

ARMA REPRESENTATION OF WIND FIELD

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ABSTRACT

The dynamic response analysis of structures subjected to a stochastic wind field is carried out in the time domain by a step-by-step integration approach. The loading is represented by simulated time histories of the aerodynamic force. The auto-regressive and moving average (ARMA) recursive models are utilized to simulate time series of wind loads. Depending on the system dynamic characteristics, the time-integration schemes require that the time increment should not exceed a prescribed value. This study focuses on the development of ARMA representation based procedures to simulate realizations of wind loading with small time increments required by the time-integration schemes. A three-stage-matching method and a scheme which combines ARMA and digital interpolation filters are presented to efficiently generate correlated time histories of wind loads at the prescribed time increments.

INTRODUCTION

The dynamic response analysis of structures is often performed in the frequency domain for the sake of computational expedience. However, there are cases such as system nonlinearities where a straightforward application of the frequency domain analysis becomes computationally prohibitive and the time domain solution provides a convenient alternative. The input to the numerical time domain solution requires corresponding time histories of the space-time variations in the loads. The simulated time series are required to match single-point power spectral density functions and multi-point correlation structure. One of the traditional approaches for simulation is to utilize a superposition of trigonometric functions, e.g., cosine functions, with statistically independent phase angles (Shinozuka, 1971). The simulated numbers can be conveniently incorporated in a Monte Carlo simulation of the response of a system utilizing a numerical integration scheme. The procedure in principle is applicable not only to scalar processes, but may be applied to multivariate and/or multidimensional fields. However, the summation of a large set of trigonometric terms involved in the simulation procedure renders this approach computationally inefficient (Shinozuka and Jan, 1972; Wittig and Sinha, 1975). In this context, it has been noted that the digital generation of the sample time histories can be carried out efficiently with the aid of the Fast Fourier Transform (FFT) algorithm.

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The use of a FFT based simulation improves the computational efficiency, but not without the expense of increased demand on computer storage. This difficulty magnifies manifold in the event that the simulated data is required over a long period or when the problem in hand involves multivariate and/or multi-dimensional processes. ARMA model representation of random fields has been utilized for simulating sample functions of target random processes and fields (e.g., Samaras, et al., 1985 and Naganuma, et al., 1987). The recursive nature of this approach substantially helps to reduce demand on memory space and CPU time for generating desired sample functions.

The parameters associated with ARMA models are determined in such a manner that the spectral description of the system response to white noise approximates the target spectral characteristics in an optimum sense (e.g., Mignolet and Spanos, 1987). The coefficient matrices in this manner may be determined by the maximum entropy method (MEM). However, for the multi-variate ARMA models MEM requires solving a large number of nonlinear equations. This has led to the development of many other techniques. The two-stage matching method is one popular approach (e.g., Spanos and Schultz, 1985). Most of the studies reported in the literature have focussed on matching a univariate or a multivariate ARMA model to a prescribed spectral description; little attention has been given to the influence of time increment on the simulation. The time-integration schemes often require a small time step to ensure that the energy content of the highest frequency component of the excitation is accurately described. This paper provides efficient simulation techniques to generate time series concerning wind field with small time steps utilizing ARMA models.

WIND FIELD & LOAD EFFECTS

Under the influence of wind, structures experience force in the alongwind, acrosswind and torsional directions (Kareem, 1987). The alongwind force is primarily resulting from drag. In order to formulate fluctuating alongwind force acting on a structure, it is necessary to establish a description of the multi-point statistics of the wind velocity fluctuations. Appropriate aerodynamic transfer functions are introduced to transform wind velocity fluctuations to corresponding wind loads. Accordingly, the wind load effects on structures can be simulated either by generating space-time structure of random wind field or by directly generating fluctuating loads induced by wind fluctuations. Additional details concerning the space time description of wind field may be found in the following: Davenport (1961), Simiu and Scanlan (1986) and Kareem (1987).

The first approach, requires generation of a vector containing time series of fluctuating wind velocities at the centroids of various sections, $\mathbf{u}(n\Delta t)$, by a multi-variate ARMA model, which is followed by computing the associated vector of the wind force time series. In the second approach, a vector of time series of wind loading, $\mathbf{F}(n\Delta t)$ is directly generated by a multi-variate ARMA model matched to the wind force spectral description.

