PERFORMANCE OF MULTIPLE MASS DAMPERS UNDER RANDOM LOADING

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ABSTRACT: The dynamic characteristics and effectiveness of multiple mass dampers (MMDs), a collection of several mass dampers with distributed natural frequencies, under random loading are investigated in this paper. The MMD attached in a parallel configuration modifies the transfer function of the damper-building system by flattening the peaks observed in a typical single tuned mass damper-building transfer function. The MMD parameters considered here include the frequency range of MMDs, damping ratio of individual dampers, and the number of dampers. Uniform and variable frequency increments in a specified frequency range and mass variation alone, and in combination, are considered. The secondary inertial effect can be represented by conventional mass dampers or liquid sloshing or oscillating liquid column dampers. A parameter study is conducted to delineate the influence of several parameters on the effectiveness and robustness of MMDs in comparison with a single tuned mass damper (TMD). The random loads considered here include wind and seismic excitations. It is demonstrated that the MMD configuration is more effective in controlling the motion of the primary system. It offers the advantages of portability and ease of installation (because of the reduced size of an individual damper), which makes it attractive not only for new installation, but also for temporary use during construction or for retrofitting existing structures.

INTRODUCTION

The utilization of tuned mass dampers (TMD) or tuned liquid dampers for controlling wind-induced motion has received much attention. The application of these systems in the United States is rather limited; however, several such systems, with some variations, have been installed in Japan [e.g., McNamara (1977), Kareem (1983), Sun and Kareem (1986), Kareem and Tamura (1994), Tamura (1990), Fujita (1991)]. These systems have also been studied for the reduction of seismic response (Kaynia et al. 1981; and Kareem and Sun 1987). The effectiveness of tuned mass or liquid dampers in controlling wind-induced motion has been demonstrated in computational models, wind tunnels, and full-scale testing. Nonetheless, in the case of seismic excitation, the effectiveness of these passive systems has not been established, because of their inability to respond, within a very short period of time, to a variety of transient base excitations. This has led to the development of active and hybrid mass damper systems that can accommodate these features. General information about these systems and their modeling and analysis may be found in, for example, Housner and Masri 1990; Soong 1990; and Suhardjo et al. (1992).

Multiple mass dampers (MMDs) can be designed in a parallel or series configuration. These can be incorporated in a structural system at one location or distributed spatially. The effectiveness of multiple dampers, spatially distributed in a structure, was investigated by Bergman et al. (1989, 1991). The analysis suggested that the damper tuned to the fundamental mode is most effective and other strategically located dampers, tuned to higher modes, facilitate improved performance. In a study by Suhardjo et al. (1992), two TMDs in passive and active modes were investigated for a 60-story building under winds. Kareem and Sun (1987) presented a complete formulation for the analysis of a multidegree of freedom primary system, combined with multiple dampers, attached in a parallel configuration at any desired location. More recently, Igusu and Xu (1991), Xu and Igusa (1992), and Yamaguchi and Harnpornchai (1993) studied the concept of distributed tuned mass dampers. In this system the multiple secondary systems have their natural frequencies distributed over a range of frequencies. Such a system promises to be more effective under excitation frequencies distributed over a wider band. Multiple dampers in the series configuration are generally limited to two secondary masses and have been shown to be more effective than a single mass damper (Korenov and Reznikov 1993). During the preparation of this paper, related studies by Fujino and Sun (1993), Abe and Fujino (1993), and Korenev and Reznikov (1993) were made available.

In this paper, following the preliminary investigation of the MMDs, their effectiveness is investigated under narrow- and wide-banded excitations representing wind loads. Their performance is also examined under stationary earthquake excitations. Additional damper configurations, such as variable mass and frequency distribution, are considered to explore the different

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possible combinations of parameters to make the MMDs in a parallel configuration more effective and robust.

MODELING OF STRUCTURE-DAMPERS SYSTEM

The structure-dampers system can be modeled as a multidegree-of-freedom system representation of the structure to which MMDs are attached as appendages at a desired location. Alternatively, the primary system can be represented by a single-degree-of-freedom system with attached MMDs. In the case of a single TMD, Kareem (1983) demonstrated that buildings with well-separated natural frequencies can be adequately represented by an equivalent single-degree-of-freedom primary system. The dynamics of a multiple-degree-of-freedom model of the primary system, with multiple inertial appendages, was reported by Kareem and Sun (1987). The analysis procedure can be considerably simplified if the contribution of the higher modes of the primary system is ignored (Kareem 1981). In this case the primary system is represented by its mode-generalized system in the fundamental mode (Fig. 1).

If the natural frequencies of the MMDs are equal, this configuration degenerates to a single damper, which, for the optimally tuned situation, has been effectively used in controlling motion. However, an additional feature of the MMDs is that their frequencies can be distributed around the natural frequency of the primary system that needs to be controlled. It is important to exercise caution so that the range of frequencies around the primary system does not become very large. As will be demonstrated later, this range in its optimal design is small; therefore, a larger range not only impairs the effectiveness of the system, but may also interfere with the higher modes of vibration of the primary system. To explore this feature a systematic variation of the frequencies of the individual dampers is considered. Fig. 2 shows this variation in terms of the nondimensional frequency of the dampers α, which is the damper frequency divided by the primary system frequency. Other parameters of interest are $\delta\alpha$, $\Delta\alpha$, and α_0, which describe the spacing of the damper frequency, the total frequency span of the MMDs, and the difference between the natural frequency of the middle damper from the primary system frequency (Fig. 2). The analysis that follows is based on an equivalent single-degree-of-freedom system (Fig. 1). The offset frequency α_0 is taken to be equal to zero, without any loss of generality. Some of the key parameters for this study are: (1) The total number of dampers N; (2) the mass ratio μ_{D_n} of the nth damper to the generalized mass of the primary system in the fundamental mode; (3) the damping ratio ξ_D of each damper; (4) the frequency spacing $\delta\alpha$ of dampers; and (5) the frequency span $\Delta\alpha$ of dampers.

The equations of motion of the combined system in Fig. 1 are given by the following matrix equation:

$$M\ddot{x} + C\dot{x} + Kx = f(t) \quad (1a)$$

where $x, \dot{x}, \ddot{x}$, = displacement, velocity, and acceleration of the combined system, with respect to a fixed reference, when $f(t)$ is an external force acting on the structural envelope (e.g., wind loading or wave loading). In the case of seismic excitation x denotes relative displacement and a dot over x represents its time derivative. Accordingly, $f(t) = -Mr\ddot{x}_g(t)$, where r represents ground displacement influence vector, and $\ddot{x}_g(t)$ is ground acceleration. The mass matrix is diagonal and both C and K matrices are given by

$$K = \begin{bmatrix} \bar{K} & \underline{k}_d^T \\ \underline{k}_d & \underline{k} \end{bmatrix}; \quad C = \begin{bmatrix} C & \underline{C}_d^T \\ \underline{C}_d & \underline{c} \end{bmatrix} \quad (1b,c)$$

where $\bar{K} = (k_b + \Sigma_{n=1}^{N} k_{d_n})$

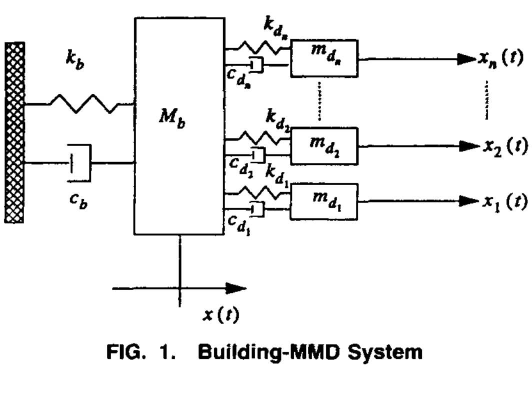

FIG. 1. Building-MMD System

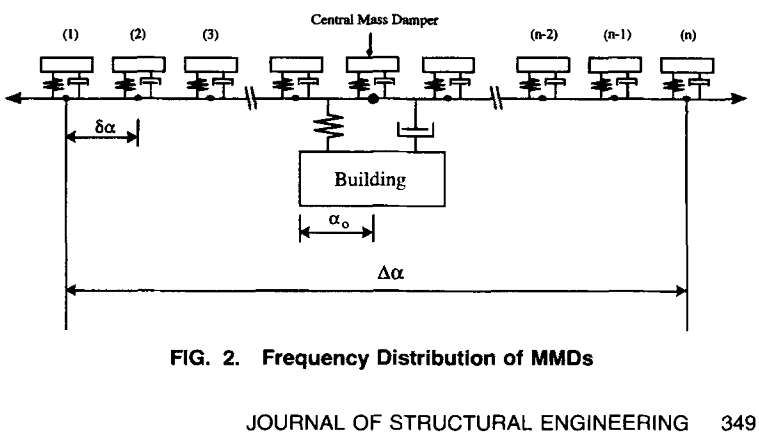

FIG. 2. Frequency Distribution of MMDs

$$\underline{k}_d = \begin{bmatrix} -k_{d_1} \\ -k_{d_2} \\ \cdot \\ -k_{d_N} \end{bmatrix}; \quad \underline{k} = \begin{bmatrix} k_{d_N} & 0 & \cdot & \cdot & 0 \\ & k_{d_N} & 0 & \cdot & 0 \\ & & \cdot & \cdot & \cdot \\ \text{sym} & \cdot & & \cdot & k_{d_N} \end{bmatrix} \quad (2a,b)$$

and the elements of matrix C follow the stiffness matrix format.

