Dynamic response of structures with uncertain damping

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The uncertainty associated with damping in structural systems is identified and discussed. A second-order perturbation technique is utilized to examine the effects of damping variability on the transient and steady-state dynamic response of structural systems. The results demonstrate that the uncertainty in damping indeed influences the system response. The effects are more pronounced for higher variability of damping values.

Keywords: damping, uncertainty, random vibration, perturbation, seismic, wind, waves, buildings

Quantification of damping is by far the most vexing problem in structural engineering. Unlike the inertial and stiffness properties of a structural system, damping does not refer to a unique physical phenomenon; increasingly, it is recognized as an important factor in the design of structures that are sensitive to wind, waves or earthquake loading, singly or in combination. Damping is of particular interest to the designers of high-rise buildings, where it plays an important role to help meet serviceability limit states from human comfort considerations. The estimates of damping in a structural system have intrinsic variability as a result of the complexity of damping mechanisms. The ability to estimate damping values accurately, at the design stage, would certainly alleviate a major source of uncertainty from the design of dynamically sensitive structures.

The objective of this study is to investigate the influence of damping uncertainty on the system dynamic response. Following a brief discussion of fundamental damping mechanisms and their mathematical modelling features, a probabilistic description of the system response in terms of the second-order statistics of the variability of damping is presented utilizing a perturbation-based approach. A numerical example of a discretized structural system is presented to illustrate the significance of damping uncertainty on the system response.

Background

Damping is a measure of structural capacity to dissipate energy in order to reach a quiescent state. The damping capacity may be defined as the ratio of the energy dissipated in one cycle of oscillation to the maximum amount of energy accumulated in the structure in that cycle. There are as many damping mechanisms as there are modes of converting mechanical energy into heat. The important damping mechanisms are material damping and interfacial damping[1,2].

The material damping contribution comes from complex molecular interactions within a material, and so the total damping of the structure is dependent on the type of material, method of manufacturing and final finishing process. The complexity of the situation is increased by the simple reality that material properties often differ from sample to sample, resulting possibly in significant differences in energy losses among members of a structural system. The equations of motion in structural dynamics usually describe macroscopic behaviour, while material damping processes arise from microscopic phenomena. It is for this reason that phenomenological theories are being utilized for achieving satisfactory representation of energy dissipation in dynamical systems.

The interfacial damping mechanisms arise from coulomb damping in the form of a frictional interface between members and connections of a structural system. Welded connections tend to reduce the contribution of interfacial damping compared with bolted connections. The soil-structure interaction may contribute towards the overall damping[3].

A structure vibrating in a fluid is subjected to fluid dynamic forces which may tend to dampen the vibration resulting from the viscous action of the surrounding fluid. This damping contribution is generally small compared with mechanical damping in aerodynamic applications whereas in hydrodynamic situations, for example, depending on the magnitude of the drag coefficient, a tension leg type platform may experience large hydro-

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dynamic damping forces[4]. It is also noted that for some structures with certain shapes the fluid dynamic forces may tend to enhance the oscillations by synchronizing with the structural motion, or in other words result in negative damping[5,6]. The effects of negative aerodynamic damping are often significant and may cause severe structural damage.

An element in the damping coefficients matrix, C_{ij}, may be defined as a force developed at coordinate i due to unit velocity at j. A consistent damping coefficient may be formulated if the distributed damping coefficient per unit length is known. The overall system damping matrix may be assembled from the element damping matrices. In the case of utility plant structures which are fabricated from a variety of materials and components fastened together by means of several complex joints, such local damping mechanisms can vary significantly in different parts of the structure, and are difficult to quantify[7]. In practice, evaluation of the distributed damping coefficient is generally impractical, and this has led to the customary practice of expressing damping in terms of measured damping ratios.

The assumption of proportional damping is often invoked to utilize modal superposition techniques which facilitate decoupling of the equations of motion with the aid of modal matrices associated with the undamped system. The most general proportional damping model is given by

$$[C] = [M] \sum_i a_i([M]^{-1}[K])^i \tag{1}$$

in which i may range from $-\infty < i < \infty$ and the summation may include as many terms as desired. For $i = 0$ and $i = 1$ in the previous equation, one obtains the Rayleigh damping. In the case of combined systems where proportional damping is generally not possible, decoupling may be accomplished utilizing a state-vector approach[8].