ARMA MODELING

An autoregressive and moving average (ARMA) model of orders P and Q is defined as a linear filter that permits simulation of the wind velocity or force vector $\mathbf{y}(\Delta t)$ ($M \times 1$) by its past time histories and the past and present white noise processes:

$$y(n\Delta t) + \sum_{r=1}^P A_r y[(n-r)\Delta t] = \sum_{r=0}^Q B_r \epsilon_{n-r}, \quad (1)$$

in which A_r and B_r are AR and MA coefficient matrices, and ϵ_{n-r} is a vector containing white noise with a zero mean and a unit variance. The coefficient matrices are determined from the specified spectral description of the random field to be simulated. A desirable feature of ARMA modeling is that the model orders and the error are small, since the total order translates into the number of multiplications and additions at each discrete time interval during the simulation procedure.

ARMA models are characterized by poles and zeroes. A low-order ARMA model could produce a small error if the coefficients are suitably selected. The application of the maximum entropy condition to a multivariate ARMA model requires the solution of a large number of nonlinear equations. Therefore, some techniques have been developed to circumvent this difficulty. One of these approaches is the two-stage-matching method, by which an ARMA model is developed based on a prior AR model. There are a number of different approaches that follow the two-stage-matching method e.g., (Samaras et al., 1984 and Spanos and Shultz, 1985). In this manner, an ARMA model can be formulated by the solution of a number of linear equations. This procedure is computationally convenient. However, ARMA model coefficients with given P and Q are not unique, since they depend on the prior AR order P'.

Altogether, there are three parameters to be selected: P', P, Q and Δt . Samaras et al. (1984) gave an empirical relationship,

$$P = Q \text{ and } P' \geq P + Q + 2. \quad (2)$$

Optimal model orders may only be obtained from a larger number of combinations of the above parameters. In the present study, an interactive computer algorithm is developed to study the effects of various combinations of the parameters on the model accuracy. An empirical relation based on this study suggests

$$P' = 3(P + Q). \quad (3)$$

Then for a fixed P+Q, a selection of P and Q can affect the model error. For the wind field, it is recommended to use

$$P \geq Q. \quad (4)$$

The time increment, Δt , also influences the model error. Approximately, the ARMA model involves a window of time period (P+Q) Δt . A decrease in Δt could increase the model error. The shape of the wind spectral density function can affect significantly the model accuracy. For example, an ARMA model used to match the wind spectrum with a finite value at the zero frequency, such as the Harris spectrum, has higher accuracy than an ARMA model for the wind spectrum with a zero value at the zero frequency, such as the Davenport spectrum. It is important to note that the ARMA matching is also sensitive to the accuracy of the correlation functions derived from a given spectral density function.

Typical examples of the comparisons between the target spectra and the estimated spectra are shown in Fig. 1, and the comparisons of the coherence functions are shown in Fig. 2. It is noted that an ARMA model can be matched to the Harris wind spectrum with almost no error in the entire frequency range. The main difficulty in matching the Davenport wind spectrum is due to the representation of the sharp drop in the spectral ordinates near the zero frequency.

In most of the above examples, the time series were simulated for large Δt (≥ 2 sec), in which the spectral density of the wind velocity at the Nyquist frequency still has significant energy. However, it is generally required to have a much smaller Δt for the following reasons: 1) The natural frequency of the structure may be high; 2) The structure may be subjected to other environmental loads, such as wave loads, earthquake, etc., which have high frequencies, hence, the time series of wind loads must have a small time increment to be consistent with the time series of other loads; 3) Depending on the numerical scheme, the accuracy and stability during the time-integration procedure requires that the time increment not exceed a prescribed maximum value. If the required time increment is 0.5 sec., then the Nyquist frequency would be 1 Hz, but generally a significant portion of wind energy is less than 0.1 Hz. Also the coherence function is often significant only within a small portion of the entire frequency range. Therefore, it becomes quite difficult to match an ARMA model to wind fields with small Δt . In the following sections, the discussion focuses on two methodologies: a three-stage-matching method and interpolation models.