The transfer function of the combined system displacement response is given by

$$H_x(if2\pi) = 1 \Bigg/ \left\{ \left[1 - \left(\frac{f}{f_b}\right)^2 + i2\xi_b \left(\frac{f}{f_b}\right) \right] - \left(\frac{f}{f_b}\right)^2 \sum_{n=1}^{N} \frac{m_{d_n}}{M_b} \left[1 + i2\xi_{d_n} \left(\frac{f}{f_{d_n}}\right) \right] \Bigg/ \left[1 - \left(\frac{f}{f_{d_n}}\right)^2 + i2\xi_{d_n} \left(\frac{f}{f_{d_n}}\right) \right] \right\} \quad (3)$$

The preceding transfer function can be expressed in the following simplified form:

$$H(i2\pi f) = \frac{1}{E + iF} \quad (4a)$$

$$E = 1 - \left(\frac{f}{f_b}\right)^2 - \left(\frac{f}{f_b}\right)^2 \sum_{n=1}^{N} \frac{m_{d_n}}{M_b} \left[\frac{1 - \left(\frac{f}{f_{d_n}}\right)^2 + 4\xi_{d_n}^2 \left(\frac{f}{f_{d_n}}\right)^2}{1 - 2\left(\frac{f}{f_{d_n}}\right)^2 + \left(\frac{f}{f_{d_n}}\right)^4 + 4\xi_{d_n}^2 \left(\frac{f}{f_{d_n}}\right)^2} \right] \quad (4b)$$

$$F = 2\xi_b \left(\frac{f}{f_b}\right)^2 - \left(\frac{f}{f_b}\right)^2 \sum_{n=1}^{N} \frac{m_{d_n}}{M_b} \left[\frac{-2\xi_n \left(\frac{f}{f_{d_n}}\right)^3}{1 - 2\left(\frac{f}{f_{d_n}}\right)^2 + \left(\frac{f}{f_{d_n}}\right)^4 + 4\xi_{d_n}^2 \left(\frac{f}{f_{d_n}}\right)^2} \right] \quad (4c)$$

where N = total number of dampers; f_b = natural frequency of the structures; f_{d_n} = natural frequency of the nth damper; M_b = generalized mass of the primary system in the first mode; m_{d_n} = mass of the nth damper; ξ_b = damping ratio of the primary structure; $\xi_{d_n} = c_{d_n}/2m_{d_n}f_{d_n}$ = damping ratio of the nth damper; and c_{d_n} = damper damping coefficient. The damper parameters can be expressed in the following nondimensional quantities: $\mu_n = m_{d_n}/M_b$ and $\alpha_n = f_{d_n}/f_b$. Depending on the frequency range, the frequency response characteristics of a MMD vary from a single peaked function to a flattened shape similar to that caused by an increase in the damping of a TMD.

The preceding modeling was based on the lumped-mass-type dampers, and a similar procedure is applicable to liquid-type dampers. For small amplitudes of structural motion one can model a liquid sloshing and oscillating liquid column-type dampers by an equivalent lumped mass system [e.g., Kareem and Sun (1987), Fujino et al. (1992), Kareem (1993)]. In the case of liquid dampers with known transfer functions $H_{d_n}(i2\pi f)$, the transfer function in (3) may be rewritten as

$$H_x(i2\pi f) = \frac{1}{\left[1 - \left(\frac{f}{f_b}\right)^2 + i2\xi_b \left(\frac{f}{f_b}\right) \right] - \left(\frac{f}{f_b}\right)^2 \sum_{n=1}^{N} \frac{m_{d_n}}{M_b} H_{d_n}(if)} \quad (5)$$

where $H_{d_n}(i2\pi f)$ = damper transfer function for base force, introduced by an input base acceleration.

LOADING DESCRIPTION

Due to wind loads, a general type of loading is utilized to examine the performance of MMDs in this study. A general spectral description of the acrosswind excitation, which includes the possibility of a secondary peak for building shapes with long after bodies, is employed here. This is of particular interest to see how a bank of mass dampers with their natural frequencies distributed over a specified band would respond to a double peaked spectrum. Details of the across-wind-loading spectral loading are not included here; additional information can be found in Kareem and Kline (1993). A limited study is also conducted to observe their behavior under stationary ground excitation. The Kanai-Tajimi spectrum was utilized with a combination of parameters, i.e., f_g = 0.5 Hz and ξ_g = 0.3; and f_g = 2.5 Hz and ξ_g = 0.6 for soft and firm soils, respectively.

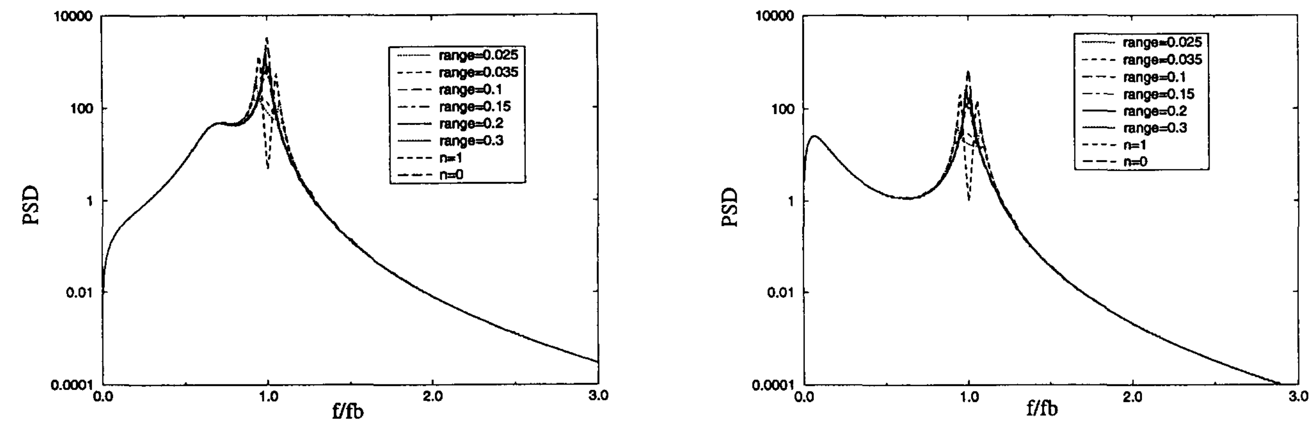

FIG. 3. PSD of Building-MMD System: (a) Square Building; (b) Rectangular Building

RESPONSE ANALYSIS

The response of a building-MMD system in wind or seismic excitation is obtained based on random viabration theory. The factors influencing the response of a structure equipped with a bank of MMDs include the structural geometry and its dynamic characteristics, number of dampers, ratio of the damper mass to structural mass, damping ratio of each damper, and the frequency range of MMDs. For the first part of this study the total mass of the dampers equals one-hundredth of the building generalized mass in the fundamental mode, with the mass evenly distributed among the dampers. Two buildings have been considered in this study. One has a square base (B = 31 m and D = 31 m) and the other has a rectangular base (B = 31 m and D = 155 m). The building density is assumed to be 192 kg/m^3. The total damper mass is equal to 1% of the building generalized mass in the fundamental mode and is equally distributed among the individual dampers, unless otherwise indicated. Each damper in the MMD configuration has damping equal to 1% of the critical. These buildings were selected to examine the effect of a unipeak spectrum and a flatter spectrum with dual peaks. The natural frequency in the across-wind direction for both of the 186-m-high buildings is 0.2 Hz. One percent damping ratio is taken for both buildings. The response estimates are nondimensionalized for comparison purposes, such that $\sigma_F^2/(2\pi f_b)^4 M_b$ is taken to be equal to unity, in which σ_F is the root-mean square (RMS) value of the forcing function. Like the wind analysis, the response due to an earthquake is nondimensionalized by taking $S_o/(2\pi f_b)^4 M_b$ equal to unity, the spectral ordinate in the Kanai-Tajimi spectrum.