The selection of an appropriate damping value is a controversial subject. Although it is a general consensus that damping values change with amplitude of motion, their functional descriptions are rather limited[9-13]. Besides the complex nature of damping, the methods employed to ascertain damping of full-scale structures and the analysis and interpretation of data introduce additional uncertainty. Information available from full-scale measurements for analysing the variability of damping has been assembled by Haviland[14], Jeary and Ellis[13], Yokoo and Akiyama[12], and Davenport and Hill-Carroll[10], among others. Haviland[14] reported a wide range of data for different response amplitude levels, structural systems and building heights. This study showed that log-normal and Gamma distributions provided the best fit to the data. The coefficient of variation (COV) of damping values varied between 42 and 87%. In a more recent study by Davenport and Hill-Carroll, the variability of damping was analysed by carefully selecting data obtained from available full-scale studies[10]. A regression analysis in their study suggested that the expected value of damping could be expressed as

$$\bar{\xi} = A\left(\frac{\Delta}{H}\right)^n \tag{2}$$

in which (Δ/H) is the ratio of the RMS amplitude in millimetres to the building height in metres, and A and n are constants ($A = 0.02$-0.03; $n = 0.075$-0.11) that depend on the building height and the type of building (e.g., steel or concrete). Although the COV of damping estimates based on the available data ranged from as low as 33% to as high as 78%, they suggested a value of 40%. The log-normal distribution was found to suitably describe the variability in damping. Based on a selected group of measurements, meeting sufficient quality criteria, Jeary has proposed a theoretical model that provides damping values which are in good agreement with measured values[9]. The damping at any structural response amplitude, X_H, may be estimated by the expression

$$\xi = \xi_o + \xi_l \frac{[X_H]}{H} \tag{3}$$

in which $\xi_o = f_o$, $\log_{10} \xi_l = \sqrt{D}/2$; D = building dimension[9]. A similar expression is also suggested in ESDU[11].

The level of damping is being currently augmented in structural systems directly by means of viscoelastic dampers, or in some cases indirectly through active or passive dynamic vibration absorbers or tuned mass dampers (TMDs)[15-20]. A viscoelastic damper is a passive discrete damping device that is capable of dissipating large amounts of energy in shear. However, the viscoelastic material properties are a function of frequency and temperature, which requires that these features be included in the performance evaluation of such a system. Currently, viscoelastic dampers have been successfully used in the World Trade Center, New York and Columbia Center Building, Seattle[17,18]. The current use of viscoelastic dampers has so far been limited to non-structural applications; extension of the concept is possible to include their contribution to the stiffness of the system. Base isolation is being extensively used to isolate buildings from earthquake-induced ground motion by means of multilayer elastomeric bearings[20]. In another application, the response of structures with liquid-containing appendages is mitigated when one of the sloshing modes of the secondary fluid appendage is tuned to the fundamental mode of the primary system[19]. Incorporation of any of the foregoing damping devices introduces additional uncertainty in estimates of damping available to the system.

The uncertainty associated with damping introduces variability in representing the response of a system. The damping uncertainty may be expressed in terms of the constants in equation (1), or alternatively uncertainty may be assigned to the critical damping ratios. For proportional damping, the damping matrix can be decoupled by the normal mode approach in terms of damping ratios. On the other hand, if the damping ratios are known, one can obtain the damping matrix[7,8]. The variability of damping can be expressed in terms of the damping ratio or the damping matrix as stated earlier. However, in the event of the damping matrix being non-proportional, one has to invoke a second-order perturbation technique to represent the uncertain response and thereby facilitate a convenient solution. Even in this situation, the analysis requires that the mean part of the damping matrix be proportional, otherwise one needs to resort to alternative techniques such as recasting the problem in $2N$ state variables[8]. In view of the impracticality of determining damping coefficients and the general engineering practice of expressing structural damping in terms of the critical damping ratios, the formulation in this study is based on uncertainty of damping expressed in terms of mean and fluctuating damping ratios.

Despite the importance of variability of damping, one should not overlook other sources of uncertainty in the characteristics of a structural system. The stiffness and mass of a structure have intrinsic variability due to a variety of reasons. Their influence on the eigenproperties has long been a subject of interest to researchers in applied mathematics as well as engineering mechanics. More recently the need to investigate the influence of these uncertainties on the system response has become a topic of considerable interest that is directed toward the development of a probabilistic structural analysis methodology.