THREE-STAGE MATCHING METHOD

A three-stage-matching method is proposed that involves the introduction of a suitable parameter α which transforms $y(\Delta nt)$ into $z(n\Delta t)$.

$$z(n\Delta t) = y(n\Delta t) - \alpha y[(n-1)\Delta t] . \quad (5)$$

Using the backshift operator B, the preceding equation reduces to

$$z(n\Delta t) = (1 - \alpha B) y(n\Delta t) . \quad (6)$$

The (i,j) th element of the spectral density matrix of $z(t)$ becomes

$$G_{z_{ij}}(f) = [1 + \alpha^2 - 2 \cos(2\pi f\Delta t)]^2 G_{y_{ij}}(f) . \quad (7)$$

If $\alpha=1$, then Eq. (6) is similar to the seasonal difference method in the Box-Jenkins approach (Box & Jenkins, 1970). However, $\alpha=1$ leads to a zero spectral ordinate in Eq. (7), and it is not invertible. Hence, α is selected such that $0 < \alpha < 1$ which ensures a slow decrease in the spectral ordinates of $z(n\Delta t)$ with an increase in frequency. For the Harris wind spectrum, it is recommended to use $\alpha=0.94\sim 0.98$.

An example of wind loads acting on a tall building with the lowest natural frequency equal to 0.2 Hz (the wind and the wind force data are listed in Table I) is used to illustrate this approach. The building face is divided into five segments. It is required to have the wind load simulated at an interval of at least 0.5 sec. Figure 3 demonstrates the spectral densities of the wind loads at the 5th level before and after the transformation with $\alpha=0.955$. It is noted in this figure that the wind load spectral ordinates are negligible

within the frequency range 0.1~1 Hz. However, after the transformation, the spectrum covers the entire frequency range up to the Nyquist frequency and decays slowly with the frequency.

$z(n\Delta t)$ can be matched by an ARMA model utilizing the two-stage-matching procedure discussed in the preceding section,

$$z(n\Delta t) + \sum_{r=1}^{Pz} A_r^z z[(n-r)\Delta t] = \sum_{r=0}^Q B_r \epsilon_{n-r}, \quad (8)$$

which, after substitution in Eq. (6), takes the form of Eq. (1), in which

$$P = P^z + 1, \quad (9)$$

and

$$A_1 = A_1^z - \begin{bmatrix} \alpha & 0 & \dots & 0 \\ 0 & \alpha & \dots & 0 \\ \vdots & \vdots & \dots & 0 \\ 0 & \dots & \dots & \alpha \end{bmatrix},$$

$$A_r = A_r^z - \alpha A_{r-1}^z \quad \text{for } r > 1. \quad (10)$$

In this example, an ARMA (4,4) model from a prior AR (30) model has been matched for $z(n\Delta t)$. Subsequently, ARMA model (5,4) for $y(n\Delta t)$ is formulated. The spectral density of the target wind load and those represented by the ARMA model are plotted in Fig. 4. The results demonstrate the closeness between the target and estimated spectral functions.

The ARMA model representation given by the three-stage-matching method is especially suitable for describing the high frequency spectra. If the low frequency wind loads are not important, it is possible to obtain a very low order ARMA model, e.g., ARMA (3,1) for the above example. The three-stage-matching method involves a selection of the parameter α , in addition to P' , P and Q . However, it does improve the shape of the spectral density functions, which facilitates a convenient modeling, but it cannot improve the shape of the coherence functions. Hence, the error in the correlation functions may not be small. Additional details may be found in Li and Kareem (1990). This method can be applied to most of the wind spectra, such as the Harris or Kareem spectra (Kareem, 1987). For some wind spectra, such as the Davenport spectrum, which has a zero ordinate at the zero frequency, this method is not suitable.