The response power spectral density for the square and rectangular buildings are shown in Figs. 3(a and b). The ordinate describes the spectra of nondimensional response and the abscissa represents the frequency normalized with the building frequency. The curves representing different configurations are identical in both cases, at frequencies far from the building natural frequency, but the effect of the dampers is evident at frequencies near the building frequency. The background response in the case of the rectangular building is quite significant in comparison with the square building, in which the resonant action is more prevalent.

Effect of Number of Dampers

The response spectra for the square building with zero, one, five, 11, and 21 dampers have been plotted while zooming on to a smaller frequency range of around f/f_b = 1.0 in Fig. 4(a–c). As one should expect for building alone, or zero damper, there is a single peak where the excitation frequency equals the building frequency. In the case of a structure with one damper, there are two distinct peaks. For increasing number of dampers, the curve flattens out when the dampers cover a certain range of frequencies. This feature indicates that the system is effective over a wide range of frequencies. A similar trend is observed for the rectangular building. The effect of increasing dampers is similar to that of adding damping to the dampers; i.e., flattening of the frequency response function.

Effect of Damping

The damping ratio of each damper is an important factor. The transfer function of the square building with 11 dampers is shown in Fig. 5(a). The curves corresponding to six different damping ratios of the dampers are included for the frequency range of 0.2 and the building damping ratio of 1% of the critical. The transfer function of the damper is smooth only when the damper damping ratio is sufficiently high, but even for larger values of damping the amplitude of the transfer function begins to increase. This observation also holds for the case in which the building

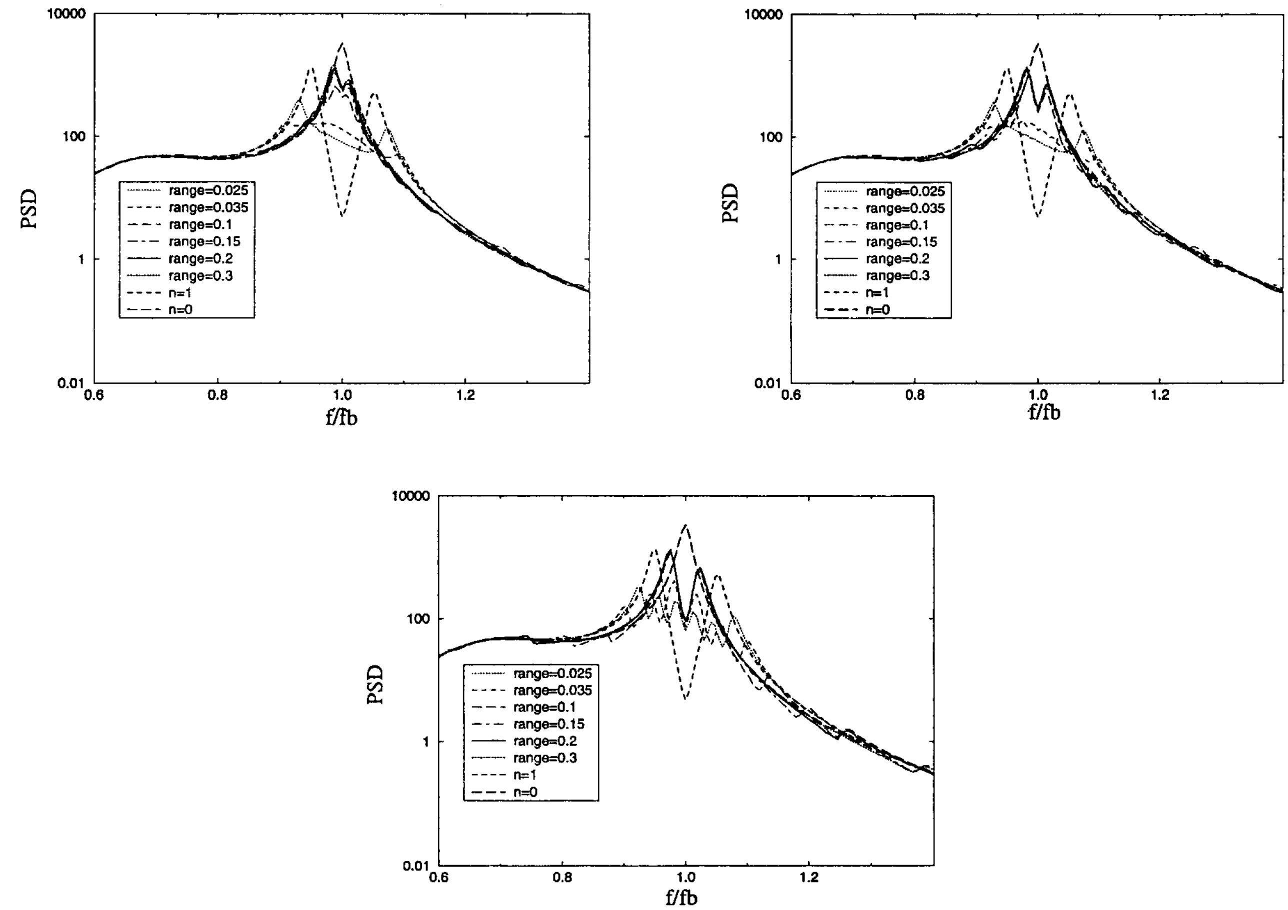

FIG. 4. PSD of Building-MMD System: (a) N = 21; (b) N = 11; (c) N = 5

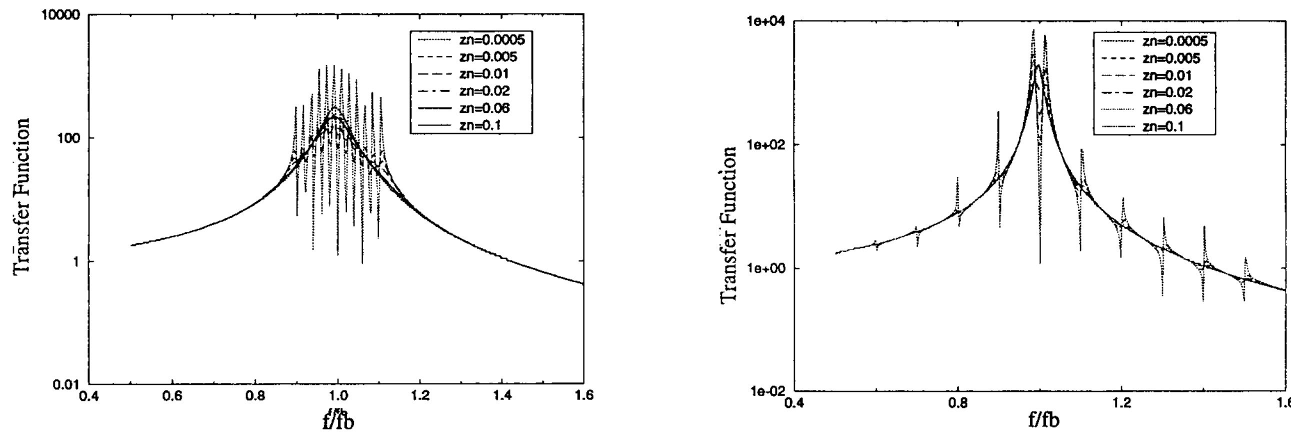

FIG. 5. Building-MMD Transfer Functions for N = 11, Variable ξ_{d_n}: (a) ξ_b = 0.01; (b) ξ_b = 0.005

damping was reduced to 0.005 [Fig. 5(b)]. In this case the damping required for a smoother transfer is lower, but the local spikes are more pronounced and cover a broader range of frequencies. The flattening of the combined transfer function is also attributed to the suppression of secondary peaks introduced by its adjacent damper. This mechanism proceeds in progression, resulting in the overall flattened transfer function of the combined system.