Propagation of uncertainty

The probabilistic dynamic response of uncertain systems is expressed in terms of uncertain parameters associated with structural characteristics and load effects. Uncertainties associated with each basic variable are propagated through the functional relationships that relate them to the response for determining the uncertainty of the dynamic response estimates. In the framework of probabilistic or stochastic finite element analysis, analytical and computational concepts such as first- or second-order Taylor series expansions, perturbation techniques, random fields, Neumann expansions and simulation techniques may be employed for the propagation of uncertainty. Some of these techniques are currently in the development stages. Literature related to these studies may be found, for example, in References 21-28. Many of these studies have focused on the treatment of uncertain stiffness of the system. The implementation of uncertainty of damping in the dynamic analysis has been addressed in References 21, 28-30. In this study, the uncertainty of damping is propagated by means of a second-order perturbation technique in both time and frequency domains to investigate its influence on the transient and steady-state vibrations.

Formulation

The equations of motion of a discretized system subjected to an external and a base excitation, respectively, are given by

$$M\ddot{X} + C\dot{X} + KX = F(t) \tag{4}$$

$$M\ddot{X} + C\dot{X} + KX = -MI\ddot{X}_g(t) \tag{5}$$

respectively, in which M, K are assembled deterministic mass and stiffness matrices, C is a proportional uncertain damping matrix, X is a response vector which represents relative displacement in the case of base excitation (equation (5)), $F(t)$ is external excitation (e.g., wind or wave loading), $\ddot{X}_g(t)$ represents base acceleration and I is an identity vector. Employing the standard transformation of coordinates involving undamped eigenvectors of the system offers the following uncoupled equations corresponding to equations (4) and (5)

$$\ddot{q}_i + 2\xi_i\omega_i\dot{q}_i + \omega_i^2 q_i = p_i(t) \tag{6}$$

$$\ddot{q}_i + 2\xi_i\omega_i\dot{q}_i + \omega_i^2 q_i = \Gamma_i\ddot{X}_g(t) \tag{7}$$

in which $p_i(t) = \Phi_i^T F(t)$, $\Gamma_i = \Phi_i^T(-M)I$, and $X = \Phi_i q_i$.

The uncertain damping ratio that corresponds to the ith mode in the previous equations may be expressed in terms of the mean and perturbed values

$$\xi_i = \xi_i^o(1 + \alpha_i) \tag{8}$$

in which ξ_i^o is the mean value of damping ratio, and α_i is a small Gaussian fluctuation. In certain physical situations such as parametric systems, it is possible to experience random fluctuation as a function of time; e.g., in aeroelasticity applications[28]. Uncertainty in damping, represented by equation (8), is propagated in this study using a second-order perturbation technique. The higher-order perturbations would require moments of order higher than fifth, which may require extensive computations. Therefore, the analysis here has been limited to second-order only to demonstrate the methodology, which can be further refined, if so desired.

Following the perturbation approach, the modal response is expressed in terms of the mean and perturbed values

$$q_i = q_i^o + q_i'\alpha_i + q_i''\alpha_i^2 \tag{9}$$

in which q_i^o, q_i' and q_i'' are various orders of perturbation. Substituting equations (8) and (9) into equations (6) and (7) and equating the same powers of α_i offers the following zeroth-, first- and second-order equations for externally and base excited systems, respectively.

Zeroth-order

$$\ddot{q}_i^o + 2\xi_i^o\omega_i\dot{q}_i^o + \omega_i^2 q_i^o = P_i(t) \tag{10}$$

$$\ddot{q}_i^o + 2\xi_i^o\omega_i\dot{q}_i^o + \omega_i^2 q_i^o = \Gamma_i\ddot{X}_g \tag{11}$$

First-order

$$\ddot{q}_i' + 2\xi_i^o\omega_i\dot{q}_i' + \omega_i^2 q_i' = -2\xi_i\omega_i\dot{q}_i^o \tag{12}$$

Second-order

$$\ddot{q}_i'' + 2\xi_i^o\omega_i\dot{q}_i'' + \omega_i^2 q_i'' = -2\xi_i\omega_i\dot{q}_i' \tag{13}$$

The equations corresponding to the first-order and second-order perturbations are similar for both types of excitations considered, except in the interpretation of q_i as stated earlier.

