Dynamic Response of High-Rise Buildings to Stochastic Wind Loads

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Abstract

This paper presents results of a recent study addressing issues concerning the dynamic response of high-rise buildings. Utilizing a high-frequency base-balance, the alongwind, acrosswind and torsional components of aerodynamic loads and their statistical correlations for a wide range of generic building shapes of different aspect ratios in two approach flow conditions are quantified. A random-vibration-based procedure for estimating response of high-rise buildings is outlined. A checking procedure for building serviceability is presented for enabling designers to assess the building serviceability performance. The methodology presented here will enable designers in the preliminary design stages to assess the serviceability of buildings, the need for a detailed wind tunnel test and to evaluate the merit of design modifications that may include a choice of an auxiliary damping system.

1. INTRODUCTION

Modern high-rise buildings designed to satisfy static lateral drift requirements may experience excessive motion during wind storms. Although these buildings are safe from structural collapse point of view, but they may cause discomfort to the building occupants, especially near the top portion of the building. Occupant comfort needs to be emphasized due to human biodynamic sensitivity to motion which is accentuated by the increase in awareness of building motion provided by visual cues of moving surroundings [Hansen et al., 1973 and Kareem, 1983].

Under the influence of dynamic wind loads, typical high-rise buildings oscillate in the alongwind, acrosswind, and torsional directions. The alongwind motion primarily results from pressure fluctuations on the windward and leeward faces, which generally follows the fluctuations in the approach flow, at least in the low frequency range. Therefore, alongwind aerodynamic loads may be quantified analytically utilizing quasi-steady and strip theories. The dynamic effects are customarily represented by a random-vibration-based "Gust Factor Approach" [Davenport, 1963]. The acrosswind motion is introduced by pressure fluctuations on the side forces which are influenced by the fluctuations in the separated shear layers and wake dynamics. This renders the applicability of the strip and quasi-steady theories rather doubtful. The wind induced torsional effects result from the unbalance in the instantaneous pressure distribution on the building surface. These loads effects are further amplified in asymmetric buildings as a result of inertial coupling [Kareem, 1985 and Tallin and Ellingwood, 1985]. Physical modelling of fluid-structure interactions provides the only viable means of obtaining information on the wind loads. Recently, research in the
area of computational fluid dynamics is making progress in numerically generating flow fields around bluff bodies exposed to turbulent flows [Murakami, 1989].

2. AERODYNAMIC LOADS

In this paper, methods to quantify aerodynamic loads on buildings are briefly described with examples. This is followed by a systematic procedure for dynamic analysis of torsionally coupled building. An example is presented in building response that results from the eccentricity between the stiffness (elastic) and mass centers of the building. In closing a brief discussion of serviceability limit states, checking procedure to evaluate building performance and framework for better decisions in the design process are presented.

The aerodynamic loads on buildings may be obtained by mapping and synthesizing the random pressure fields acting on building envelope. The structure of random pressure fields through simultaneously monitored multiple-point realizations of pressure fluctuations, and measurement of the local averages of the space-time random pressure fields by means of spatial and temporal averaging techniques can be mapped [e.g., Kareem, 1989]. The spatial averaging procedure may employ local averaging of the random pressure field utilizing an electronic summation circuitry, a pneumatic manifolding device, or a pressure-sensitive surface element like PVDF [Kareem, 1989]. In this study, multi-level loads on a prismatic building with a rectangular shaped plan were measured by means of pneumatic manifolding devices utilizing uniform and weighted discrete matrices of pressure taps. Multi-level alongwind, acrosswind and torsional loads were ascertained.

Alternatively, high-frequency force balance may be used for determining the dynamic wind induced structural loads from scale models of buildings and structures [e.g., Kareem and Cermak, 1979; Tschanz and Davenport, 1983 and Boggs and Peterka, 1989]. These techniques have dramatically reduced both the time and cost required to obtain estimates of wind loads and structural response levels. The force balance provides dynamic load information for a specific building geometry and setting which may be used to calculate loads and response levels for a wide range of structural characteristics, damping values, and building masses.