INTERPOLATION MODELS

A more general procedure to simulate time series of wind field involves a combination of an ARMA model and the interpolation model. Let the time increment in the ARMA model be Δt , such that the corresponding Nyquist frequency is a little larger than the frequency beyond which the wind has insignificant energy. Suppose that the time-integration scheme for the solution of the dynamic system requires a much smaller time increment δt , where $\Delta t / \delta t$ is an integer S . It is required to formulate time series $y[(nS+\beta)\delta t]$ from $y(n\Delta t)$ in which $\beta < S$. There are a number of interpolation methods

available in the literature (e.g., Oppenheim & Schafer, 1989). Recently, new interpolation techniques with applications in engineering mechanics have been developed (Li, 1988). Their details will be reported elsewhere. Here some concepts relevant to wind engineering are introduced. First, it is important that the interpolation following an ARMA model satisfy the following requirements:

Local interpolation: $y[(nS+\beta)\delta t]$ is simulated from $y(m\Delta t)$ with $m= n-Q_1^-, n-Q_1^-+1, \dots, n, \dots, n+Q_1^+$, in which Q_1^- and Q_1^+ are small integer numbers. The conventional global interpolation involving the total time series is not suitable for the present application.

Stability: The interpolation is said to be stable if a bounded time series after interpolation still remains bounded.

Accuracy: The spectral density functions represented by $y[(nS+\beta)\delta t]$ are the same as those of $y(n\Delta t)$ when the frequency is less than $1/2\Delta t$, and zeros in the frequency range $1/2\Delta t \sim 1/2\delta t$.

The interpolation techniques may be classified as linear, polynomial and trigonometric. Frequently, a piecewise linear interpolation of the discrete data is utilized. This method is the simplest, but it may introduce a large error in the spectral density function. The polynomial interpolation is the next level of interpolation. For example, Li (1988) developed a cubic polynomial interpolation, which results in a process continuous at its first-order time derivative, involving three multiplications and additions at each discrete time interval. The trigonometric interpolation suggested by Saunders and Collings (1980), is further developed in this study, and it is perceived to be the most suitable choice for wind engineering applications. In the following a basic concept of the trigonometric interpolation developed herein is presented:

The discrete time series can be viewed as a process, $y^{\Delta t}(t)$, consisting of numerous pulses:

$$y^{\Delta t}(t) = \sum_{n=-\infty}^{\infty} y(n\Delta t) \Delta t \delta(t - n\Delta t), \quad (11)$$

in which $\delta(t-n\Delta t)$ is the Dirac delta function. Its Fourier transform is written as

$$Y^{\Delta t}(f) = \sum_{n=-\infty}^{\infty} y(n\Delta t) \exp(-j2\pi f n\Delta t) \Delta t, \quad (12)$$

which is a discrete Fourier transformation. The preceding equation shows that $Y^{\Delta t}(f)$ is cyclic with a cycle $1/\Delta t$, and $Y^{\Delta t}(f) = \overline{Y^{\Delta t}(1/\Delta t - f)}$ in which the overbar is the conjugate sign. Similarly, the time series with a time increment δt can be viewed as a pulse process $y^{\delta t}(t)$, and the corresponding Fourier transformation becomes

$$Y^{\delta t}(f) = \sum_{m=-\infty}^{\infty} y(m\delta t) \exp(-j 2\pi f m\delta t) \delta t. \quad (13)$$

Define a transfer function $H(f)$ which satisfies

$$H(f) = \frac{Y^{\delta t}(f)}{Y^{\Delta t}(f)}. \quad (14)$$

For the ideal case, $H(f)=1$ for $f < 1/2\Delta t$ and $H(f)=0$ for $1/2\delta t < f < 1/2\Delta t$. Additional details are available in Li and Kareem (1990). The final expression for $y^{\delta t}(t)$ is given by

$$y^{\delta t}[m\delta t] = \sum_{r=-SQ^I+1}^{SQ^I} h(r\delta t) y^{\Delta t}[(m-r)\delta t], \quad (15)$$

in which Q^I is the interpolation order, and $h(r\Delta t)$ the convolution kernel for interpolation is given by

$$h(r\delta t) = \sum_{m=-SQ^I+1}^{SQ^I} H(m\Delta f) \exp\left(j\pi \frac{mr}{SQ^I}\right) / (2SQ^I), \quad (16)$$

As an example, Fig. 5 demonstrates a comparison of an ideal transfer function, with the transfer function given by Saunders and Collings (1980) and the present study. It is noted that with the same interpolation model order, the transfer function represented by the interpolation form introduced in the present study is closer to the ideal transfer function.