Nonuniform Mass Distrubution

In the preceding section it was assumed that the total mass of the dampers was 1% of the building mass, spread evenly among the dampers. The performance of MMDs with an uneven distribution of mass among the dampers is examined. Fig. 6(a) shows the response spectra for the square building with 21 dampers. In this configuration the mass of the central damper is equal to half of 1% of the building generalized mass in the fundamental mode, and the remaining

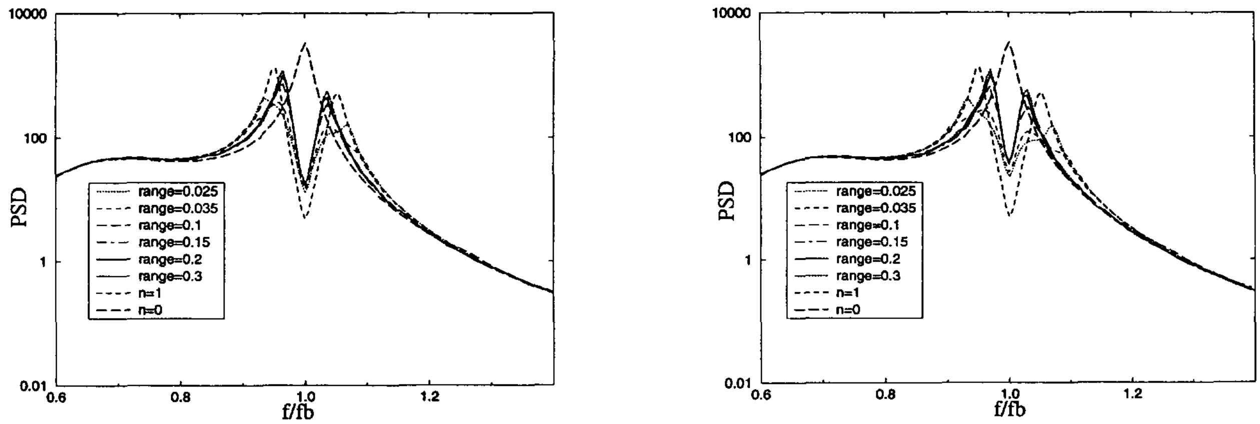

FIG. 6. Effect of Total Number of Dampers with Nonuniform Mass Distribution, m_d (central = $0.01M_b/2$: (a) N = 21, m_d (central) = $0.01M_b/3$; (b) N = 21

half is evenly spread among the other dampers. In subsequent references this configuration will be referred to as a nonuniform mass distribution of dampers. A comparison of Fig. 6(a) with Fig. 4 shows that the valley between the peaks, which tends to flatten and broaden the range of effectiveness of the MMDs or widen the span of frequencies, is missing in Fig. 6(a). The central damper with half of the mass controls the response trend because the valley does not get flattened, and as such it reflects the characteristics of a single TMD. However, the effect of an MMD is evident under secondary peaks, which tend to be flattened for a certain range of frequencies. The rectangular building showed similar trends in the results. These trends are present even in the case in which the central damper mass is kept at one-third of 1% of the building generalized mass, and the remaining two-thirds is spread evenly among the dampers [Fig. 6(b)].

Nonuniform Frequency Spacing

In the preceding analysis, the frequencies of the dampers are evenly spaced throughout the indicated range. This spacing is varied to study its impact on the MMD system performance. One possibility is a system with five dampers in which the dampers are spaced adjacent to the central damper, as done previously (spacing = range/4), but the remaining two dampers are spaced at one-half this spacing (spacing = 0.5 range/4). The response spectra of a square building with this system is given in Fig. 7(a). The response spectra of a similar system, in which the exterior dampers are spaced at three-fourths of the usual spacing (spacing = 0.75 range/4), are given in Fig. 7(b). Another possibility for a system with 11 dampers is considered, in which the dampers adjacent to the central damper are spaced in the usual manner (spacing = range/10), but the dampers adjacent to these are spaced at nine-tenths of the previous (spacing = 0.9 range/10), and subsequent dampers are spaced with decreasing increments until the exterior-most dampers are spaced at only six-tenths of the usual spacing. This is shown in Fig. 7(c).

The response spectra of the aforementioned three variable frequency spacing systems are repeated for a case of variable damper mass, in which half of the total damper mass is assigned to the central damper while the remaining mass is equally distributed. By comparing the results obtained in the uniform mass and frequency distribution cases, in Figs. 4(b and c), the effects of variable distribution are hardly discernable.

Effect of Number of Dampers for Fixed Frequency Range and Fixed Frequency Spacing

The effect of the number of dampers on the response of the system can be seen in Figs. 8(a,b), in which the frequency range covered by the dampers is fixed at 0.2 Hz, centered at the building frequency, and evenly spaced. Thus, the frequency increment between the dampers decreases as the number of dampers increases. A comparison of the system with varying numbers of dampers is also shown in Figs. 8(c, d), in which the frequency increment between all adjacent dampers is fixed at 0.01 Hz. Therefore, the frequency range of the dampers, increases as the number of dampers increases. In Fig. 8(d), the center damper contains half of the total damper mass, and the remaining mass is evenly distributed among the other dampers. The difference between Figs. 8(a,b) and 8(c,d) stem from the change in the frequency range.

Equivalent Effective Damping

As indicated earlier, a secondary inertial system indirectly imparts additional damping to the system through the modification of the building transfer function. The effectiveness of a damper

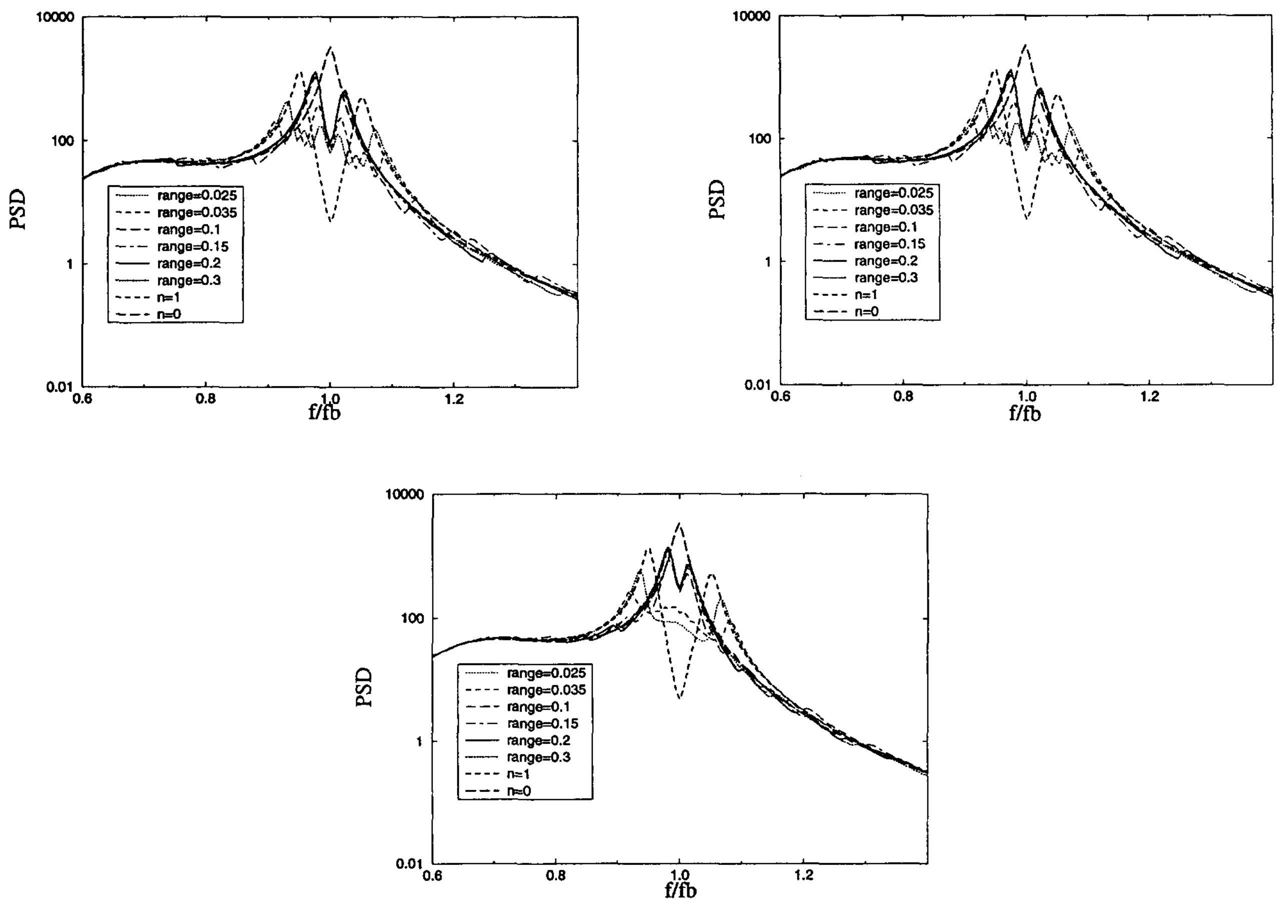

FIG. 7. Effect of Nonuniform Spacing of Damper Frequencies: (a) N = 5, $\delta\alpha$ = 0.5, 1.0, 0.5 times $\Delta\alpha/(N - 1)$; (b) N = 5, $\delta\alpha$ = 0.75, 1.0, 1.0, 0.75 times $\Delta\alpha/(N - 1)$; (c) N = 11, $\delta\alpha$ = 0.6, 0.7, 0.8, 0.9, 1.0, 0.9, 0.8, 0.7, 0.6 times $\Delta\alpha/(N - 1)$

system can be assessed by considering the amount of extra damping imparted to the system [e.g., McNamara (1977), Kareem (1983)]. The critical damping ratio in a single-degree-of-freedom model of a building alone, which makes it equivalent to the response of the building-damper system, is referred to as an equivalent effective damping. The equivalent effective damping can be estimated by equating the response of a building alone with effective damping to the response of a building-damper system.

