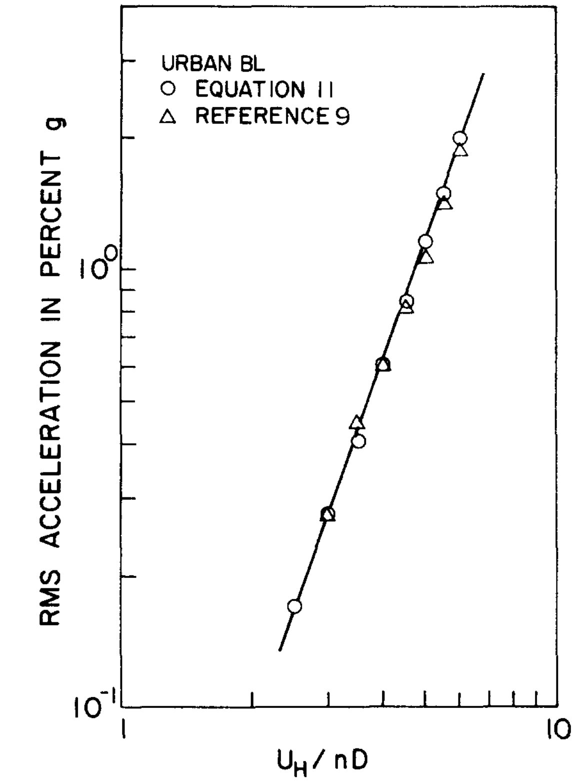

Figure 8 Comparison of predicted rms acceleration response

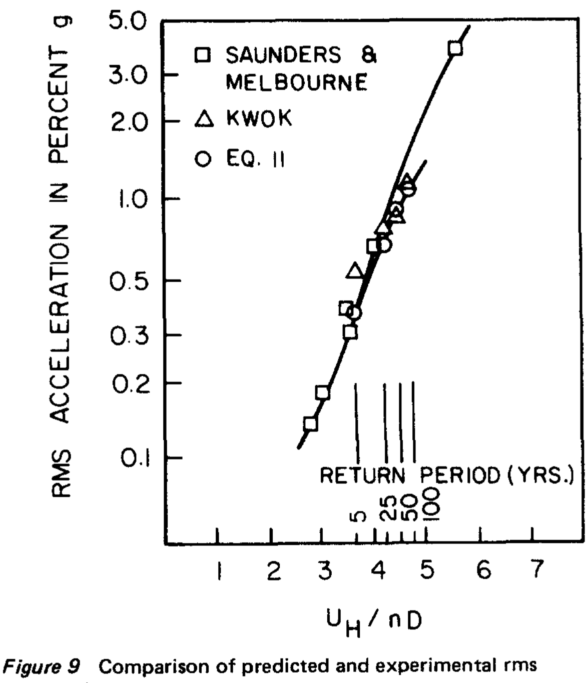

Figure 9 Comparison of predicted and experimental rms acceleration response

Conclusions

The analytical expression for the acrosswind spectrum reported here provides a basic formulation of the forcing function for isolated square cross-section buildings of aspect ratio equal or greater than 1:4 for typical design wind speeds.

The model provides flexibility in the selection of appropriate input parameters, broadening the scope of its application, and providing a useful tool for tailoring the preliminary design of isolated tall buildings in any desired flow condition. The response predictions based on the proposed model offer excellent agreement with earlier studies.[8,9,15]

The study indicates that the mode-generalized spectra for typical tall buildings can be adjusted for different mode shapes by a simple transformation.

It is also concluded that the simplification which leads to the formulation of the acrosswind forcing spectrum, obtained through the introduction of correlation length scale, may lead to significant overestimation of the structural response.

Acknowledgments

The author thanks Dr J. E. Cermak for his encouragement and invaluable suggestions during the experimental part of the work, and Mr Chii-Ming Cheng for assistance with the computer work. This work was partly supported by the National Science Foundation under Grant Nos. ENG-76-03135 and CEE-8019392. Their support is gratefully acknowledged.

References

1 Simiu, E. 'Modern developments in wind engineering – Part IV', *Eng. Struct*., 1983, **5**, 27

2 Kareem, A. and Cermak, J. E. 'Wind tunnel simulation of wind structure interactions', *ISA Trans*. 1979, **18** (4),

3 Tschanz, T. 'Measurement of total dynamic loads using elastic models with high natural frequencies', *Prepr., Int. Workshop on Wind Tunnel Modeling Criteria and Techniques for Civil Engineering Applications*, NBS, 14–16 April 1982

4 Kareem, A. 'Measurement of total loads using surface pressures', *Prepr., Int. Workshop on Wind Tunnel Modeling Criteria and Techniques for Civil Engineering Applications*, NBS, 14–16 April 1982. (Also published by Cambridge University Press)

5 Reinhold, T. 'Measurement of simultaneous fluctuating loads at multiple levels on a model of a tall building in a simulated urban boundary layer', *PhD Thesis*, Virginia Polytechnic Institute and State University, Blaksburg, Virginia, 1977

6 Saunders, J. W. and Melbourne, W. H. 'Tall rectangular building response to cross-wind excitation', *Proc. Fourth Int. Conf. Wind Effects on Buildings and Structures*, Heathrow, England, 1975

7 Vickery, B. J. 'Load fluctuations in turbulent flow', *Proc. Am. Soc. Civ. Eng., J. Eng. Mech. Div*. 1968, **108** (EM1), 94

8 Kwok, K. C. S. 'Cross-wind response of tall buildings', *Eng. Struct*. 1982, **4**, 256

9 Kareem, A. 'Acrosswind response of buildings', *J. Struct. Div., ASCE* 1982, **108** (ST4)

10 Hansen, S. O. 'Cross-wind vibration of a 130-m tapered concrete chimney, *J. Wind Eng. and Industr. Aerodynamics* 1981, **18**

11 Davenport, A. G. 'The influence of turbulence on the aeroelastic response of tall structures to wind', *IAHR-IUTAM Symp. Practical Experiences with Flow-Induced Vibrations*, Karlsruhe, 1979, Springer-Verlag, 1980

12 Vickery, B. J. 'A model for prediction of the response of chimneys to vortex excitation', *3rd Int. Symp. Chimneys* Oct. 1978, Munich, Germany

13 Kareem, A. and Cermak, J. E. 'Pressure fluctuations on a square building model in boundary layer flows', *J. Wind Eng. and Indust. Aerodynamics* (to be published, 1984)

14 Kareem, A. 'Wind excited motion of buildings', *Thesis*, Colorado State University, Fort Collins, Colorado, 1978

15 Saunders, J. W. and Melbourne, W. H. 'Wind-excited buildings – design sway stiffness', Royal Melbourne Institute of Technology, Department of Mechanical and Production Engineering, Australia, Nov. 1977

16 Kareem, A. and Hseih, J. 'Reliability of concrete chimneys under winds', Department of Civil Engineering, University of Houston, *Report No. CER83-UH83-AK-JH1*, May 1983