The force balance technique has some shortcomings, e.g., only approximate estimates of the mode-generalized torsional moments are obtained and the lateral loads may be inaccurate if the sway mode shapes of the structure differ significantly from a linear mode shape. Therefore, the mode-generalized spectrum obtained from a force-balance study requires adjustments if the building mode shapes depart from those implied in the derivation of the force balance theory [e.g., Vickery et.al., 1985; and Boggs and Peterka, 1989]. This is especially true for the torsional loads. A second generation of force balances permits overcoming the aforementioned limitations [Reinhold & Kareem, 1986].

The loads obtained by the base-balance approach do not include motion-induced aerodynamic loads. It is a general consensus that in most tall buildings the influence of motion-induced loading is insignificant for typical design wind speeds. For exceptionally slender, flexible and lightly damped structures one may resort to aeroelastic models or use special motion induced force measurement systems in conjunction with a force balance.

In this study, a wide range of ultra-light weight models were fabricated and used in conjunction with a sensitive base balance. A host of generic buildings with a wide range of shapes and modular combinations were utilized to include mid-rise to super
tall buildings (Fig. 1). Each prismatic building was tested for three different heights to delineate the influence of aspect ratio on the aerodynamic loads. The tests were conducted for two approach flow characteristics representing urban and open country flow conditions, and for different angles of approach to examine sensitivity of loading to the direction of wind. This study has made possible a unique collection of data set that can be used for estimating response of a wide range of buildings. A sample of results obtained from this study are reported here for a square cross-section building in Fig. 2. In addition to this information, correlation between these loads is also evaluated in terms of coherence and phase angle. A sample of these results for the square cross-section building are given in Fig. 3. Although the limitation of space here precludes a detailed discussion of the nine prismatic shapes and seven combinations of generic prismatic modules a brief discussion of results concerning a square cross-section building is presented here.

![Image](image1.png)

Figure 1. A view of the building models used in this study.

Typically, for serviceability analysis the reduced frequency of interest ranges between 0.15-0.5. Previous studies concerning acrosswind load spectra have shown
large scatter in the measured values in this range of frequencies. The steepness of the acrosswind force spectrum in this range coupled with uncertainties associated with the mean wind speed at the building height and the aforementioned scatter may introduce significant variability in the estimated acrosswind response. The observed scatter may be attributed in part to a number of factors, namely, difference in turbulence intensity in the simulated flow conditions, model aspect ratio and errors in measurements. Saunders & Melbourne (1975) and Kwok (1982) summarized data for a square cross-section building. It is noted from their results that the aspect ratio does affect the spectral amplitude and shape in the frequency range of interest for serviceability. However, the measurements conducted in this study tend to suggest that the aspect ratio between 1:4 and 1:6 for a square cross-section building does not affect the acrosswind spectrum significantly. The trend remains unchanged for both open country and urban flow conditions.

![Figure 2. Measured accrosswind and torsional force spectra for a square cross-section building. (Aspect ratio 1:6) in open country flow for different wind directions (Note: ordinate adjustment is necessary when considering different wind directions).](image)

Earlier studies have suggested that the correlation between alongwind and acrosswind or torsional forces is negligible [Kareem, 1982, Reinhold 1983]. This observation is also corroborated by the present measurements. However, it needs to
be emphasized that the correlation between the acrosswind and torsional force components is very significant in both approach flow conditions [Fig. 4].

Figure 3. Coherence and phase between different aerodynamic force components. (solid lines - open country; broken lines - urban).

3. DYNAMIC RESPONSE ANALYSIS

A typical high-rise building can be modelled using commercially available finite-element-based codes. Following the assembly of all the structural components the global coordinate system is condensed to two translations and one rotation at each story. For a torsionally coupled building the center of mass and center of resistance at each story may not be coincident. This will generally lead to coupled mode shapes. The equations of motion of a typical N-story building can be written in terms of 3N equations.

\[
[M] \{\ddot{d}\} + [C] \{\dot{d}\} + [K] \{d\} = \{f\}
\]