The transient and steady-state modal response at each order may be obtained following the procedures of random vibration theory. The transient modal response under zero initial conditions, i.e., $\{x(t = 0)\} = 0$ and $\{\dot{x}(t = 0)\} = 0$, is given by

$$q_i^o(t) = \int_0^t h(\tau)p_i(t - \tau)\mathrm{d}\tau \tag{14}$$

$$q_i^o(t) = \Gamma_i\int_0^t h(\tau)\ddot{X}_g(t - \tau)\mathrm{d}\tau \tag{15}$$

in which $h(\tau)$ represents the impulse response function. The external excitation $p_i(t)$ can be due to wind or wave load fluctuations on structures, whereas the base excitation may result from earthquakes. In either case the loading conforms to a filtered white noise characterizing the corresponding load effects[31-34]. The implementation of filtered white noise adds to the computational effort considerably. Symbolic manipulation by means of MACSYMA may help to reduce the overall computations[35]. However, in this study, for the sake of illustration, the excitation is assumed to be represented by a Gaussian white noise; even though this idealization may not accurately reflect the features of the physical random event over the entire spectrum, it has been widely used to represent physical random events. In the following analysis, for the sake of brevity only, the formulation is restricted to the base excitation. Extension to the external excitation is straightforward. The zeroth-order velocity response is given by

$$\dot{q}_i^o(t) = \int_0^t \dot{h}(\tau)\Gamma_i \ddot{X}_g(t-\tau)\,d\tau \tag{16}$$

in which $\dot{h}(\tau)$ represents the impulse response function for a velocity component of response. The mean square value of the zeroth-order displacement response is given by

$$\overline{q_i^{o2}(t)} = \Gamma_i^2 \int_0^t \int_0^t h(\tau)\overline{\ddot{X}_g(t-\tau)\ddot{X}_g(t-\tau')}h(\tau')d\tau'\,d\tau \tag{17}$$

By virtue of $\ddot{X}_g$ being represented by a white noise process, $\overline{\ddot{X}_g(t)\ddot{X}_g(t+\tau)} = 2\pi S_o\delta(\tau)$, where S_o is the intensity of white noise. This reduces equation (17) to the following form

$$\overline{q_i^{o2}(t)} = 2\pi S_o \Gamma_i^2 \int_0^t \int_0^t h(\tau)\delta(\tau-\tau')h(\tau')d\tau\,d\tau' \tag{18}$$

$$\overline{q_i^{o2}(t)} = 2\pi S_o \Gamma_i^2 \int_0^t h^2(\tau)d\tau \tag{19}$$

Similarly, the first-order displacement and velocity response components are given by

$$q_i'(t) = -2\xi_i^o\omega_i\Gamma_i \int_0^t \int_0^{t-\tau} h(\tau)\dot{h}(\tau')\ddot{X}_g(t-\tau-\tau')d\tau'\,d\tau \tag{20}$$

$$\dot{q}_i'(t) = -2\xi_i^o\omega_i\Gamma_i \int_0^t \int_0^{t-\tau} \dot{h}(\tau)\dot{h}(\tau')\ddot{X}_g(t-\tau-\tau')d\tau\,d\tau' \tag{21}$$

The time-dependent mean square value of the first-order displacement may be given by

$$\overline{q_i'^2(t)} = 8\pi S_o \xi_i^{o2}\omega_i^2\Gamma_i^2 \int_0^t \int_0^{t-\tau} \times \int_0^{\tau+\tau'} h(\tau)\dot{h}(\tau')h(\gamma)h(\tau-\tau'-\gamma)d\gamma\,d\tau'\,d\tau \tag{22}$$

The nonstationary second-order displacement component is given by

$$q_i''(t) = 4\xi_i^{o2}\omega_i^2\Gamma_i^2 \int_0^t \int_0^{t-\tau} \int_0^{t-\tau-\gamma} h(\tau)\dot{h}(\gamma) \times \dot{h}(\gamma')\ddot{X}_g(t-\tau-\gamma-\gamma')d\gamma'\,d\gamma\,d\tau \tag{23}$$

The mean square value of the second-order terms can be obtained from equation (23).

For time t approaching infinity, the previous equations provide the statistics of the steady-state response at various orders.

The modal response of a system with uncertain damping subjected to random initial conditions may be evaluated following the foregoing procedure.