Considering that $y^{\Delta t}(m\delta t)=0$ for $m\delta t \neq n\Delta t$ in which n is an arbitrary integer number, Eq. (15) can be rewritten as

$$y^{\delta t}[(Sn+\beta)\delta t] = \sum_{r=-Q^I+1}^{Q^I} h_{\beta r} y^{\Delta t}[(n+r)\Delta t], \quad (17)$$

in which $h_{\beta r}$ is a rearrangement of $h(r\delta t)$. Hence, virtually, the interpolation is carried out at each time interval $(Sn+\beta)\delta t$ by $2Q^I$ multiplications and additions,

$$y[(Sn+\beta)\delta t] = \sum_{r=-Q^I+1}^{Q^I} h_{\beta r} y[(n+r)\Delta t]. \quad (18)$$

An example of this interpolation model is considered here. A fluctuating component of wind velocity according to the Davenport spectrum is simulated by an ARMA model with $\Delta t=2$ sec., and is further interpolated to form the wind velocity with $\delta t=0.5$ sec. This interpolation is performed by the above described interpolation model of order 8. In order to validate the interpolation model, the discrete time series before interpolation is taken from a continuous time function instead of an ARMA model. In Fig. 6, the continuous time function and the discrete time series before and after the interpolation are plotted. It is noted in this figure that the continuous function and the time series simulated by interpolation are almost coincident.

CONCLUDING REMARKS

The simulation of wind velocity and wind force fields can be performed by ARMA models. These models are developed to match the prescribed spectral description utilizing,

among others, the two-stage-matching procedures. However, the nature of the dynamic systems and the numerical schemes utilized for the solution of the dynamic response often require a small time increment, which renders a straightforward application of ARMA models difficult. A three-stage-matching method is developed, in which a multivariate ARMA model of small time increments can be matched to a target wind spectrum. A salient feature of this method is that despite the low order of the model the spectral density function of the simulated data is in good agreement with the target spectral density function.

A more general technique utilizing an interpolation approach is presented. First, the time series is generated from an ARMA model with the time increment selected according to the maximum frequency of interest. Then by the interpolation technique the generated time series with large time increment is transformed into a time series with a smaller time increment. In this research a digital filter based on trigonometric interpolation is developed utilizing discrete convolution of finite and infinite waveforms. The two-stage proposed approach based on an ARMA model combined with an interpolation scheme offers a computationally efficient simulation procedure.

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TABLE I. WIND AND WIND LOADING CONDITIONS

WIND PARAMETERS

SPECTRUM	HARRIS
MEAN WIND VELOCITY AT 32.8 ft HIGH	29.84 ft/sec
POWER EXPONENT	0.34
LENGTH SCALE	4000 ft.
WIND SURFACE STRESS COEFFICIENT	0.05
DECAY PARAMETER (HORIZONTAL & VERTICAL).....	10, 16
AIR DENSITY	0.0019 Slug/ft ³

WIND DATA ON EACH SECTION

NO	z(ft)	U (ft/s)	C_u (ft ² /s ²)					
1	540	77.34	287.1	168.2	125.6	98.17	76.01	
2	420	71.01	168.2	287.1	162.7	118.4	87.74	
3	300	63.33	125.6	162.7	287.1	154.8	105.9	
4	180	53.23	98.17	118.4	154.8	287.1	144.9	
5	60.	36.64	76.01	87.74	105.9	144.9	287.1	

WIND FORCE DATA ON EACH SECTION

NO	C_d	W(ft)	H(ft)	MEAN (kN)	C_F (kN ²)					
1	1.2	100	120	81.83	855.8	566.2	394.9	265.6	142.5	
2	1.2	100	120	68.98	566.2	705.8	444.3	284.6	147.5	
3	1.2	100	120	54.87	394.9	444.3	544.6	312.2	153.0	
4	1.2	100	120	38.77	265.6	284.6	312.2	373.3	170.6	
5	1.2	100	120	18.37	142.5	147.5	153.0	170.6	158.3	