(1)

in which \(M\), \(C\), and \(K\) are assembled real matrices of the discretized building system, \(d\) and \(f\) are the response and loading vectors. For a building having coincident resis-
tance and mass centers, the preceding set of 3N equations can be separated into three sets of N uncoupled equations. For a building with eccentric mass and resistance centers, generally referred to as torsionally coupled, the preceding simplification is not possible and the analysis becomes computationally more involved. The mass and stiffness matrices in Eq.1 are given by, in a general case:

\[
M = \begin{bmatrix}
m & 0 \\
m' & m' \\
0 & m
\end{bmatrix};
\]

(2)

The submatrices in the above equation are defined as:

\[
m = \begin{bmatrix}
m_1 \\
m_2 \\
\vdots \\
m_N
\end{bmatrix}; \quad K_{ii} = \begin{bmatrix}
k_{i1} + k_{i2} & -k_{i2} & 0 & \cdots & 0 \\
-k_{i2} & k_{i2} + k_{i3} & -k_{i3} & 0 & \cdots & 0 \\
0 & \ddots & \ddots & \ddots & \ddots & \ddots \\
0 & \cdots & 0 & k_{iN}
\end{bmatrix}
\]

(3)

\[
k_{x0} = \begin{bmatrix}
e_{y1}k_{x1} + e_{y2}k_{x2} & -e_{y2}k_{x2} & 0 & \cdots & 0 \\
-e_{y2}k_{x2} & e_{y2}k_{x2} + e_{y3}k_{x3} & -e_{y3}k_{x3} & 0 & \cdots & 0 \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots \\
0 & \cdots & 0 & e_{yN}k_{xN}
\end{bmatrix}
\]

(4)

\[
k_{y0} = \begin{bmatrix}
e_{x1}k_{y1} + e_{x2}k_{y2} & -e_{x2}k_{y2} & 0 & \cdots & 0 \\
-e_{x2}k_{y2} & e_{x2}k_{y2} + e_{x3}k_{y3} & -e_{x3}k_{y3} & 0 & \cdots & 0 \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots \\
0 & \cdots & 0 & e_{xN}k_{yN}
\end{bmatrix}
\]

(5)

in which \(\vec{d} = [\vec{d}_x, \vec{d}_\theta, \vec{d}_y]^T\) = displacement vector \(\vec{d}_x = [x_1, x_2, \ldots, x_N]^T\), \(\vec{d}_\theta = [\theta_1, \theta_2, \ldots, \theta_N]^T\), \(\vec{d}_y = [y_1, y_2, \ldots, y_N]^T\); \(\vec{d}_0 = [\theta_1, \theta_2, \ldots, \theta_N]^T\), \(\vec{\gamma} = \) radius of gyration vector = \(\vec{\gamma}_1, \vec{\gamma}_2, \ldots, \vec{\gamma}_N]^T\); \(m_i = \) lumped mass at floor \(i\), \(x_i = \) x-axis translational displacement at floor \(i\), \(y_i = \) y-axis translational displacement at floor \(i\), \(\theta_i = \) rotation at floor \(i\), \(\gamma_i = \) radius of gyration of floor \(i\), \(k_{i_i} = \) stiffness of the \(i\)th storey, in \(i\)th direction \((i = x, \theta, y)\), \(e_{xi}, e_{yi} = \) eccentricities between the centres of resistance and mass at the \(i\)th floor and \(f\) is the excitation vector. The loading vector \(f\) corresponding to the lateral and ro-
tational directions is expressed as \( \{f\} = \{f_x, f_\theta, f_y\}^T \) in which \( f_x, f_\theta, f_y \) represent loading vectors in the alongwind, torsional and acrosswind directions, respectively. For this class of buildings it is essential that a multi-level second-generation force balance or a multi-channel simultaneous pressure measurement system is used to ascertain aerodynamic loading. Typically such a system is limited to a small number of levels of measurements, therefore the building structural model needs to be condensed to conform with the measurement levels.