In the frequency domain, the mean square modal response is obtained through the integration of the power spectral density (PSD) function of response, which is given by the product of the system transfer function and the PSD function of the excitation, for example

$$\sigma_{q_i^{(r)}}^2 = \int_0^\infty |H_{q_i^{(r)}}(\Omega)|^2 S(\Omega)d\Omega \tag{24}$$

in which $\sigma_{q_i^{(r)}}^2$ is the variance of the rth-order modal response in the ith mode, $|H_{q_i^{(r)}}(\Omega)|^2$ is the squared modulus of the system transfer function, and $S(\Omega)$ is the excitation PSD. Generally, the integration in the preceding equation is performed numerically; however, for white noise excitation and a class of filtered white noise excitation closed form integrals are available[36–38]. Alternatively, numerical quadrature or symbolic manipulation may be utilized to integrate equation (24)[35].

The transfer functions corresponding to equations (8), (9) and (10) are given below.

Zeroth-order

$$H_{q_i^o}(\Omega) = \Gamma_i H_i(\Omega) \tag{25}$$

$$H_i(\Omega) = \frac{1}{\Omega^2 + \omega_i^2 + 2j\Omega\omega_i\xi_i^o} \tag{26}$$

First-order

$$H_{q_i'}(\Omega) = -2\xi_i^o\omega_i j\Omega\Gamma_i H_i^2(\Omega) \tag{27}$$

Second-order

$$H_{q_i''}(\Omega) = -2\xi_i^o\omega_i j\Omega H_{q_i'}(\Omega)H_i(\Omega) \tag{28}$$

By idealizing the base-excitation spectra with a white noise process, the variance of the modal response in the various orders of perturbation may be obtained following the residue theorem[35,36]

Zeroth-order

$$\sigma_{q_i^o}^2 = \frac{\Gamma_i^2\pi S_o}{2\xi_i^o\omega_i^3} \tag{29}$$

First-order

$$\sigma_{q_i'}^2 = \frac{A_3B_1^2}{A_1(A_2A_3 - A_1A_4) - A_0A_3^2} \tag{30}$$

Second-order

$$\sigma_{q_i''}^2 = \frac{\pi S_o a_0 b_3(a_0a_3a_5 + a_1^2a_6 - a_1a_2a_5)}{a_0(a_0^2a_5^3 + \cdots - a_1a_2a_3a_4a_5)} \tag{31}$$

in which

$$B_1 = -2\xi_i^o\omega_i\Gamma_i$$

$$A_0 = \omega_i^4$$

$$A_1 = 4\xi^o\omega_i^3$$

$$A_2 = 2\omega_i^2 + 4\omega_i^2\xi_i^2$$

$$A_3 = 4\omega_i\xi_i^o$$

$$A_4 = 1$$

$$b_3 = (4\xi_i^{o2}\omega_i^2\Gamma_i)^2$$

$$a_0 = 1$$

$$a_1 = 6\xi_i^{o^2}\omega_i$$
$$a_2 = 3\omega_i^2(1 + 4\xi^{o^2})$$
$$a_3 = 4\xi_i\omega_i^3(2\xi_i^2 + 3)$$
$$a_4 = 3\omega_i^4(1 + 4\xi^{o^2})$$
$$a_5 = 6\xi_i^o\omega_i^5$$
$$a_6 = \omega_i^6.$$

In the preceding section, both the time and frequency domain modal response analyses are presented. The response in the physical coordinates at the nth node is given by

$$X_n(t) = \sum_{i=1}^{M} \Phi_{ni} q_i(t) \tag{32}$$

in which Φ_{ni} denotes the nth element of the ith mode shape. In view of equation (9), $X_n(t)$ may be expressed in matrix form as

$$X_n(t) = \boldsymbol{\Phi}^T \boldsymbol{\theta} \tag{33}$$

in which $\boldsymbol{\Phi}^T = [\Phi_n^T \ \Phi_n^T \ \Phi_n^T]$ and $\boldsymbol{\theta} = [\mathbf{q}^o \ \alpha\mathbf{q}^1 \ \alpha^2\mathbf{q}'']^T$. The covariance of X_n is expressed as

$$\textstyle\sum_{X_n} = \boldsymbol{\Phi}^T \sum_{\theta\theta} \boldsymbol{\Phi} \tag{34}$$