The equivalent damping ratio for the several cases of MMDs considered here are reported in Tables 1 and 2. The results demonstrate the effectiveness of the dampers as the building response decreases with the addition of dampers. The comparison with a single TMD highlights the superiority of the multiple dampers over a single TMD. The equivalent effective damping is as high as 0.025 with multiple dampers as opposed to 0.01 for the building alone, indicating an increase in damping by a factor of 2.5. Additional discussion on the sensitivity of effective damping on several system parameters is given in the following section.

OPTIMAL PERFORMANCE AND ROBUSTNESS OF MMDs

Optimal Performance

To ascertain the most effective system for a given structure, the optimal number of dampers, frequency distribution of dampers, and damping ratios must be determined. In this study the H_2 norm of the system is used as a measure of the optimal performance.

Effect of Damping Ratio

First, the effect of the damping ratio of the MMDs for different frequency ranges on the building-dampers response is examined. The RMS value of the response is plotted against the frequency range of MMDs in Figs. 9(a–d) for four different damping ratios of the dampers. These results clearly point out that there is a well-defined optimum frequency range. For the square building this range varies from 0.025 Hz to 0.035 Hz as the number of dampers increases from five to 21. Except for the very high damping value of 0.04, the optimum frequency range of the MMDs remains unaffected by a change in damper dampings. This trend is slightly affected

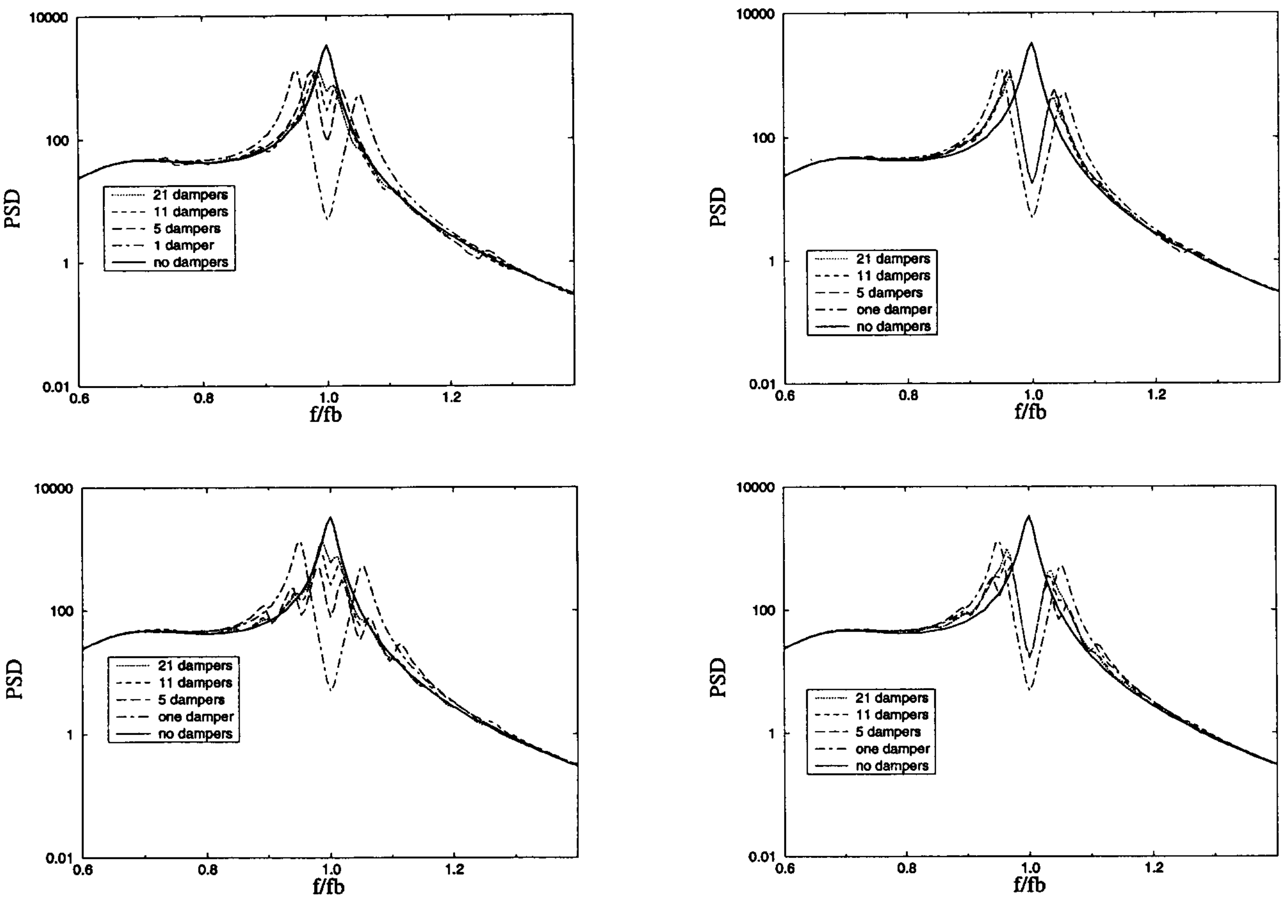

FIG. 8. Effect of Fixed Frequency Range with Different Number of Dampers with Uniform and Nonuniform Mass Distribution: (a) $\Delta\alpha$ = 0.2, Uniform Mass; (b) $\Delta\alpha$ = 0.2, m_d (center) = 0.01M_b/; (c) $\Delta\alpha$ = 0.01, Uniform Mass; (d) $\delta\alpha$ = 0.01, m_d (center) = 0.01$M_{b/2}$

TABLE 1. Summary of MMDs Performance

Number of damper (1)	Frequency ranges ($\Delta\alpha$) (2)	Equivalent Effective Damping: Uniform mass distribution (3)	Equivalent Effective Damping: Nonuniform mass distribution (4)
21	0.025	0.024	0.021
21	0.035	0.025	0.022
21	0.100	0.018	0.019
21	0.150	0.016	0.018
21	0.200	0.014	0.017
21	0.300	0.013	0.016
11	0.025	0.024	0.022
11	0.035	0.025	0.022
11	0.100	0.017	0.019
11	0.150	0.015	0.017
11	0.200	0.014	0.016
11	0.300	0.013	0.015
5	0.025	0.024	0.022
5	0.035	0.023	0.022
5	0.100	0.016	0.016
5	0.150	0.015	0.015
5	0.200	0.014	0.015
5	0.300	0.014	0.015
1	0.000	0.015	0.015
0	0.000	0.01	0.01

as the number of dampers increases. In addition, the effectiveness of dampers with lower damping value increases as the number of dampers increases. The secondary peaks in the transfer function lead to a higher RMS value and (in this situation) these peaks are suppressed by an increase in the number of dampers, or the same effect can be reached by increasing the damping of individual dampers. For high values of damping it is observed that the optimal frequency range shifts towards a lower value. Asymptotically, for overdamped MMDs the optimal frequency range

TABLE 2. Summary of MMDs Performance (Uneven Frequency Spacing)

Number of damper (1)	Frequency range ($\Delta\alpha$) (2)	Equivalent Effective Damping: Uniform mass distribution (3)	Equivalent Effective Damping: Nonuniform mass distribution (4)
N = 11; frequency spacing 0.6, 0.7, 0.8, 0.9, 1.0, 0.9, 0.8, 0.7, 0.6 of normal spacing = $\Delta\alpha/(N-1)$	0.025	0.022	0.020
	0.035	0.025	0.023
	0.100	0.018	0.019
	0.150	0.015	0.017
	0.200	0.014	0.016
	0.300	0.013	0.015
N = 5; frequency spacing 0.75, 1.0, 1.0, 0.75 of normal spacing = $\Delta\alpha/(N-1)$	0.025	0.023	0.022
	0.035	0.023	0.023
	0.100	0.016	0.016
	0.150	0.015	0.015
	0.200	0.014	0.015
	0.300	0.014	0.015
N = 5; frequency spacing 0.5, 1.0, 1.0, 0.5 of normal spacing = $\Delta\alpha/(N-1)$	0.025	0.022	0.021
	0.035	0.023	0.023
	0.100	0.015	0.016
	0.150	0.015	0.015
	0.200	0.014	0.015
	0.300	0.014	0.015
N = 1	0.000	0.015	0.015
N = 0	0.000	0.01	0.01