The dynamic analysis can be accomplished in either time domain or frequency domain. Generally, frequency domain approach is utilized which has an inherent computational advantage for linear systems. In the frequency domain analysis, the normal mode approach is utilized wherein the undamped eigenvectors facilitate decoupling of the system of equations employing standard transformation of coordinates. The following uncoupled system of equations is obtained

\[
\ddot{q}_i + 2\xi_i \omega_i \dot{q}_i + \omega_i^2 q_i = P_i
\]  

in which \( P_i = \{\phi^{(i)}\}^T \{f\} \) and \( \xi_i \) is the critical damping ratio in the \( i \)th mode. The transformation used herein leads to a mode displacement method in which the system coordinates are related to modal coordinates by \( \{d\} = [\Phi] \{q\} \), in which \( \Phi \) is the modal matrix containing coupled mode shapes, \( \phi \).

The mean square value of response components due to wind loads is given by

\[
\sigma^2_{d(r)} = \sum \{\Phi \} [\int G_{q(r)}(f) \, df] \{\Phi\}^T
\]  

\[
G_{q(r)}(f) = [H^{(r)}(i2\pi f)] \ast [G_F(f)] [H^{(r)}(i2\pi f)]
\]  

\[
G_F(f) = [\Phi]^T [S_F(f)] [\Phi]
\]  

\[
[S_F(f)] = \begin{bmatrix}
S_{F_x}(f) \\
\vdots \\
S_{F_y}(f)
\end{bmatrix}
\]  

\[
[H^{(r)}(i2\pi f)] = \frac{(2\pi)^r}{(2\pi)^2 \left[(\hat{f}_n^2 - f_n^2) + 2i\xi f_n\right]}
\]
in which \([H^{(r)}(i2\pi\nu)]\) is a diagonal matrix of modal transfer functions; \([S_F(\nu)]\) is the cross power spectral density matrix of the forcing function; and \(\Phi\) is the modal matrix that is normalized with respect to the mass matrix. The superscript, \(r\), denotes higher derivatives, e.g., \(r\) equal to 1, 2, and 3 represents velocity, acceleration and jerk, respectively. The preceding analysis simplifies for mechanically uncoupled buildings. This reduces 3N equations of motion (Eq. 1) to 3 decoupled set of equations, in \(x\), \(y\) and \(\Theta\) directions, each set now consists of \(N\) uncoupled equations of motion. Now the off-diagonal submatrices in Eq. 4 are reduced to zero due to a lack of eccentricity in mass and resistance centers. Accordingly, the \(\Phi\) matrix contains uncoupled modes.

The preceding idealization is possible based on features characteristic of many conventional high-rise buildings, i.e., all floors of the building have the same geometry in plan, and all stories have the same stiffness ratio in the \(x\) and \(y\) directions [Kan and Chopra, 1977 and Kareem, 1985]. In this case the 3N degrees-of-freedom system is reduced to an uncoupled system with \(N\) degrees-of-freedom in each respective direction. Now the analysis is reduced to solving the \(j\)th order eigenvalue problem for first \(J\) modes in each direction. This is followed by defining a mode-generalized system in \(j\)th mode associated with the \(j\)th group of the three uncoupled vibration modes as having the properties given by the following relationship for \(j = 1, N\),

generalized mass \(m_{xj}, m_{yj}\), and \(m_{\Theta j}\), generalized stiffness \(K_{xj} = \omega_{xj}^2 m_{xj}, K_{yj} = \omega_{yj}^2 m_{yj}\) and \(K_{\Theta j} = \omega_{\Theta j}^2 m_{\Theta j}\gamma^2\)

\[e_{xj} = \frac{\{\psi_{xj}\}^T[K_{xj}]\{\psi_{xj}\}}{\omega_{xj}^2 m_{xj}}; \text{ and } e_{yj} = \frac{\{\psi_{yj}\}^T[K_{yj}]\{\psi_{yj}\}}{\omega_{yj}^2 m_{yj}} \tag{12}\]

where \(\psi_{xj}, \psi_{yj}\) and \(\psi_{\Theta j}\) are the \(j\)th mode shapes in each respective direction, and \(\gamma\) is the radius of gyration of the floor deck. Now the \(j\)th mode-generalized system has three degree-of-freedom and the corresponding equations of motion are given by