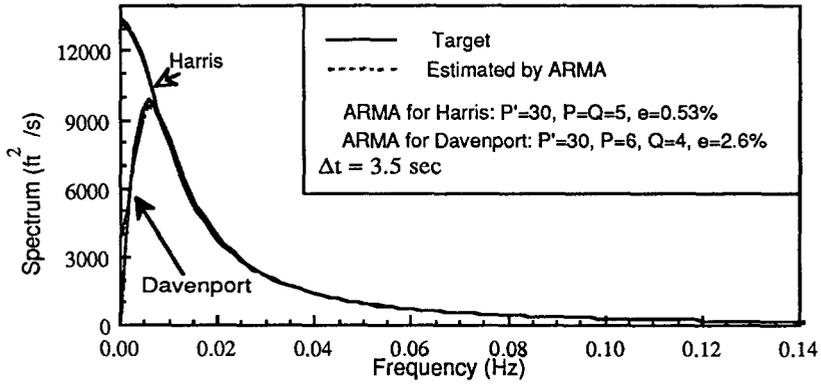


Fig. 1 Comparison of Estimated and Target Spectral Density Functions

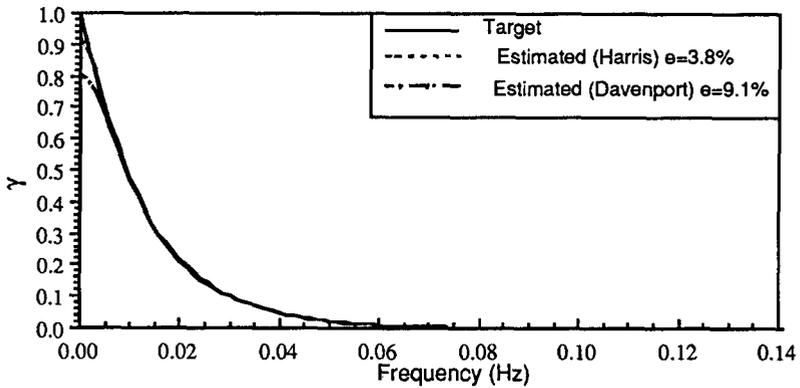


Fig. 2 Comparison of Estimated and Target Coherence Functions

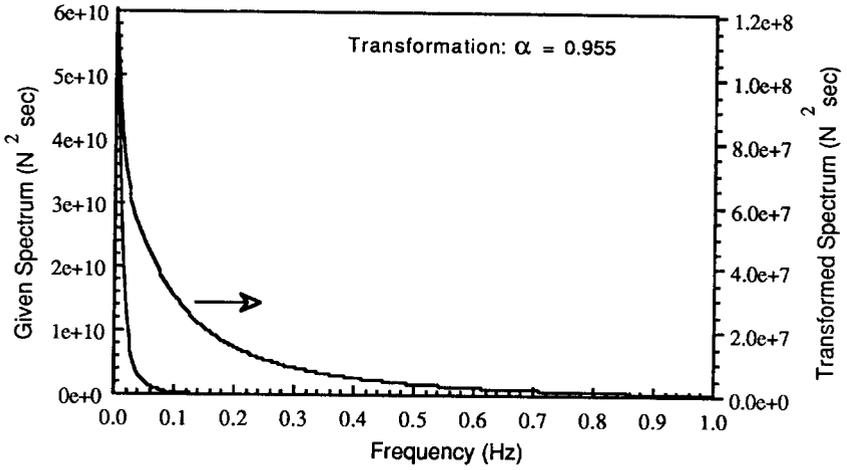


Fig. 3 Target Spectra Before and After Pre-Transformation