in which the covariance matrix $\sum_{\theta\theta}$ is given by

$$\sum_{\theta\theta} = \begin{bmatrix} [E(\mathbf{q}^o\mathbf{q}^{o^T})] & [E(\mathbf{q}^o\alpha\mathbf{q}'^T)] & [E(\mathbf{q}^o\alpha^2\mathbf{q}''^T)] \\ & [E(\alpha\mathbf{q}'\alpha\mathbf{q}'^T)] & [E(\alpha\mathbf{q}'\alpha^2\mathbf{q}''^T)] \\ \text{Symmetrical} & & [E(\alpha^2\mathbf{q}''\alpha^2\mathbf{q}''^T)] \end{bmatrix} \tag{35}$$

in which the operator E( ) denotes expected values. In view of the implicit independence of α_i and q_i and its components, and utilizing relationships for the higher-order moments of Gaussian processes[39,40], the terms in the preceding equation can be evaluated. Some of these terms vanish by virtue of Gaussianity. The contribution of off-diagonal terms is relatively small in relation to the main diagonal terms. As an approximation, by ignoring them, the mean square value of the response at the nth node is given by

$$\sigma_{X_n}^2 = \sum_{i}^{M} \Phi_{ni}^2[\sigma_{q_i^o}^2 + \sigma_{q_i}^2\sigma_{\alpha_i}^2 + 3\sigma_{q_i''}^2\sigma_{\alpha_i}^4] \tag{36}$$

in which σ^2 with a subscript denotes the mean square value of the prescribed variable.

Example

A numerical example utilizing a five-storey building modelled as a lumped-mass system is considered to illustrate the methodology presented herein and to examine quantitatively the influence of uncertainty level in the damping on the overall system response to white noise base excitation. Applications to wind- or wave-induced loading are straightforward. However, for filtered white noise representation, numerical quadrature or symbolic manipulations are necessary. The lumped mass at each floor, $m_1 = m_2, \ldots, m_5 = 41.67$ lb-s^2/in and interstorey stiffness between each level is equal to $k_1 = k_2, \ldots, k_5 =$ 96049 lb/in. The intensity of white noise excitation is 8.0 in^2/s^3. The mean value of the critical damping ratio in each mode was assumed in this study to be equal, although a different value for each mode may be used as an input. The mean values of the modal damping ratio ξ examined in this study are 1%, 3% and 5%. Each value of the modal damping was assigned a COV that varied from 10 to 40% with uniform increments of 10%. In *Table 1*, the variance of the displacement at each node is presented in terms of the zeroth-, first-, and second-order perturbations for $\xi = 5\%$. The results suggest that the second-order contribution is insignificant for small values of the COV of damping. However, its contribution increases with an increase in the COV of the damping ratio (*Figure 1*). The building response at node 5 for different values of the mean and COV of damping are plotted in *Figure 2*.

Concluding remarks

The uncertainty associated with damping in structural systems is identified and discussed. A second-order perturbation technique is utilized to examine the effects of damping variability on the transient and steady-state

Table 1 RMS displacement response of a building with uncertain damping ($\xi^o = 5\%$)

	Node					
	1	2	3	4	5	Ω_ξ
Zeroth-order	0.1136	0.2159	0.3003	0.3611	0.3935	
First-order	0.1152	0.2191	0.3046	0.3663	0.3954	10%
Second-order	0.1152	0.2192	0.3047	0.3664	0.3993	
First-order	0.1201	0.2283	0.3162	0.3816	0.4158	20%
Second-order	0.1205	0.2291	0.3184	0.3830	0.4173	
First-order	0.1276	0.2426	0.3373	0.4058	0.4422	30%
Second-order	0.1297	0.2467	0.3429	0.4124	0.4494	
First-order	0.1376	0.2615	0.3636	0.4373	0.4765	40%
Second-order	0.1437	0.2732	0.3794	0.4567	0.4978	

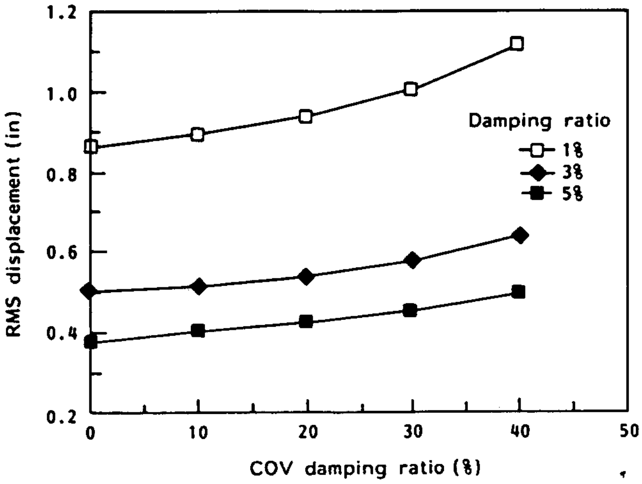

Figure 1 Displacement at node 5 in terms of first- and second-order perturbations (damping ratio 5%)