\[
\begin{bmatrix}
    m_j & \gamma^2 & 0 \\
    \gamma^2 & m_j & 0 \\
    0 & 0 & m_{\Theta j}
\end{bmatrix}
\begin{bmatrix}
    \dot{x}^{(j)} \\
    \dot{\theta}^{(j)} \\
    \dot{y}^{(j)}
\end{bmatrix} + 
\begin{bmatrix}
    K_{xj} & -e_y K_{xj} & 0 \\
    -e_y K_{xj} & K_\Theta & e_x K_{xj} \\
    0 & e_x K_{yj} & K_{yj}
\end{bmatrix}
\begin{bmatrix}
    x^{(j)} \\
    \theta^{(j)} \\
    y^{(j)}
\end{bmatrix} = 
\begin{bmatrix}
    f_x^{(j)}(f) \\
    f_\Theta^{(j)}(f) \\
    f_y^{(j)}(f)
\end{bmatrix} \tag{13}\]

The preceding formulation is ideally suited when only the base balance measurements are available. However, the building structural system needs to meet the requirements stated earlier. Now in the case of a base-balance study \(j = 1\).

The response spectral matrix is given by

\[
[S_R(n)] = [H(i2\pi n)] [S_F(n)] [H^*(i2\pi n)]^T;
\]

\[
[H^{(r)}(i2\pi n)] = (2\pi n)^r - (2\pi n)^2 [M] + i2\pi n [C] + [K]^{-1};
\]

\[
[S_F(n)] = 
\begin{bmatrix}
    S_{Fx}(n) & S_{Fx\Theta}(n) & S_{Fx\gamma}(n) \\
    S_{Fx\Theta}(n) & S_{F\Theta}(n) & S_{F\Theta\gamma}(n) \\
    S_{Fx\gamma}(n) & S_{F\Theta\gamma}(n) & S_{F\gamma}(n)
\end{bmatrix} \tag{14}\]
in which $[H^{(r)}(i2\pi n)] = \text{matrix of frequency response function with } ^* \text{ denoting the complex conjugate;} [S_R(n)] = \text{cross spectral density matrix of response, and} [S_F(n)] = \text{cross spectral density matrix of forcing function derived from a set of base-balance measurements;} [M], [K] = \text{mass and stiffness matrices (Eq. 13):} [C] = \text{damping matrix. The matrix of mean square response is given by}$

$$\sigma^n_R = \int_0^\infty [S_R(n)] dn; \quad [S_R(n)] = \begin{bmatrix} S_x(n) & S_{x\theta}(n) & S_{xy}(n) \\ S_{x\theta}(n) & S_{\theta}(n) & S_{\theta y}(n) \\ S_{xy}(n) & S_{\theta y}(n) & S_y(n) \end{bmatrix}$$

As indicated earlier, the corners of a building experience considerable effects of torsional response, i.e., excessive stresses and human discomfort. The rms response of the building corner at $(D/2, B/2)$ is given by

$$\sigma_x^{(p)} = \sqrt{\sigma_x^{(p)} + \frac{B^2}{4} \sigma_\theta^{(p)^2} - B\sigma_x^{(p)^2}}; \quad \text{and } \sigma_y^{(p)} = \sqrt{\sigma_y^{(p)^2} + \frac{D^2}{4} \sigma_\theta^{(p)^2} + D\sigma_y^{(p)^2}}$$

and which $\sigma_x^2, \sigma_y^2, \sigma_\theta^2 = \text{mean square values of the alongwind, acrosswind and torsional response at the building center and } \sigma_{x\theta}^2 \text{ and } \sigma_{y\theta}^2 \text{ are the response covariances.} \ B \ & \ D \text{ represent building width and depth. The response at any other location can be obtained accordingly.}$

4. EXAMPLE

A 31 m square in plan and 183 m tall building with the fundamental torsional frequency of 0.35 hz, fundamental lateral frequencies in the x and y directions of 0.2 hz, each, building density of 192 Kg/m$^3$ and damping equal to 0.1% in all the three principle directions of motion was utilized to demonstrate the effect of lateral-torsional coupling. The building was modelled according to the procedure presented in this paper by considering only the fundamental mode associated with each direction and base-balance derived aerodynamic loads. The building was first analyzed by considering that the mass and elastic centers were coincident. This was followed by introducing positive and negative eccentricities in both x and y directions systematically. Also, a special case of a building was considered in which lateral and torsional fundamental frequencies were all set equal.