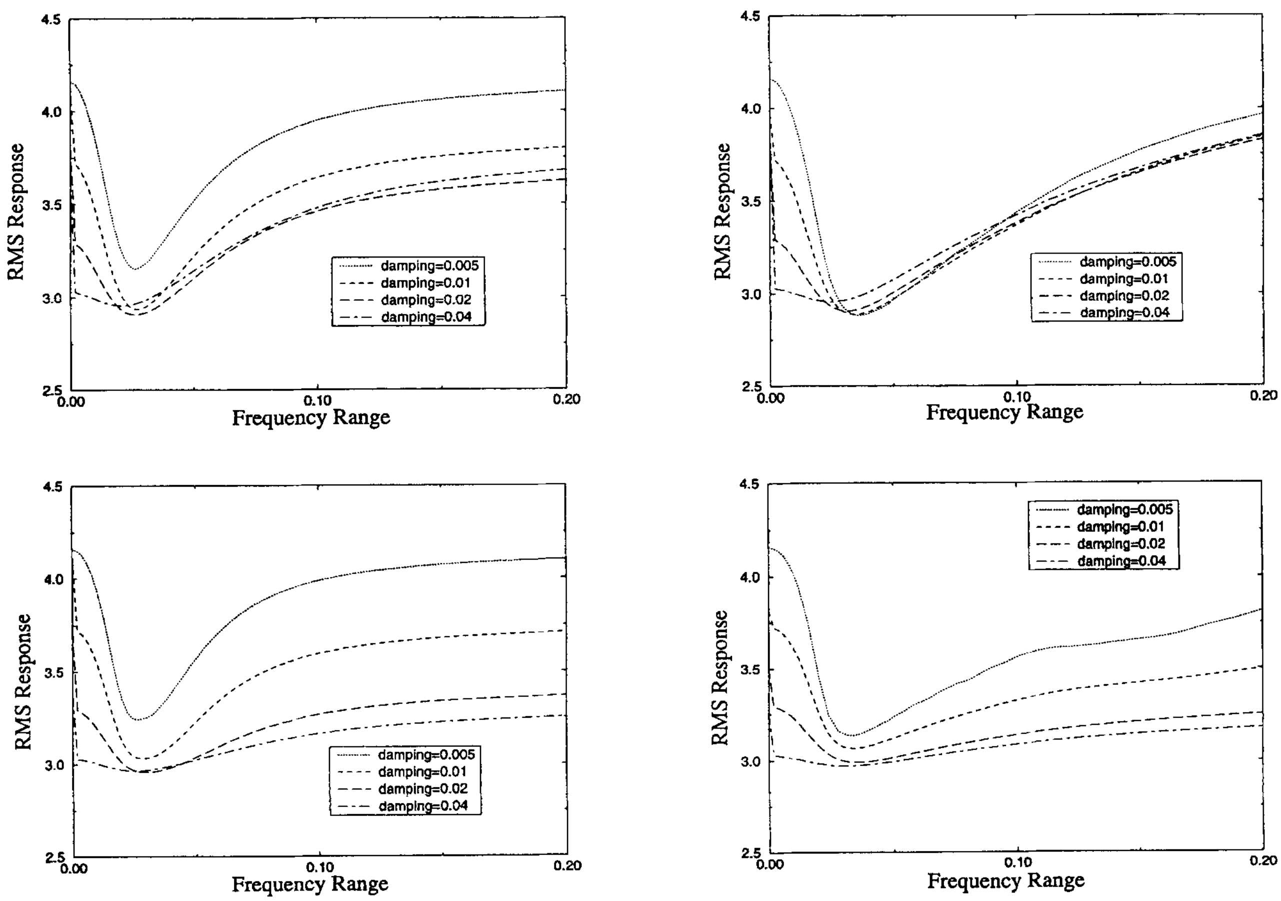

FIG. 9. Effectiveness of MMDs versus Frequency Range for Different Damper Ratios: (a) N = 5, Uniform Mass; (b) N = 21, Uniform Mass; (c) N = 5, m_d (center) = $0.01M_b/2$; (d) N = 21, m_d (center) = $0.01M_b/2$

would approach the zero frequency range, implying a single damper. Therefore, damping above a certain level may offset the beneficial features of MMDs and reduce these to a single TMD. For the case in which the central damper mass is half of the total damper mass, the results generally follow the trend observed in the previous case, with an equally distributed damper mass. One notable exception concerns the better effectiveness of this damper configuration over the previous one, even for a larger frequency range.

The influence of the damper damping ratio on the performance of the MMD is presented in Figs. 10(a–d) for six ranges of frequency, which include the range that represented the optimal values in Fig. 9. For the optimal frequency range, the RMS response reaches the minimum at the lowest damping ratio. This trend is not observed for responses above and below this range. For the range below the optimal value the minimum response is obtained at damping ratios much higher than the optimal, and for ranges above the optimal value the results tend to approach the optimal damping of a single TMD. With an increase in the number of dampers, there is a concomitant decrease in the optimal damping ratio. The basic trends are similar in the configuration involving the central damper equal to half the mass of the MMDs. In summary, the performance of the MMDs is influenced by the frequency range, the total number of dampers, and the damping ratio. For an optimal selection of these parameters an optimal system of MMDs can be designed. One can draw a parallel between the performance of an optimal single TMD and optimal MMDs. The TMD involves an optimal tuning and damping ratio; whereas optimal MMDs involve a frequency range and a combination of damping ratio and number of dampers. The damping ratio does not significantly influence the effectiveness of the damper system, provided it is small. An increase in the number of dampers is very similar to the effects of introducing additional damping in the dampers. Thus, the frequency range remains the most important design variable.

COMPARISON OF EFFECTIVENESS OF MMDS WITH TMD

To compare the effectiveness of a TMD with a MMD, the variation of the RMS response is studied with changes in the damping ratio of MMDs. In Figs. 11(a,b), these results are plotted for a TMD and MMDs tuned according to the results presented in Fig. 10. These parameters may not represent an optimally tuned system, but for the purpose of the comparison these values are used based on the optimization of individual parameters. These results suggest that as the number of dampers increases, not only is the combined response reduced, but this is accomplished at lower damping values. The curve for a single TMD provides an optimal damping