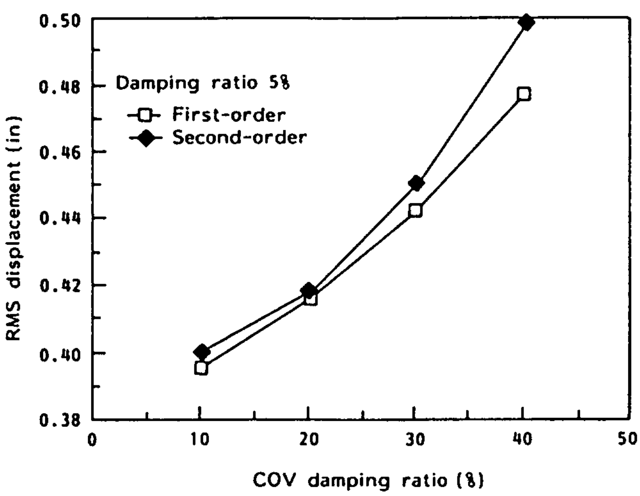

Figure 2 Response of building with uncertain damping (node 5)

dynamic response of structural systems. The formulation also permits the evaluation of the system response to random initial conditions. In the analysis presented in the paper, the excitation is assumed to be represented by a Gaussian white noise. This representation permits a closed-form solution to the problem. However, the procedure is equally applicable to wind, wave and earthquake-type loading, singly or in combination, which are characterized by a filtered white noise process. In this case, a numerical quadrature or symbolic manipulations may become necessary. The results demonstrate that the uncertainty in damping indeed influences the system response. Depending on the mean value of the damping ratio, the effects are more pronounced for higher variability of damping values.

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References

1 Lazan, B.J. *Damping of Materials and Members in Structural Mechanics*, Pergamon Press, Oxford, UK, 1968
2 Nashif, A.D., Jones, D.I.G. and Henderson, J.P. *Vibration Damping*, John Wiley & Sons, Chichester, UK, 1984
3 Novak, M. and Hifnawy, L. El 'Structural response to wind with soil-structure interaction', *Proc. 7th Int. Conf. on Wind Engng*, Aachen, West Germany, 6-10 July 1987
4 Kareem, A. 'Wind induced response analysis of tension leg platforms', *J. Struct. Engng, ASCE* 1985, **111**, No. 1, 37-55
5 Vickery, B.J. and Basu, R.J. 'Acrosswind vibrations of structures of circular cross-section: Part I development of a mathematical model for two-dimensional conditions', *J. Wind Engng and Ind. Aerodynamics* 1983, **12**, 49-73
6 Kareem, A. 'Acrosswind response of buildings', *J. Struct. Engng, ASCE* 1982, **108**, No. ST4, 869-887
7 Kana, D.D. 'Energy methods for damping synthesis in seismic design of complex structural systems', *J. Engng Mech. Div., ASCE* 1981, **107**, EM1, 255-259
8 Meirovitch, L. *Analytical Methods in Vibrations*, Macmillan, New York, 1967
9 Jeary, A.P. 'Damping in tall buildings: a mechanism and a predictor', *Earthquake Engng and Struct. Dyn.* 1986, **14**, 733-750
10 Davenport, A.G. and Hill-Carroll, P. 'Damping in tall buildings; its variability and treatment in design', *Building Motion in Wind, Proc. ASCE Convention*, Seattle, WA, 1986
11 *Damping of Structures, Part 1: Tall Buildings*, Engineering Science Data Units, Item No. 83009, London, Sept. 1983
12 Yokoo, Y. and Akiyama, H. 'Lateral vibration and damping due to wind and earthquake effects', *Proc. Int. Conf. on Planning and Design of Tall Buildings*, **II- 17**, ASCE, NY, 1972
13 Jeary, A.P. and Ellis, B.R. 'Vibration tests of structures at varied amplitudes', *Proc. ASCE/EMD Specialty Conf. on Dynamic Response of Structures*, Atlanta, GA, 1981
14 Haviland, R. 'A study of the uncertainties in the fundamental translational periods and damping values for real buildings', *Res. Rep. No. 5, Pub. No. R76-12, Dept of Civ. Engng*, MIT, Cambridge, MA, 1976
15 Kareem, A. 'Mitigation of wind induced motion of tall buildings', *J. Wind Engng and Ind. Aerodynamics* 1983, **11**, Nos 1-3, 273-284
16 McNamara, R.J. 'Tune mass dampers for buildings', *J. Struct. Div., ASCE* 1977, **103**, No. ST9
17 Mahmoodi, P. 'Structural dampers', *J. Struct. Div., ASCE* 1969, **95**, No. ST8
18 Keel, C.J. and Mahmoodi, P. 'Design of viscoelastic dampers for Columbia Center Building', *Building Motion in Wind, Proc. ASCE Convention*, Seattle, WA, 8 April 1986
19 Kareem, A. and Sun, Wei-Joe 'Stochastic response of structures with fluid containing appendages', *J. Sound and Vibration* 1987, 389-408
20 *Proc. 8th World Conf. on Earthquake Engng*, Earthquake Engineering Research Institute, Prentice-Hall, 1984
21 Kareem, A. and Sun, Wei-Joe, 'Probabilistic response of structures with parametric uncertainties', *Proc. 5th Int. Conf. on Applications of Statistics and Probability in Soil and Structural Engineering*, Vancouver, Canada, 25-29 May 1987
22 Contreras, H. 'The stochastic finite element method', *Computers and Structures* 1980, **12**, 341-348
23 Handa, K. and Andersson, K. 'Application of finite element methods in the statistical analysis of structures', *Proc. 3rd Int. Conf. on Structural Safety and Reliability*, 1981
24 Shinozuka, M. and Dasgupta, G. 'Stochastic finite element methods in dynamics', *Proc. 3rd Conf. on the Dynamic Response of Structures*, ASCE, NY, 1986
25 Sun, T.C. 'A finite element method for random differential equations with random coefficients', *SIAM J. of Numerical Analysis* 1979, **16**, 1019-1035
26 Lawrence, M., Lui, W.K. and Belytschko, T. 'Stochastic finite element analysis for linear and nonlinear structural response', *Structural Congress '86, Volume of Abstracts*, ASCE, NY, 1986
27 Vanmarcke, E.H. *et al.* 'Random fields and stochastic finite elements', *Structural Safety* 1986, 3
28 Ibrahim, R.A. and Heo, H. 'Stochastic response of nonlinear