The influence of eccentricity between the mass and elastic center was examined by expressing the building response as a ratio of the building corner response to the building response at the center of the uncoupled building in the respective directions in the absence of torsional loads. This ratio in the x and y directions is defined as $R_x$ and $R_y$, respectively.
First, eccentricity in the $y$-direction, $e_y$, was set equal to zero and $e_x$ was varied in both positive and negative directions between 0 to 10% of the building width. It was noted that the negative building eccentricity in the $x$-direction amplified response in the $y$-direction significantly as compared to the positive eccentricity which limits the response in the $y$-direction. The response in the $x$-direction is increased in case the torsional frequency is equal to the lateral frequencies, regardless of the nature of eccentricity. Similar trend was noted for eccentricity in the $x$-direction. For equal eccentricities in $x$ and $y$ directions, the response in $x$ direction is significantly amplified, and it is more pronounced for the building with equal frequencies in the lateral and torsional directions. The results for equal eccentricities are summarized in Fig. 4.

5. SERVICEABILITY LIMIT STATES

The building serviceability checking procedure from human comfort conditions may be expressed in terms of structural motion and acceptable threshold of motion. The following tentative criterion for limiting motion of high-rise buildings is being examined for design applications. This follows the format of a similar criterion proposed by Hansen, et al. (1973). The mean recurrence interval for storms causing a rms acceleration at the building top, which exceeds a pre-established limit, shall not be less than $N$ number of years. The acceleration limit is proposed to be between 8 and 10 mg and $N$ equal to 10 years. Currently used design values for peak acceleration are typically 20-25 mg for a ten year return period. Following the dynamic analysis procedure given in the preceding section, serviceability checking can be evaluated based on the preceding criterion. An effort is being made to quantify
human biodynamic response in terms of a frequency dependent transfer function. The product of this function with the power spectrum of acceleration response will provide a perception variable which will lead to a more consistent criterion.

Alternatively, an outcrossing analysis may be utilized in which the mean number of times the acceleration exceeds a threshold value can be computed for a given wind speed and direction over a specified time. The expected time during which the acceleration response exceeds the threshold acceleration within a specified period can also be evaluated. In such an excursion analysis outcrossings of a two dimensional process are employed. Details concerning this excursion analysis, dependence of outcrossing rate on wind speed and direction, and the influence of uncertainty in the parameters will be reported elsewhere.

The importance of knowing the risk of failure inherent in any design or in an existing structure is becoming increasingly critical to the engineering profession. Therefore, the need to go beyond the traditional deterministic approach is becoming increasingly important. Intrinsic variability associated with both structural resistance and wind load effects can seriously affect the performance of buildings. Recent advances in the area of probabilistic methods and statistical inference offer a mathematical framework that will enable designers to implement uncertainties arising from a variety of sources more effectively into their design process [e.g., Kareem, 1987]. In the context of serviceability of buildings, the attainment of a limit state from human comfort consideration is termed "failure", although this event necessarily does not entail loss of strength or collapse of structure. The annual probability of failure is used as a measure of the consequence of a design choice. There is often no unique solution to the design decision problem. The structural reliability analysis provides a framework for better decision making in the design process. The consequences of the design decision may be analyzed by means of utility theory. Utility may be defined as a true measure of value to the decision maker and facilities a basis for evaluation of alternatives. Different design alternatives may be evaluated in terms of the probability of failure and subsequently expressed as a function of utility. The low and high values of the probability of failure lead to an overdesign or unsafe design respectively. Based on the maximum utility, an optimum design decision may be reached that meets the accepted measures of human comfort, financial, and construction schedule constraints.

6. CONCLUDING REMARKS

This paper summarizes findings of a study that focuses on the dynamic response of high-rise buildings from serviceability point of view. The procedure presented here will enable designers in the preliminary design stages to assess the serviceability of buildings, the need for detailed wind tunnel tests and design modifications that may include a choice of a damping system, e.g., tuned sloshing dampers.

7. ACKNOWLEDGEMENTS

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8. REFERENCES