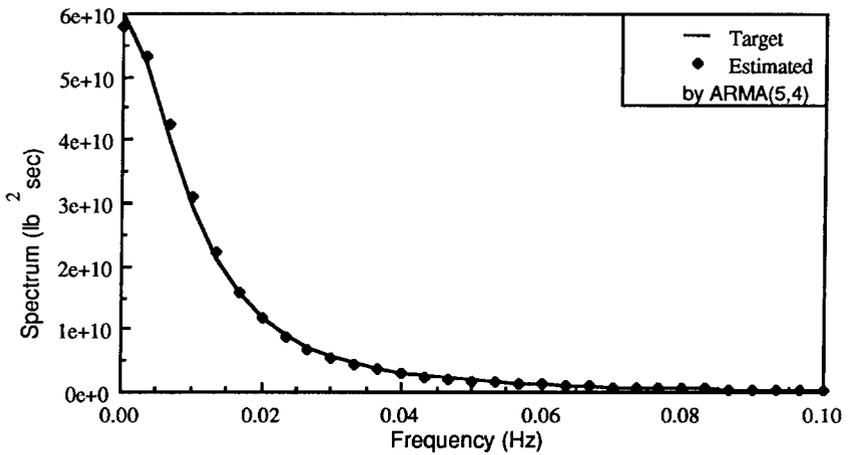


Fig. 4 Target and Estimated Wind Load Spectra on the 5th Floor

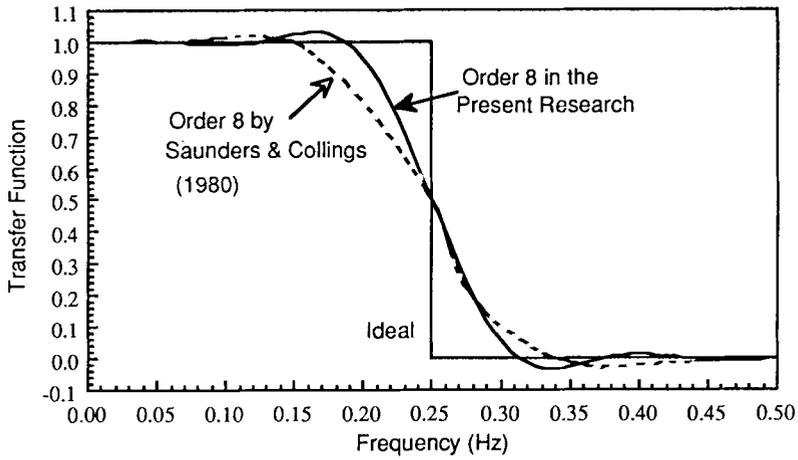


Fig. 5 Transfer Functions of Interpolation Models

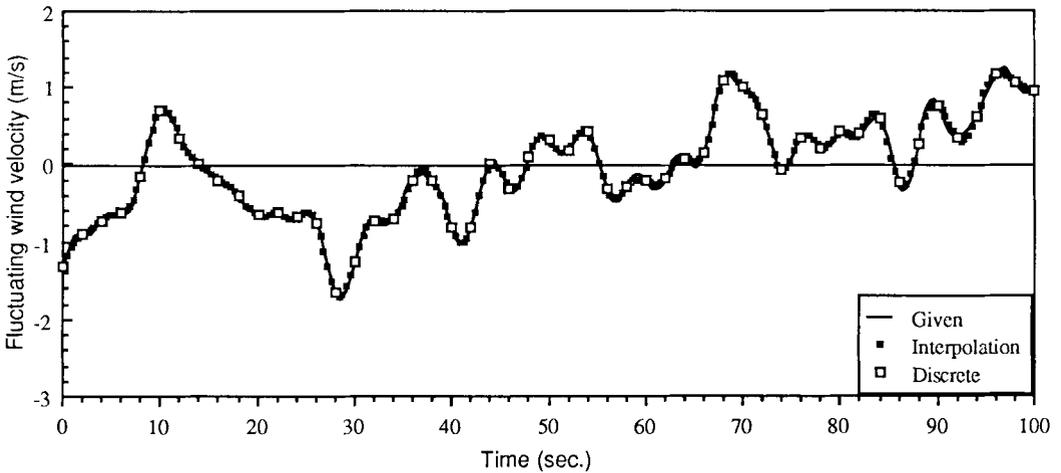


Fig. 6 Continuous, Discrete and Interpolated Time Series