structures with parameter random fluctuations', *AIAA J.* 1987, **25**, No. 2, 331-338
29 Caravani, P. and Thomson, W.T. 'Frequency response of a dynamic system with statistical damping', *AIAA J.* 1973, **11**, No. 2, 170-173
30 Nakagiri, S., Hisada, T. and Toshimitsu, K. 'Stochastic time-history analysis of structural vibration with uncertain damping', *Probabilistic Structural Analysis*, ASME, PVP., 93, 1984
31 Kareem, A. 'Dynamic response of structures under stochastic environmental loads', *J. Struct. Engng, SERC* 1987, **1**, 1-8
32 Simiu, E. and Scanlan, R. H. *Wind Effects on Structures*, John Wiley and Sons, Chichester, UK, 1986
33 Sarpkaya, T. and Issacson, M. *Mechanics of Wave Forces on Offshore Structures*, Van Nostrand Reinhold Company, 1981
34 Weigel, R.L. (Ed.) *Earthquake Engineering*, Prentice-Hall, 1970
35 Pavelle, R. 'MACSYMA: capabilities and application to problems in engineering and the sciences', in *Applications of Computer Algebra*, Kluwer Academic Publishers, MA, 1985
36 Crandall, S.H. and Mark, W.D. *Random Vibration in Mechanical Systems*, Academic Press, 1973
37 James, H.M. *et al. Theory of Servomechanisms*, MIT Radiation Laboratory Series, Vol. 25, McGraw-Hill, New York, 1947
38 Spanos, P.D. 'On the calculation of a class of improper integrals', *Proc. 5th Conf. on Applications of Statistics and Probability in Soil and Structural Engineering*, Vancouver, Canada, 25-29 May 1987
39 Isserlis, L. 'On a formula for the product-moment coefficient in any number of variables', *Biometrica* 1918, **12**, Nos. 1-2
40 Lin, Y.K. *Probabilistic Theory of Structural Dynamics*, Krieger Publishing Company, New York, 1976