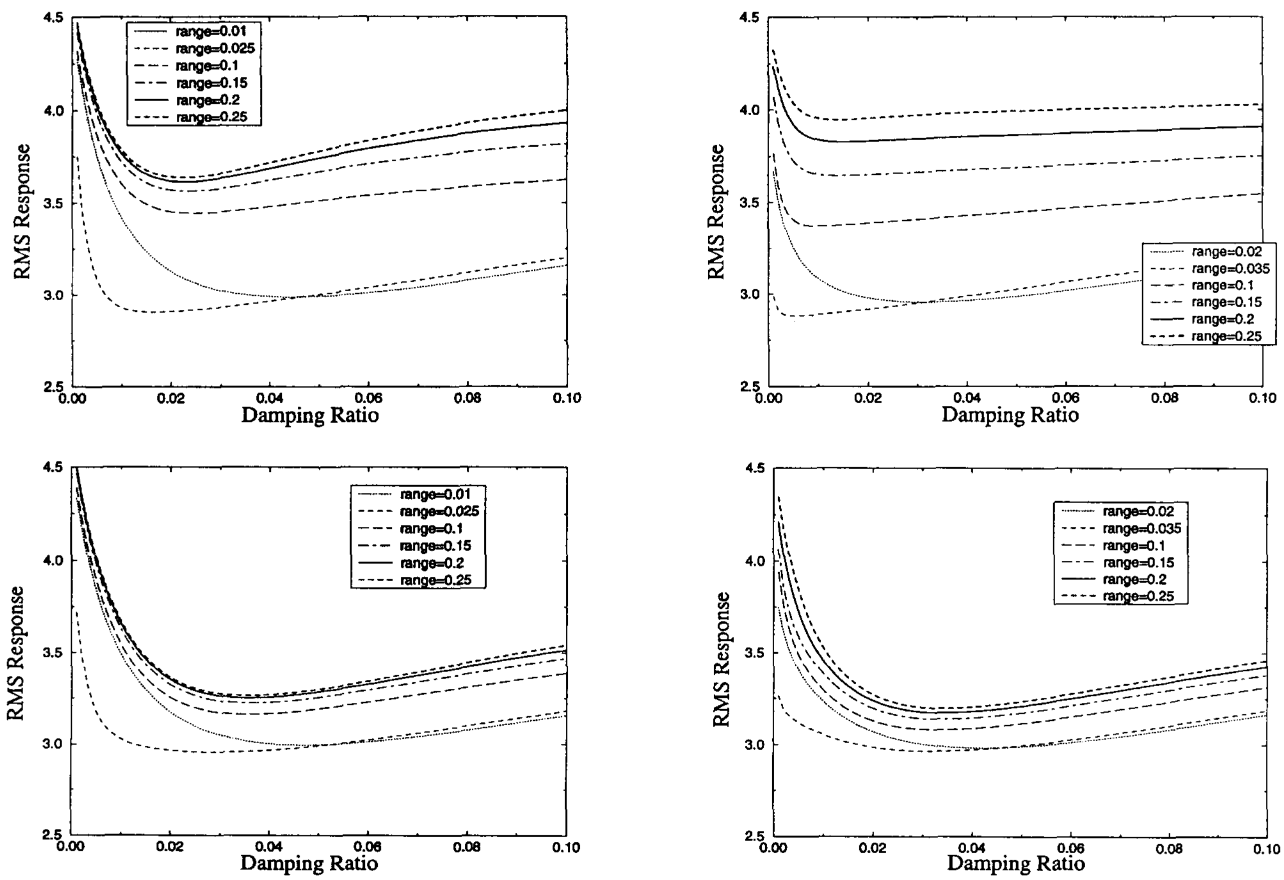

FIG. 10. Influence of Damping Ratio on Performance of MMDs with Different Frequency Ranges: (a) N = 5, Uniform Mass; (b) N = 21, Uniform Mass; (c) N = 5, m_d (center) = $0.01M_b/2$; (d) N = 21, m_d (center) = $0.01M_b/2$

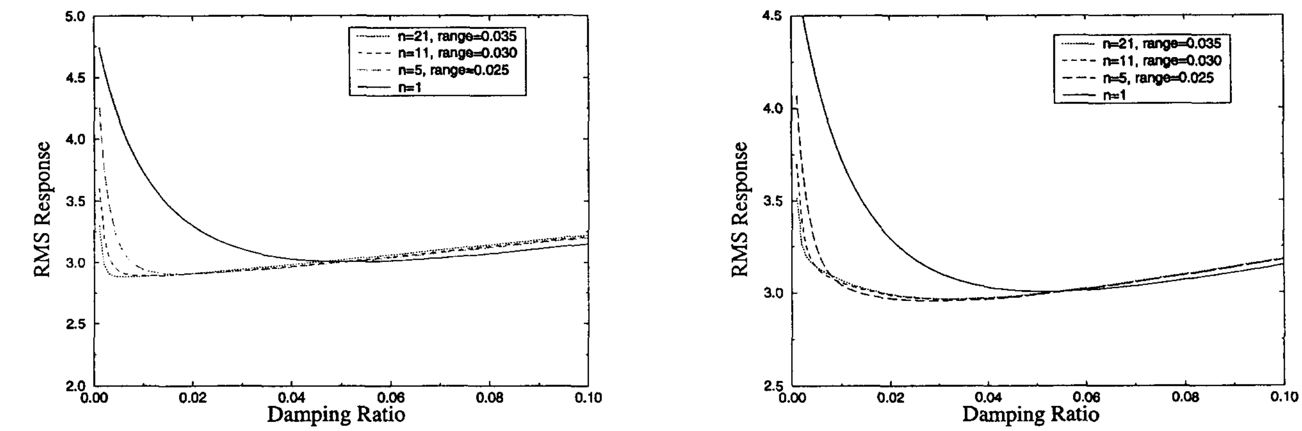

FIG. 11. Performance and Robustness of MMDs against Change in Damper Damping: (a) Uniform Mass; (b) m_d (center) = $0.01M_b/2$

value of approximately 0.05, which agrees with the damping given by Warburton (1981) for random excitation. This value differs from that derived on the basis of harmonic excitation given by Den Hartog (1956), which for the present value of μ is equal to 0.06. For optimal parameters the system response is minimum; correspondingly, the equivalent damping is maximum. These results are qualitatively summarized here. For an increase in the frequency range above the optimal and a decrease below it, the equivalent damping tends to decrease. The equivalent damping is also sensitive to the damper damping ratio for a smaller number of dampers. This trend is reduced as the number of dampers is increased. For 21 dampers, it is found that the dependence on damper damping is minimized for frequency ranges both above and below the optimal value. For the optimal and near-optimal frequency ranges, the equivalent damping is insensitive to the damping ratio of the dampers except for values less than 0.5%.

The results in Figs. 11(a,b) suggest that the optimal damping for the MMDs is much lower than that needed for a single TMD. This optimal damping value is close to the damping available in sloshing liquid unless auxilliary devices such as screens, surface contaminations, or very shallow water dampers are utilized. Clearly, there is a distinct advantage in using liquid dampers in a multiple container configuration with slight detuning. This configuration would eliminate the need to enhance damping to the level required for an optimal single-tuned liquid damper. However, one should not overlook the presence of nonlinearities in the sloshing mechanism for low liquid damping. In the case of liquid column oscillators, this may not be of serious concern. In the case of mechanical dampers, low optimal values of damping in MMDs may cause difficulty in the excessive displacement of secondary masses.

An examination of the plots in Figs. 11(a,b) show that at damping values lower than the optimal, the RMS response increases sharply. This increase is more drastic in the case of the MMDs, which renders these dampers less robust in comparison with the TMDs. Nevertheless, the MMDs are relatively more effective than a TMD for a large range of damping values. The MMD configuration in which the central damper has half the damper mass exhibits slightly improved robustness features as it has some attributes of a TMD. Caution must be exercised in the selection of damper damping so that it does not fall below the values suggested in these figures.

The effect of an estimation error in the primary system natural frequency is examined. Figs. 12(a,b) show the RMS value of the response against error or against the change in building frequency. As noted for a single TMD, a slight detuning of the central damper results in minimizing the response. According to Den Hartog's (1956) criteria, the offset between the damper building frequency should be between −0.01 and 0.0073. The observed offset for optimal tuning in Fig. 12(a) is approximately equal to 0.006. The effectiveness of a TMD is significantly impaired by an error in estimation of building frequency, as can be noted from the sharp increase in the response curve. For systems with a frequency range below optimal (i.e., in this example less than 0.03) the RMS response tends to approach that of a single TMD. However, for a frequency range above the optimal, the robustness in performance against detuning increases significantly. For frequency ranges far exceeding the optimal value, the effectiveness of the damper may be impaired because only a few dampers will be effective. The limiting case represents only one damper tuned with the primary system, which, because of a small mass as compared with the total MMDs, is rather ineffective. Despite this compromise in effectiveness, the performance of the MMDs is still better than a single TMD. This suggests that MMDs may perform better in an earthquake where there is a degradation of the stiffness of the primary structure stiffness, or in severe wind conditions where the racking of buildings may result in changes in their natural frequency.

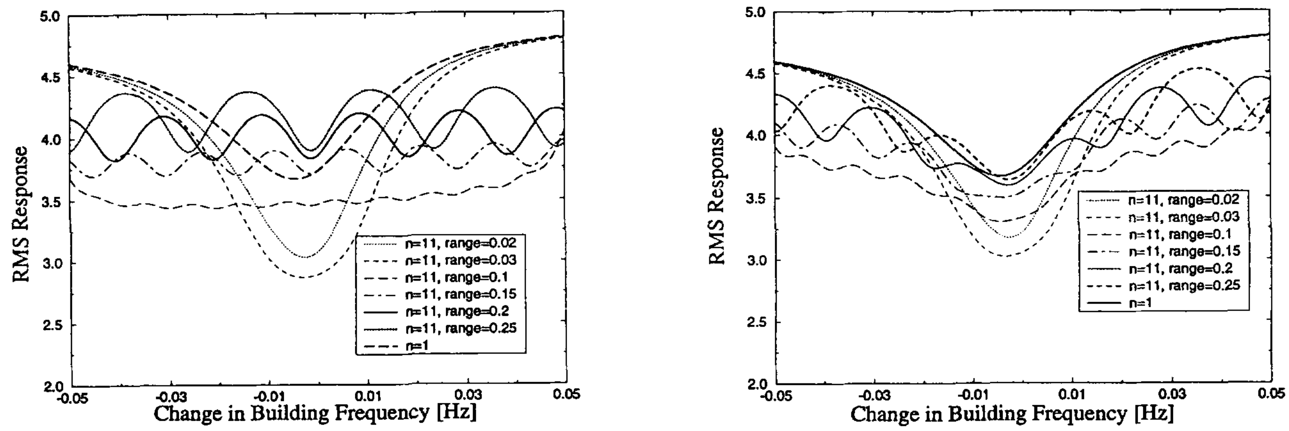

FIG. 12. Performance and Robustness of MMDs against Changes in Building Fundamental Frequency: (a) Uniform Damper Mass; (b) Nonuniform Damper Mass [m_d (center) = $0.01M_b/2$]

TABLE 3. Summary of MMDs Performance under Earthquake

Number of damper (1)	Frequency ranges ($\Delta\alpha$) (2)	Equivalent Effective Damping: f_g = 2.5 and ξ_g = 0.6 (3)	Equivalent Effective Damping: f_g = 0.5 and ξ_g = 0.3 (4)
21	0.025	0.024	0.024
21	0.035	0.028	0.029
21	0.055	0.035	0.036
21	0.065	0.036	0.038
21	0.075	0.036	0.037
21	0.1	0.033	0.034
11	0.025	0.024	0.025
11	0.035	0.029	0.029
11	0.055	0.036	0.036
11	0.065	0.036	0.035
11	0.075	0.035	0.033
11	0.1	0.032	0.033
5	0.025	0.025	0.026
5	0.035	0.030	0.031
5	0.055	0.034	0.036
5	0.065	0.033	0.035
5	0.075	0.031	0.033
5	0.1	0.027	0.028
1	0.000	0.019	0.02
0	0.000	0.01	0.010

RESPONSE UNDER SEISMIC LOADING

In the case of seismic loading, a 31m by 31m square building 93m high, with natural frequency and damping ratio equal to 0.4 and 0.01 Hz, respectively, is utilized. Two site conditions are considered, namely firm and loose soils. In the case of loose soil conditions the natural frequency of the building is closer to the dominant frequency of the ground excitation, which results in a higher response. The equivalent damping for both cases is reported in Table 3. The optimal frequency range is different from the wind case as the building frequency and the excitation frequency contents have changed. The effectiveness of MMDs under seismic loading like wind loading is higher in comparison with that of a single TMD.

CONCLUDING REMARKS

This study has demonstrated the effectiveness of the MMD system in reducing the structural response under narrow- and wide-banded excitations represented by wind and seismic loads. The findings of this study are in general qualitative agreement with the study by Yamaguchi and Harnpornchai (1993) for harmonic excitations, though quantitative variations exist due to the nature of the loading considered and additional concepts involving variable mass and frequency spacing configurations examined here. The following key results are presented:

Depending on the frequency range, the frequency response characteristics of MMDs vary from a single peaked function to a flattened shape, similar to that caused by an increase in damping for a TMD.

An optimal MMD system can be designed by an optimal selection of the frequency range, total number of dampers, and damping ratio, and it is found to be more effective for the same total mass ratio.

For the optimal and near-optimal frequency ranges, the equivalent damping is insensitive to the damping ratio of the dampers, except for very low values.

The MMD systems with variable mass dampers or variable frequency spacing alone, or their combinations, do not offer any distinct advantage or disadvantage over uniformly distributed mass or frequency systems.

Like a single TMD, MMDs are unrobust under variations in both the damping ratio and the primary structure's natural frequency. The MMDs with an optimal frequency range are more effective than single TMDs for a wide range of damping. This makes them more attractive for liquid dampers. The performance of MMDs can be enhanced against an error in the estimation of the primary system's frequency or changes in stiffness under loads by selecting a frequency range different from the optimal.

The frequency range of MMDs is the most important parameter as it influences their robustness and effectiveness. The damping ratio and total number of dampers play a secondary role in the design of a MMD.

The MMDs offer a smaller size for an individual damper than one massive TMD, which improves their constructability and maintainability. These features enhance portability and the ease of installation in existing systems and offer a range of possible spatial distributions in a structure. The overall effectiveness of the system may not be seriously compromised if one or more of the dampers fail to function because of mechanical or other reasons.

ACKNOWLEDGMENTS

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APPENDIX. REFERENCES

Abe, M., and Fujino, Y. (1993). "Dynamic characterization of multiple tuned mass dampers and some design formulas." *Earthquake Engrg. and Struct. Dynamics*, 23(8), 813–836.

Bergman, L. A., McFarland, D. M., Hall, J. K., Johnson, E. A., and Kareem, A. (1989). "Optimal distribution of tuned-mass dampers in wind sensitive structures." *Proc., 5th Int. Conf. on Struct. Safety and Reliability, (ICOSSAR)*, ASCE, New York, N.Y.

Bergman, L. A., McFarland, D. M., and Kareem, A. (1991). "Coupled passive control of tall buildings." *Struct. Abstracts*, ASCE, New York, N.Y.

Den Hartog, J. P. (1956). *Mechanical vibrations*. McGraw-Hill Book Co., Inc., New York, N.Y.

Fujino, Y. et al. (1992). "Tuned liquid damper (TLD) for suppressing horizontal motion of structures." *J. Engrg. Mech.*, ASCE, 118(10), 2017–2030.

Fujino, Y., and Sun, L. (1993). "Vibration control by multiple tuned liquid dampers." *J. Struct. Engrg.*, 119(12).

Fujita, T. (1991). "Seismic isolation and response control of nuclear and non-nuclear structures." *Proc., 11th Int. Conf. on Struct. Mech. in Reactor Technol., (SMIRT11)*.

Housner, G. W., and Masri, S. F., eds. (1990). *Proc., U.S. Nat. Workshop on Struct. Control Res.*, Univ. of Southern California.

Igusa, T., and Xu, K. (1991). "Vibration reduction characteristics of distributed tuned mass dampers." *Proc., 4th Intl. Conf. Struct. Dynamics: Recent Advances*.

Kareem, A. (1975). "Reduction of wind induced motion of tall buildings," MSc thesis, Civ. Engrg. Dept., Univ. of Hawaii, Honolulu, Hawaii.

Kareem, A. (1981). "Wind excited response of buildings in higher modes." *J. Struct. Engrg.*, ASCE, 107(4), 701–706.

Kareem, A. (1983). "Mitigation of wind induced motion of tall buildings." *J. Wind Engrg. and Industrial Aerodynamics*, Vol. 11, 273–284.

Kareem, A., and Sun, W.-J. (1987). "Stochastic response of structures with fluid-containing appendages." *J. Sound and Vibration*, 119(3), 389–408.

Kareem, A. (1987). "Wind effects on structures: a probabilistic viewpoint." *Probabilistic engineering mechanics*, 2(4), 166–200.

Kareem, A., and Kline, S. (1993). "Performance of multiple mass dampers under random loading." *Tech. Rep.* No. 93-004, Dept. of Civ. Engrg. and Geological Sci., Univ. of Notre Dame, Ind.

Kareem, A. (1993). "Liquid tuned mass dampers: past, present and future." *Proc., 7th U.S. Nat. Conf. on Wind Engrg., Vol. I*.

Kareem, A., and Tamura, Y. (1994). "Damping systems for controlling wind induced motions of structures." *Proc., ASCE Struct. Congr.*, XII, ASCE, New York, N.Y.

Kaynia, A. M., Venziano, D., and Biggs, J. M. (1981). "Seismic effectiveness of tuned mass dampers." *J. Struct. Engrg.*, ASCE, 107(8).

Korenev, B. G., and Reznikov, L. M. (1993). *Dynamic vibration absorbers*. John Wiley, New York, N.Y.

McNamara, R. J. (1977). "Tuned mass dampers for buildings." *J. Struct. Engrg.*, ASCE, 103(9), 1785–1798.

Soong, T. T. (1990). *Active structural control: theory and practice*. John Wiley and Sons Inc., New York, N.Y.

Suhardjo, J., Spencer, B. F. Jr., and Kareem, A. (1992). "Frequency domain optimal control of wind excited buildings." *J. Engrg. Mech.*, ASCE, 118(12).

Tamura, Y. (1990). "Suppression of wind-induced vibrations of buildings." *J. Wind Engrg.*, (JAWE), 44(7), 71–84.

Warburton, G. B. (1981). "Optimum absorber parameters for various combination of response and excitation parameters." *Earthquake Engrg. and Struct. Dynamics*, Vol. 9, 251–262.
Xu, K., and Igusa, T. (1992). "Dynamic characteristics of multiple substructures with closely spaced frequencies." *Earthquake Engrg. and Struct. Dynamics*, Vol. 21, 1059–1070.
Yamaguchi, H., and Harnpornchai, N. (1993). "Fundamental characteristics of multiple tuned mass dampers for suppressing harmonically forced oscillation." *J. Earthquake Engrg. and Struct. Dynamics*, Vol. 22, 51–62.